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Qualitative and Numerical Study of a Zika Epidemic Model with Fractal-Fractional Operators

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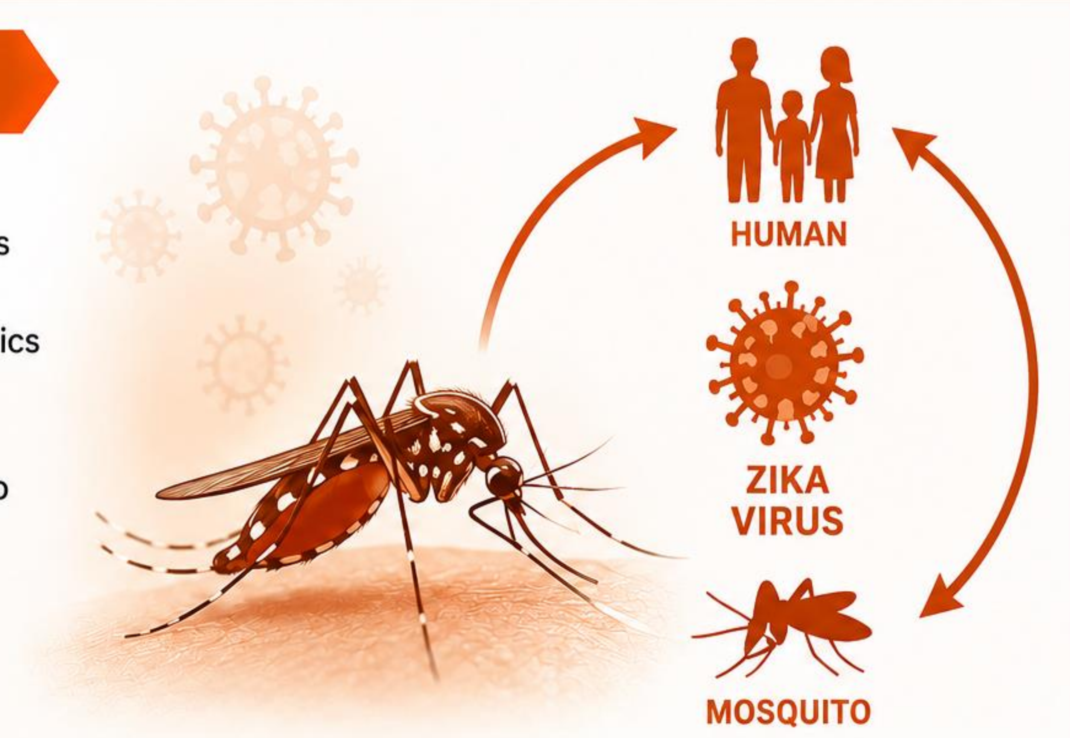
INTRODUCTION & AIM

INTRODUCTION

The Zika virus is a mosquito-borne disease characterized by rapid transmission and serious health complications. Mathematical modeling helps in understanding the transmission dynamics between human and mosquito populations. In this study, a fractal-fractional Zika epidemic model is investigated using the Caputo–Fabrizio (CF) and Atangana–Baleanu (AB) operators to compare the effects of different non-singular memory kernels on disease dynamics.



AIM OF THE STUDY

- To develop a fractal-fractional Zika epidemic model using CF and AB operators;
- To investigate the existence, uniqueness, and stability of the proposed model;
- To perform numerical simulations for different fractional orders;
- To compare the effects of CF and AB operators on Zika disease dynamics.

METHOD

METHODOLOGY

The Zika epidemic model is formulated for four compartments: susceptible humans S_h, infected humans I_h, susceptible mosquitoes S_n, and infected mosquitoes I_n.

$$\begin{aligned} D_t S_h &= \Lambda_h - \beta_1 S_h I_h - \beta_2 S_h I_n - k_1 S_h, \\ D_t I_h &= \beta_1 S_h I_h + \beta_2 S_h I_n - k_1 I_h, \\ D_t S_n &= \Lambda_n - \mu S_n I_h - k_2 S_n, \\ D_t I_n &= \mu S_n I_h - k_2 I_n. \end{aligned}$$

The classical derivative D_t is replaced by fractal-fractional operators in the Caputo sense.

