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Computing the Dynamics of Optical Breathers in a Coupled Nonlinear Schrödinger Model with Dispersion Variations

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1. INTRODUCTION

- Wave propagation in inhomogeneous nonlinear systems with varying dispersion and nonlinearities are significant to explore real-world applications.
- The considered model, variable-coefficient general coupled nonlinear Schrödinger (vc-GCNLS) equation, consists of varying dispersion effect along with self-phase–cross-phase modulations and four-wave mixing nonlinearities.
- Optical breathers are important in the context of localized nonlinear wave structures in optical media.
- Analysis on localized and periodic wave provide essential knowledge in understanding their dynamics in various optical systems.

2. OBJECTIVES

- To study vector breather dynamics in inhomogeneous optical media using the vc-GCNLS equation.
- To derive Akhmediev and Kuznetsov–Ma type breather solutions using similarity and Darboux transformations.
- To analyze the effects of varying dispersion on the dynamics of breathers and to identify their modulations.
- To explore potential applications in optical communication and nonlinear photonic crystals.

3. THE MODEL & METHOD

- **The vc-GCNLS equation:**

$$i\frac{\partial p_1}{\partial z}+\frac{\delta(z)}{2}\frac{\partial^2 p_1}{\partial t^2}+\sigma(z)\left(\epsilon|p_1|^2+c|p_2|^2+bp_1p_2^*+b^*p_1^*p_2\right)p_1+(\Omega_1(z)t+\Omega_2(z)t^2+i\Omega_3(z))p_1=0,$$

$$i\frac{\partial p_2}{\partial z}+\frac{\delta(z)}{2}\frac{\partial^2 p_2}{\partial t^2}+\sigma(z)\left(\epsilon|p_1|^2+c|p_2|^2+bp_1p_2^*+b^*p_1^*p_2\right)p_2+(\Omega_1(z)t+\Omega_2(z)t^2+i\Omega_3(z))p_2=0.$$

- $\delta(z)$: dispersion, $\sigma(z)$: nonlinearity,
- $\Omega_1(z)$, $\Omega_2(z)$, and $\Omega_3(z)$ external potential & gain/loss.
- **Three-layer method:**
  1. Similarity transformation
  2. Superposition
  3. Darboux transformation

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- *1.Similarity transformation*

