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DEGENERATE MITTAG-LEFFLER FUNCTION VIA THE DEGENERATE POCHHAMMER SYMBOL

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INTRODUCTION & AIM

Motivation

- Prabhakar/Mittag-Leffler kernels are central to fractional calculus and non-Debye relaxation models.
- The paper introduces a coefficient-degenerate analogue by replacing the classical rising factorial with the degenerate Pochhammer coefficient.
- The denominator is kept classical, preserving clean transform and fractional-calculus structure.
- Aim: prove analytic properties and build a tractable one-parameter deformation of the Havriliak-Negami transfer function.

Main contribution

A new entire coefficient-degenerate Mittag-Leffler function that continuously recovers the classical Prabhakar function in the classical limit and enables closed-form relaxation kernels.

$E^{[\lambda]}_{\alpha,\beta;\gamma}(z)$ $\quad \lambda \to 0^+$ classical recovery

Analytic scope

- Entire convergence for all complex arguments under nonresonance/pole-avoidance.
- Fox-Wright representation and sharp growth/order estimates.
- Riemann-Liouville fractional integral/derivative beta-shift identities.
- Laplace transform pair leading to a Type-B HN deformation.

METHOD

Definition

$$E^{[\lambda]}_{\alpha,\beta;\gamma}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{\lambda,n}}{n!\,\Gamma(\alpha n+\beta)} z^n, \quad \mathrm{Re}(\alpha) > 0$$

Coefficient recursion

$$(\gamma)_{\lambda,n+1} = \frac{\gamma+n}{1-\lambda(\gamma+n+1)} (\gamma)_{\lambda,n}$$

Evaluation workflow

- Use the recursion above to avoid repeated degenerate-gamma evaluations.
- Compute the partial sum and monitor a term-ratio remainder criterion.
- Stop when the certified remainder is below the prescribed tolerance; experiments use 10^{-12}.

$$S_N(z) = \sum_{n=0}^{N} u_n(z)$$

$$r_N(z) = \sup_{n \ge N} \left| \frac{u_{n+1}}{u_n} \right|, \qquad |R_N(z)| \le \frac{|u_{N+1}(z)|}{1-r_N(z)}$$

Laplace transform pair

$$\mathcal{L}\left\{ t^{\beta-1} E^{[\lambda]}_{\alpha,\beta;\gamma}(\omega t^{\alpha}) \right\}(s) = s^{-\beta}\, {}_1F_0^{(\lambda)}((\gamma)_\lambda;\ -;\ \omega s^{-\alpha})$$

Model application

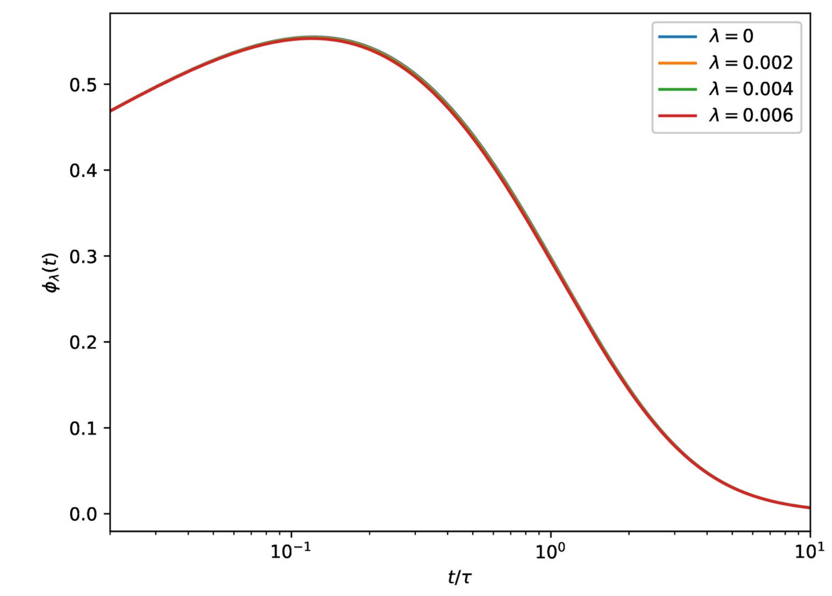
$$H_\lambda(s) = \tau^{-\alpha\gamma} s^{-\alpha\gamma}\, {}_1F_0^{(\lambda)}((\gamma)_\lambda;\ -;\ -(s\tau)^{-\alpha})$$