$${}^{FF-CF}D_{0,t}^{\sigma 1,\sigma 2}\,\Psi(t) = \frac{M(\sigma_1)}{1-\sigma_1}\int_0^t \frac{d\Psi(s)}{ds^{\sigma 2}}\, e^{-\frac{\sigma_1}{1-\sigma_1}(t-s)}\, ds$$

$${}^{FF-AB}D_{0,t}^{\sigma 1,\sigma 2}\,\Psi(t) = \frac{h(\sigma_1)}{1-\sigma_1}\int_0^t \frac{d\Psi(s)}{ds^{\sigma 2}}\, E_{\sigma_1}\left[-\frac{\sigma_1}{1-\sigma_1}(t-s)^{\sigma 1}\right] ds$$

The CF operator uses an exponential kernel, while the AB operator uses a Mittag–Leffler kernel. Positivity, boundedness, existence, uniqueness, and Hyers–Ulam stability are investigated.

SIMULATION APPROACH

Numerical simulations are performed using the same initial values $(S_h(0), I_h(0), S_n(0), I_n(0)) = (800, 200, 600, 300)$ for different fractional orders σ_1 and fractal orders σ_2. The model is solved under both fractal-fractional Caputo–Fabrizio (FF–CF) and Atangana–Baleanu (FF–AB) operators. The aim is to compare the memory effects induced by the exponential kernel (CF) and the Mittag–Leffler kernel (AB) on the dynamics of susceptible and infected human and mosquito populations.

In the simulations, the fractional order σ_1 controls the strength of the memory througth the kernel, while the fractal order σ_2 governs the local irregularity of the system. By varying these orders, we observe how memory characteristics influence the rate of infection spread, peak magnitude, and long-term behavior of the populations.

The simulations reveal that the CF operator leads to faster stabilization of the populations, indicating short-memory behavior, whereas the AB operator shows slower decay and long-lasting persistence, reflecting strong memory effects. These findings highlight the importance of choosing an appropriate fractal-fractional operator for accurately capturing the underlying memory mechanisms in Zika epidemiology.

FRACTAL INTERPRETATION

$$\frac{d\Psi(t)}{dt^{\sigma 2}} = \lim_{t'\to t} \frac{\Psi(t') - \Psi(t)}{t'^{\sigma 2} - t^{\sigma 2}}$$

The fractal order σ_2 describes the irregularity and local geometric structure of the epidemic process, while the fractional order σ_1 characterizes memory effects in disease transmission dynamics.

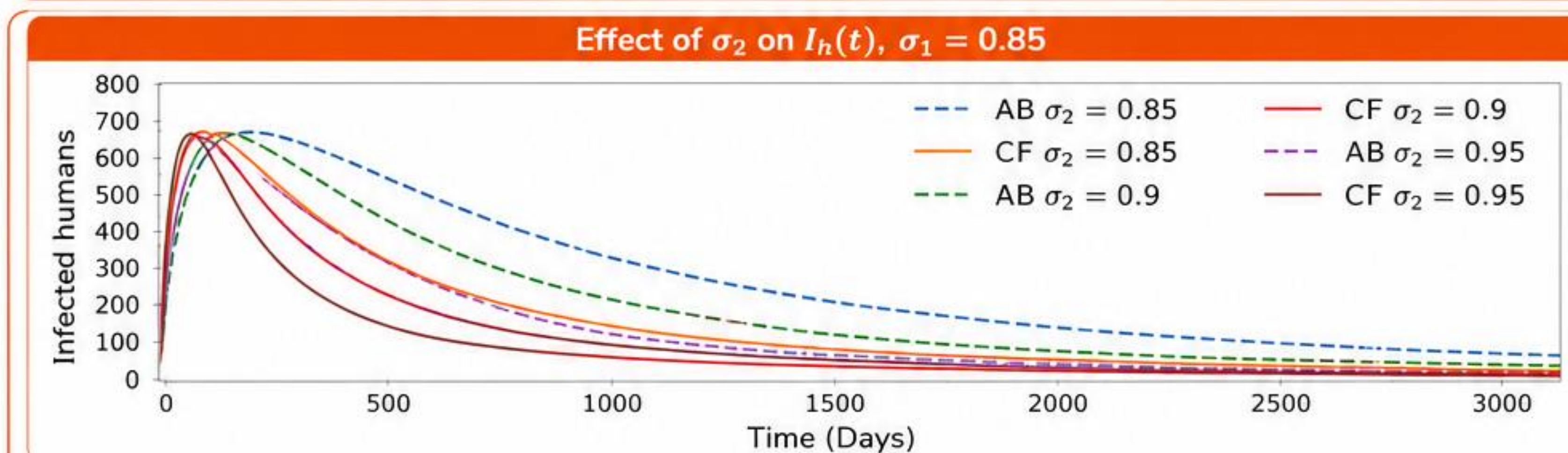
- Fractional order σ_1: controls memory strength.
- Fractal order σ_2: controls local irregularity.
- CF operator: short-memory behavior.
- AB operator: long-memory behavior.