$$p_j(z,t)=A(z)Q_j[\xi(t,z),\tau(z)]e^{i\psi(t,z)},$$

- *2. Linear Superposition*

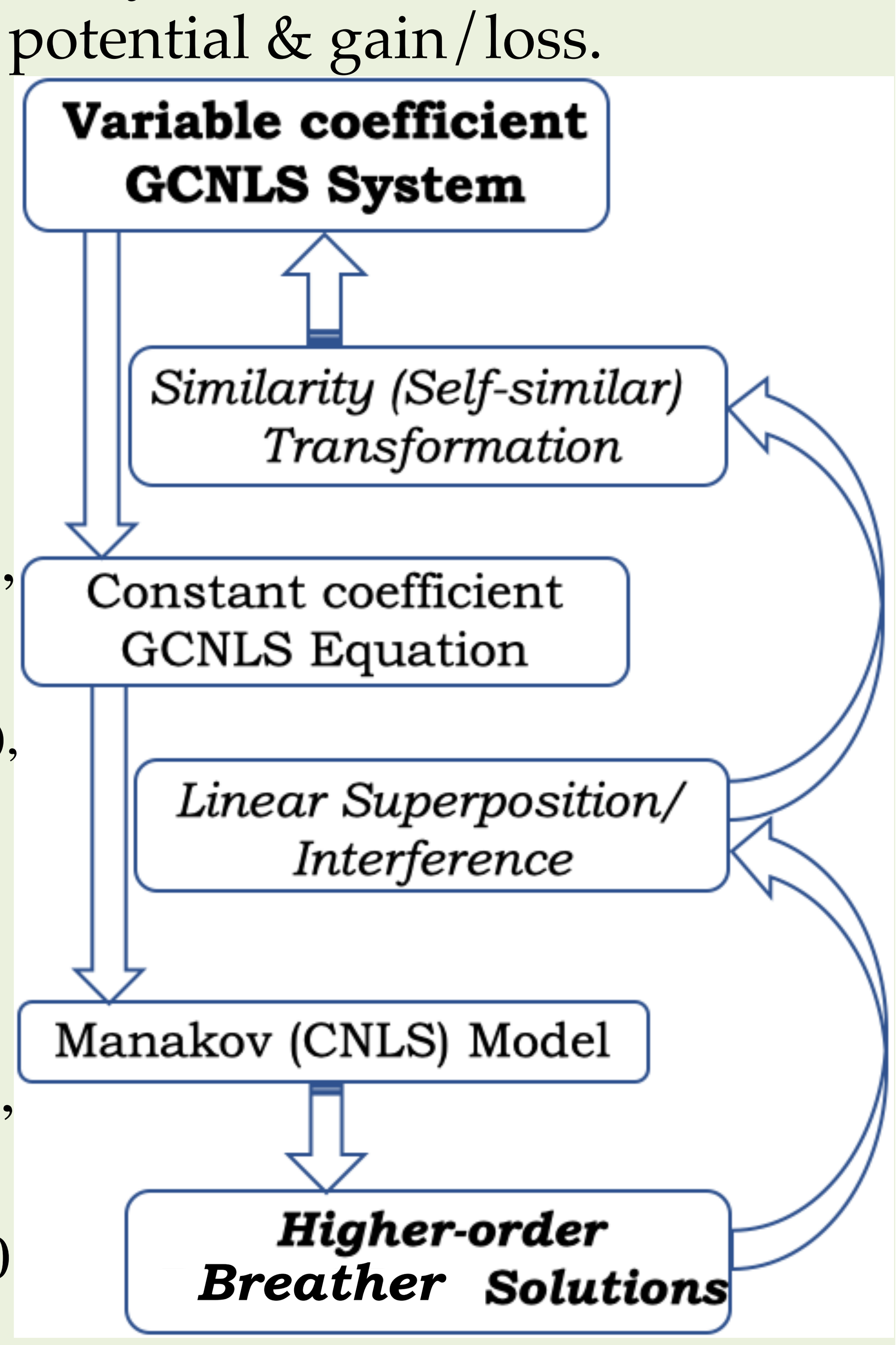
$$Q_1(\xi,\tau)=\frac{\alpha_1}{\sqrt{\rho}}q_1(\xi,\tau)-\frac{\alpha_1^*b^*+\alpha_2^*c}{\sqrt{\rho d}}q_2(\xi,\tau),$$

$$Q_2(\xi,\tau)=\frac{\alpha_2}{\sqrt{\rho}}q_1(\xi,\tau)+\frac{\alpha_1^*\epsilon+\alpha_2^*b}{\sqrt{\rho d}}q_2(\xi,\tau).$$

- *Manakov model*

$$i\frac{\partial q_1}{\partial\tau}+\frac{1}{2}\frac{\partial^2 q_1}{\partial\xi^2}+\left(|q_1|^2+|q_2|^2\right)q_1=0,$$

$$i\frac{\partial q_2}{\partial\tau}+\frac{1}{2}\frac{\partial^2 q_2}{\partial\xi^2}+\left(|q_1|^2+|q_2|^2\right)q_2=0$$



- *3.Solution through Darboux transformation*

4. RESULTS & DISCUSSION

- Different types of breather profiles can be obtained for vector breathers of the vc-GCNLS equation in inhomogeneous optical media.
- Akhmediev breathers are localized along the propagation direction *z* and periodic along the transverse direction *t*.
- Ma breathers or Kuznetsov-Ma solitons are periodic along the propagation direction *z* and localized in the transverse coordinate *t*.
- Akhmediev & Ma breathers exhibit bright, gray and dark type profiles for different choices of solution parameters.

