IOCMA 2026 Conference
The 2nd International Online Conference on Mathematics and Applications
10–12 June 2026 | Online
MDPI

Optimal Vaccination Strategies for an SEIR Epidemic Model with Application to Influenza

Patience Mabuza
Department of Applied Mathematics
University of South Africa · 24698768@mylife.unisa.ac.za

INTRODUCTION & AIM

Influenza remains a major global public health burden, placing heavy strain on healthcare systems through rapid transmission and seasonal recurrence. Mathematical models provide a rigorous framework for analysing disease dynamics and comparing intervention strategies before deployment.
While vaccination is highly effective, resource constraints in real-world settings demand carefully timed and cost-efficient immunisation policies. This study develops a controlled SEIR model with vaccine efficacy and nonlinear costs to find optimal time-varying vaccination strategies.

S → E → I → R   u(t) ↓ vaccination

MATHEMATICAL MODEL

Compartments S, E, I, R with control $u(t) \in [0,1]$ representing the time-varying vaccination rate
Vaccine efficacy $\varepsilon \in (0,1]$ captures imperfect immunological protection
$R_0 \approx 1.8$ derived via the Next Generation Matrix approach
Positivity and boundedness of all solutions rigorously established

$R_0 = \beta / (\gamma + \mu) \approx 1.8$ (influenza-calibrated)

METHOD

Forward-Backward Sweep Method iterates state and adjoint ODEs until convergence of optimal $u^*(t)$
4th-order Runge–Kutta discretisation applied to both forward and backward systems
Convergence: sup-norm difference between successive iterates $< 10^{-6}$

RESULTS & DISCUSSION

OPTIMAL CONTROL FORMULATION

The objective functional minimises the cumulative infectious burden and vaccination cost over a fixed horizon T:

$J[u] = \int_0^T [\, A \cdot I(t) + \tfrac{1}{2} B \cdot u^2(t) \,]\, dt$ → minimize

Quadratic cost $\tfrac{1}{2}Bu^2(t)$ reflects nonlinear costs and diminishing returns in mass immunisation campaigns
Pontryagin's Maximum Principle yields necessary optimality conditions and optimal control characterisation
Adjoint system solved simultaneously with state equations; transversality conditions imposed at $t = T$

KEY RESULTS

62% Peak infections reduced vs. uncontrolled
47% Fewer cumulative cases vs. uncontrolled
23% Lower vaccination cost vs. constant policy

SIMULATION DETAILS

$\beta = 0.5$ day^{-1}, $\sigma = 0.33$ day^{-1}, $\gamma = 0.25$ day^{-1}, $\mu = 0.0001$ day^{-1}
Vaccine efficacy $\varepsilon = 0.85$; weights A and B balance epidemiological and economic objectives
Horizon T = 100 days; small infectious seed in a fully susceptible population

CONCLUSION

Dynamic vaccination strategies substantially outperform static policies in suppressing influenza spread
Incorporating vaccine efficacy and nonlinear costs produces more realistic and implementable public health guidance
The framework efficiently allocates limited vaccination resources across an epidemic wave
Results generalise naturally to multi-strain or multi-dose vaccine settings

FUTURE WORK / REFERENCES

[1] Pontryagin et al. (1962). The Mathematical Theory of Optimal Processes. Wiley.
Keywords: SEIR Model · Optimal Control · Vaccination Strategies · R_0 · Epidemic Modelling · Pontryagin's Maximum Principle · Influenza