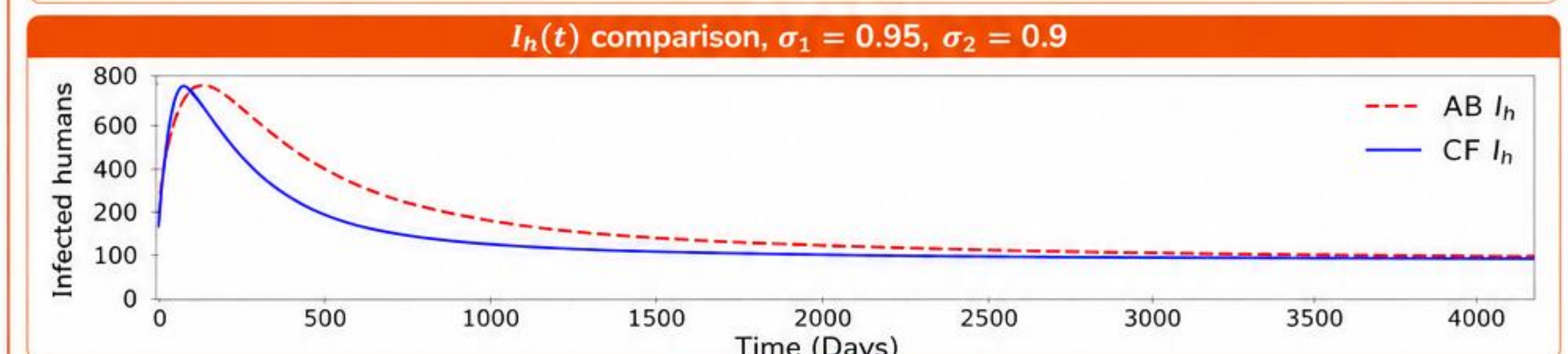
RESULTS & DISCUSSION

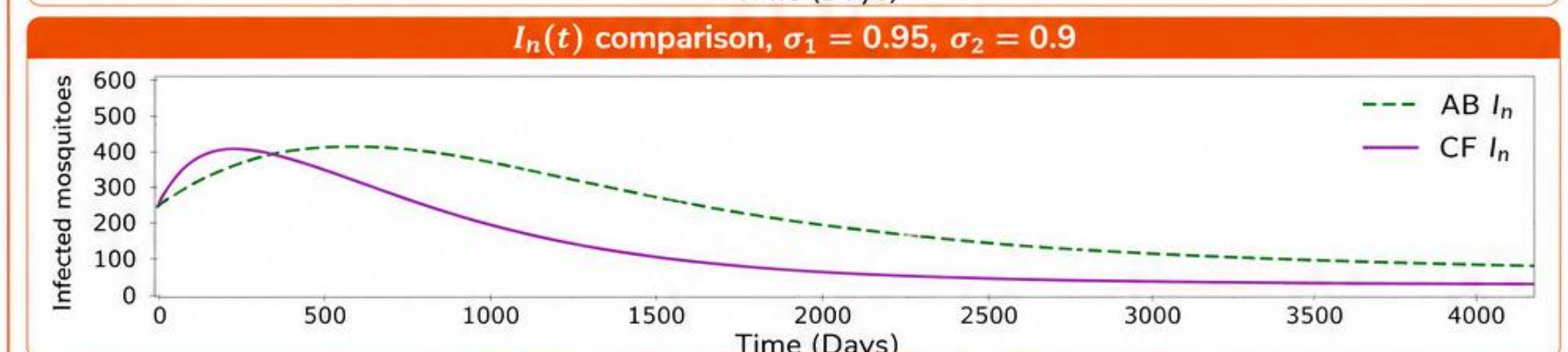
The numerical results show that both the Atangana–Baleanu (AB) and Caputo–Fabrizio (CF) fractal–fractional operators successfully describe the transmission dynamics of the Zika virus model. The susceptible human population initially decreases due to the spread of infection, then stabilizes. In contrast, the infected human population increases rapidly during the early stage of the epidemic before gradually decreasing toward equilibrium. Similar behavior is observed for the mosquito population.