$$\lambda \to 0^+: \quad H_0(s) = (1+(s\tau)^{\alpha})^{-\gamma}$$

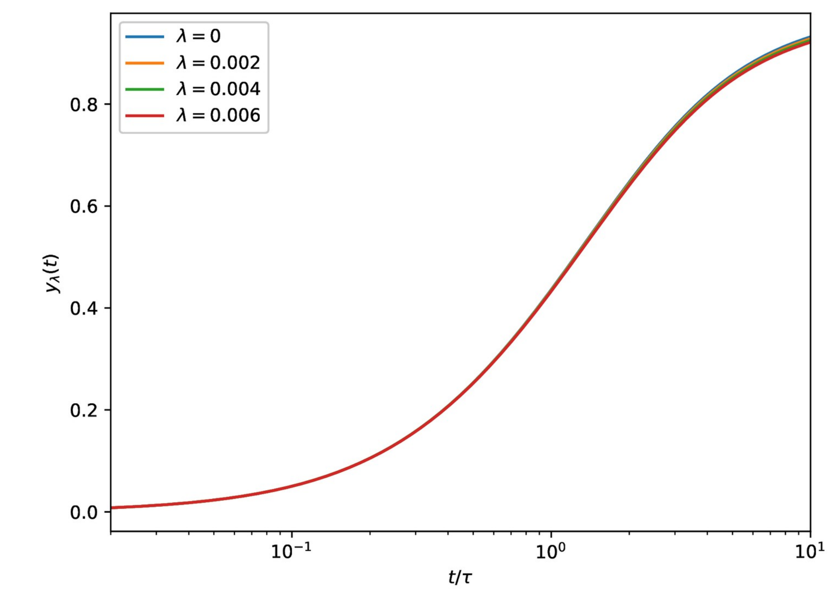
Implementation detail

- Numerics use adaptive truncation and the classical limit as a validation check.
- Figures use the Type-B kernels and step responses from the paper.

RESULTS & DISCUSSION



Kernel sweep for the Type-B deformation



Step-response sweep for the same parameters

Key analytic findings

- The degenerate Mittag-Leffler function is entire under nonresonance.
- For positive alpha, the entire-function order is
- Beta-shift fractional-calculus identities mirror the classical Prabhakar calculus.
- The Type-B deformation preserves the closed-form HN limit.

$$\rho = \frac{1}{1+\alpha}$$

Numerical interpretation

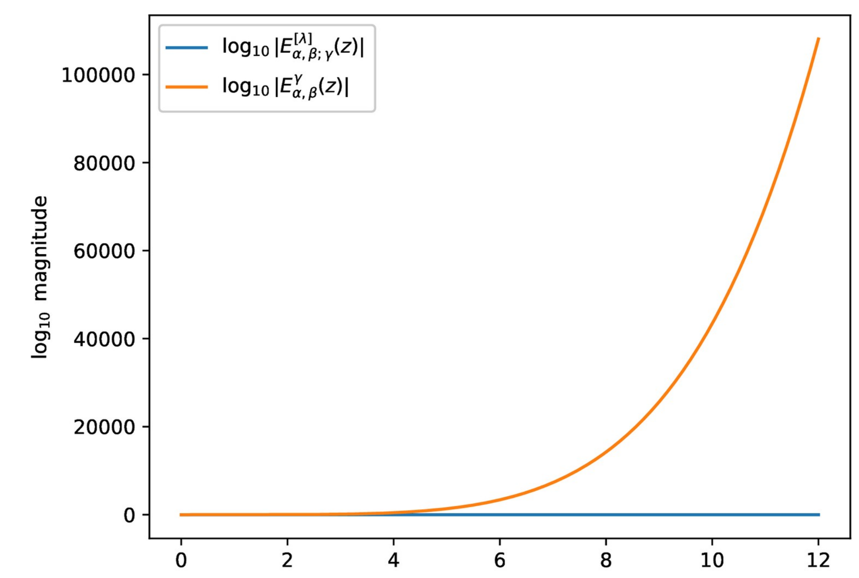
For the baseline parameters shown below, the degeneracy parameter is a stable one-dimensional refinement knob. The sweep alters intermediate-time behavior while keeping deviations from the classical HN baseline small.