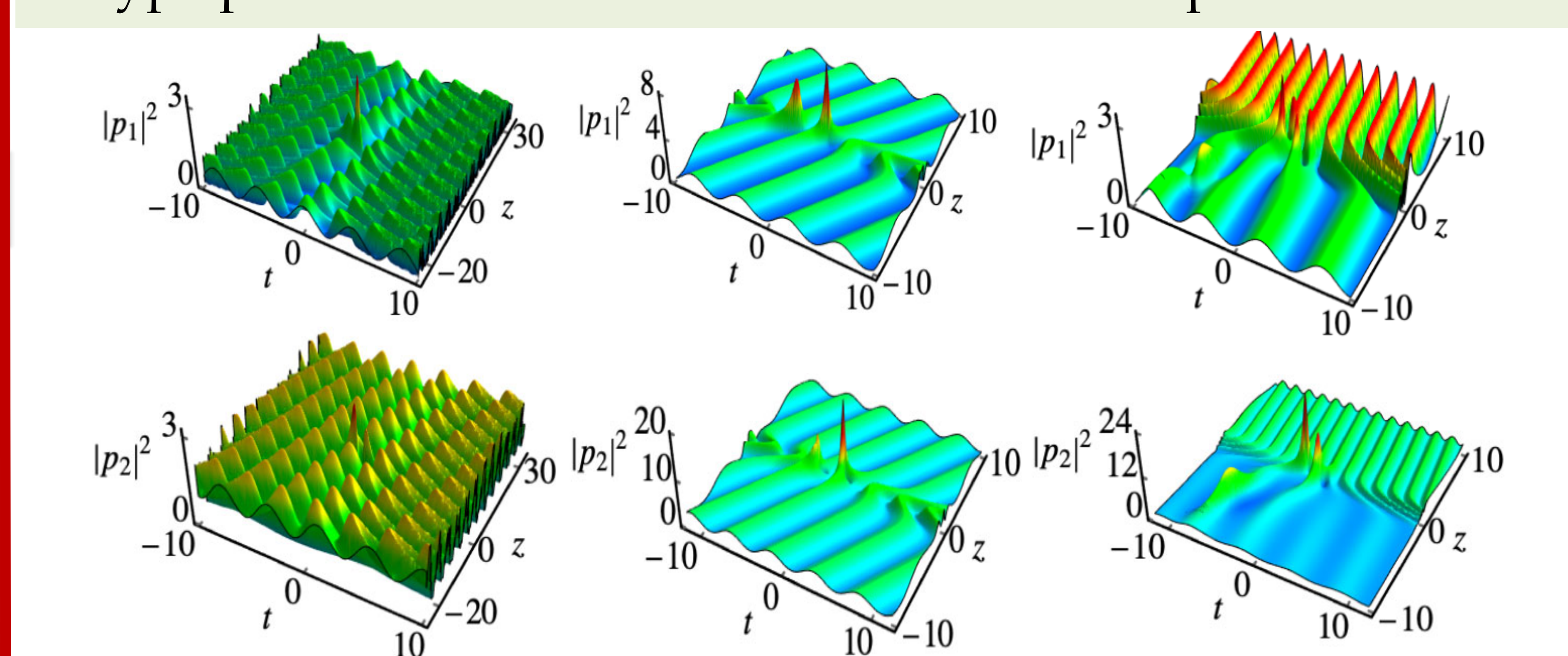


Fig. Doubly-periodic structures, cross-over interaction, amplification with compression of breathers due to inhomogeneous nature of the medium.

- *Periodic modulation:* $\sigma(z)=\sigma_0+\sigma_1\sin(\sigma_2 z+\sigma_3)$
- *Step-like modulation:* $\sigma(z)=\sigma_0+\sigma_1\tanh(\sigma_2 z+\sigma_3)$
- *Localized soliton modulation:* $\sigma(z)=\sigma_0+\sigma_1\text{sech}(\sigma_2 z+\sigma_3)$

- Inhomogeneous nature of the model is implemented through periodic, step-like tan-hyperbolic, localized soliton-type varying nonlinearities.
- Modulation in the identities of breathers occur, like localization breaking, doubly-periodic structures, amplification and compression, generation of sideband oscillating tails, cross- over and exciton formation.

5. CONCLUSION

- Studied vector breather dynamics of the vc-GCNLS equation arising in inhomogeneous optical media.
- Derived explicit breather solutions using a three-layer method involving similarity and Darboux transformations.
- Analyzed the effects of varying dispersion on the dynamics of breathers and identified their modulations.
- Potential applications include inhomogeneous type optical communication systems and nonlinear photonic crystals.

FUTURE WORK & REFERENCES

- Investigations on various other nonlinear waves in closely related vc-CNLS type systems are significant open problems to address in the near future.
- For further reading and related references, Nonlinear Dyn **113** (2025) 33777-33804. https://doi.org/10.1007/s11071-025-11749-1

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