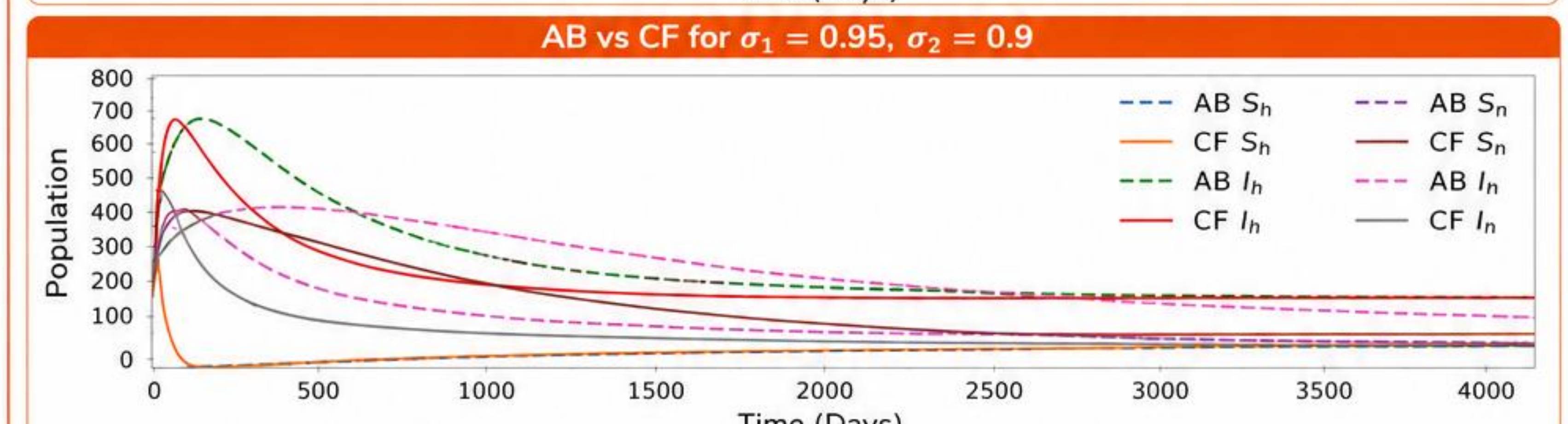
The comparison plots indicate that the AB operator produces slower decay of infected populations because of its stronger memory effect associated with the Mittag–Leffler kernel. In contrast, the CF operator shows faster stabilization due to its exponential fading memory kernel. Furthermore, decreasing the fractal order reduces the number of infected humans and mosquitoes, demonstrating the significant influence of fractal memory on disease transmission dynamics.

Overall, the obtained results confirm that fractal–fractional operators provide an effective framework for capturing memory and hereditary effects in epidemiological systems.









KEY TAKEAWAY: The AB operator, with its stronger memory, results in slower decay and longer persistence of infection, while the CF operator, with its exponential memory, leads to faster stabilization.
→ Lower fractal orders (σ_2) reduce infection levels, highlighting the crucial role of fractal memory in controlling disease spread.

CONCLUSION

- The FF Zika virus model successfully captures the transmission dynamics between humans and mosquitoes, demonstrating that memory effects significantly influence the spread and persistence of the disease.
- The comparison between the AB and CF fractal–fractional operators shows that the AB operator exhibits stronger memory effects and slower infection decay, whereas the CF operator produces faster stabilization due to its exponential fading memory behavior.

FUTURE WORK / REFERENCES

- Future work may focus on extending the proposed model to include variable-order fractal–fractional operators, mixed fractional derivatives, and directional memory effects to capture more realistic and complex epidemic transmission dynamics.
- Explore the incorporation of control measures and advanced fractal–fractional modeling techniques to improve the prediction and management of Zika virus outbreaks.

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