$$(\alpha, \gamma, \tau) = (0.8, 1.5, 1)$$
$$\lambda \in \{0, 0.002, 0.004, 0.006\}$$

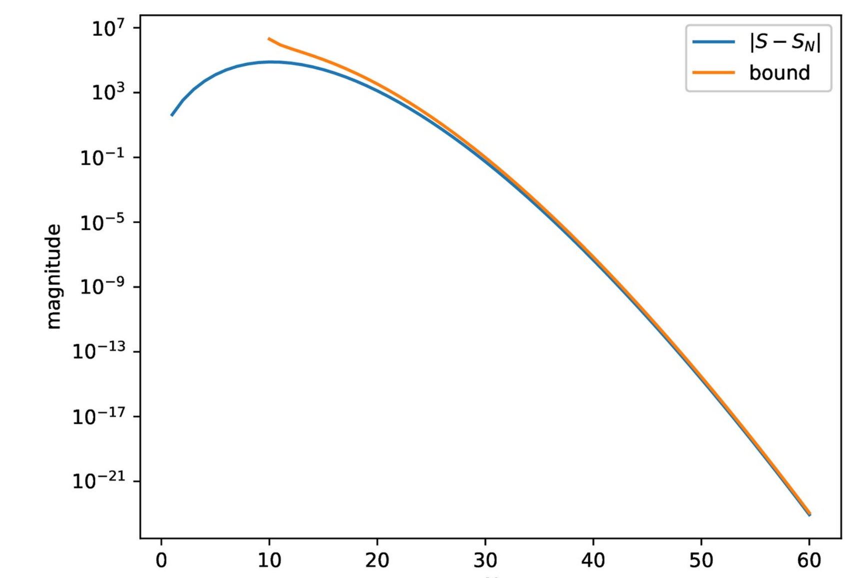
Peak remains at t*/τ = 0.120; the peak amplitude increases from 0.555116 to 0.559430 over the sweep.

Sensitivity of the Type-B deformation

λ	t*/τ	φλ(t*)	yλ(10τ)	εφ	εy
0.000	0.120	0.555116	0.931864	0.000e+00	0.000e+00
0.002	0.120	0.556551	0.931823	1.601e-03	2.358e-04
0.004	0.120	0.557989	0.931782	3.204e-03	4.711e-04
0.006	0.120	0.559430	0.931740	4.809e-03	7.058e-04



Degenerate/classical magnitude comparison



Actual truncation error and a posteriori bound

Truncation validation

$$N = 50: \quad |S - S_N| \approx 2.06 \times 10^{-15}, \qquad \text{bound} \approx 2.85 \times 10^{-15}$$

CONCLUSION

- The proposed coefficient-degenerate Mittag-Leffler function is analytically tractable and entire under transparent nonresonance conditions.
- It supplies a closed-form, one-parameter Type-B deformation of the Havriliak-Negami law, suitable for convolution-based identification and relaxation fitting.
- Adaptive truncation with rigorous a posteriori bounds gives stable numerical evaluation across the reported parameter regimes.

FUTURE WORK / REFERENCES

- Future work: large-argument asymptotics; complete monotonicity/passivity conditions; additional transform pairs and parameter-estimation workflows.
- Selected references: Garra & Garrappa (2018); Gorenflo et al. (2014); Havriliak & Negami (1967); Kim & Kim (2017); Yağcı & Şahin (degenerate Pochhammer functions).