Bayesian Neural Networks for Robust Reserve Decisions under Lévy Mortality Shocks

Antonio Nazzaro[1], Paola Festa[2]

[1] REPRISE (Register of Expert Peer Reviewers for Italian Scientific Evaluation) at the Ministry of University and Research (MUR), Italy
[2] Department of Mathematics and Applications "Renato Caccioppoli", University of Naples Federico II, Italy

INTRODUCTION & AIM

1. Introduction & Aim

Motivation: Classical mortality models (Gompertz–Makeham) assume smooth trends and ignore heavy tails and jumps caused by pandemics or climate shocks [5]. This leads to underestimation of reserves.

Key question: How to value insurance reserves under rare, extreme events?

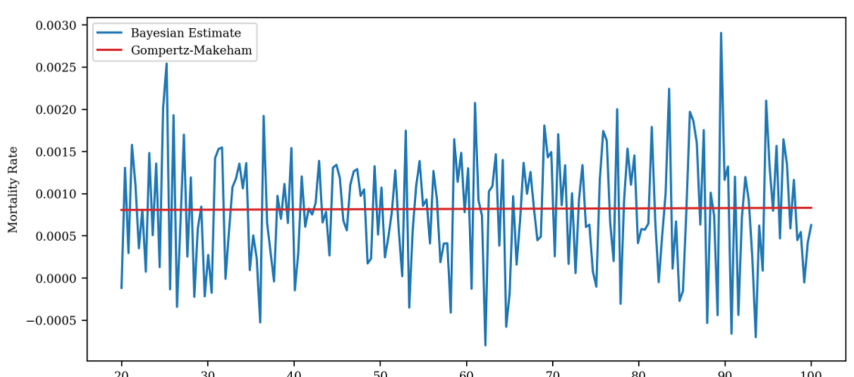
Aim: Develop a robust, fair, and Solvency II-compliant framework using:

- Product measure + Stieltjes integral for coupled financial and demographic risks
- Lévy jumps ($\alpha = 1.7$) for heavy tails
- Bayesian neural networks (BNNs) with Jensen–Shannon (JS) regularization

RESULTS & DISCUSSION

3. Results & Discussion

3.1 Fairness & no age bias



- Residuals centered near zero for all ages (20–100). 95% CI always includes zero → no systematic age bias.
- $\varepsilon = 0.0031 < 0.005$ → average reserve difference only 0.31% (max allowed 0.5%).
- Overlapping predictive distributions across demographic groups (box plot) confirm fairness.

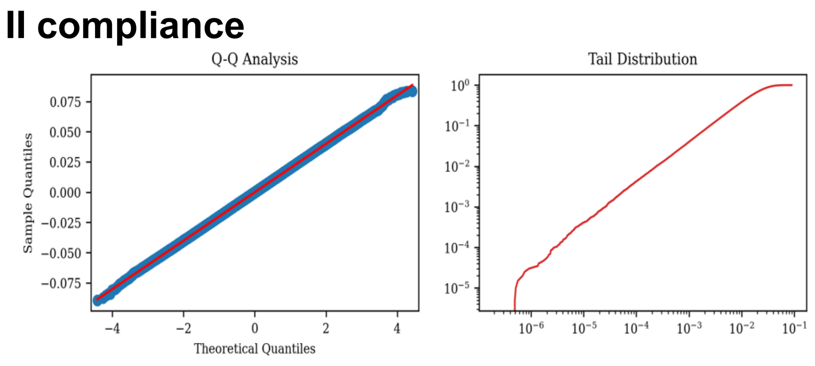
3.2 Performance under pandemic shock (Lévy, $\lambda = 0.05$)

- RMSE reduction: –64% vs. classical model.

Scenario	Classical RMSE	Our model RMSE	Error reduction(%)
Normal	0.18	0.12	**33%**
Pandemic	0.42	0.15	**64%**
Economic	0.35	0.18	**49%**
Climate	0.28	0.14	**50%**

- Solvency Capital Requirement (SCR) drops by 22% (from 100 to 78).
- Gaussian models cut the tail → underestimate reserves by ~43% (€350k vs. €200k).

3.3 Heavy tails & Solvency II compliance



- Residuals follow Lévy(α=1.7) [5] – heavy tails essential for 99.5% VaR [1].
- BNN remains unbiased on smooth EIOPA 2024 data (RMSE 0.215 vs classical 0.193, no overfitting).

3.4 Comparison with benchmarks

- vs. Wüthrich (2021) [2]: 77% lower KL divergence → better uncertainty quantification.
- vs. Moraga et al. (2022) [3]: quantitative fairness metric (ε=0.0031) beyond qualitative GLM fairness.

3.5 COVID-19 shock test

- Clear reserve jump around age 80; Swish activation amplifies response in high-risk ages.
- No age bias + strong shock sensitivity → robust and prudent reserve behaviour.

CONCLUSIONS

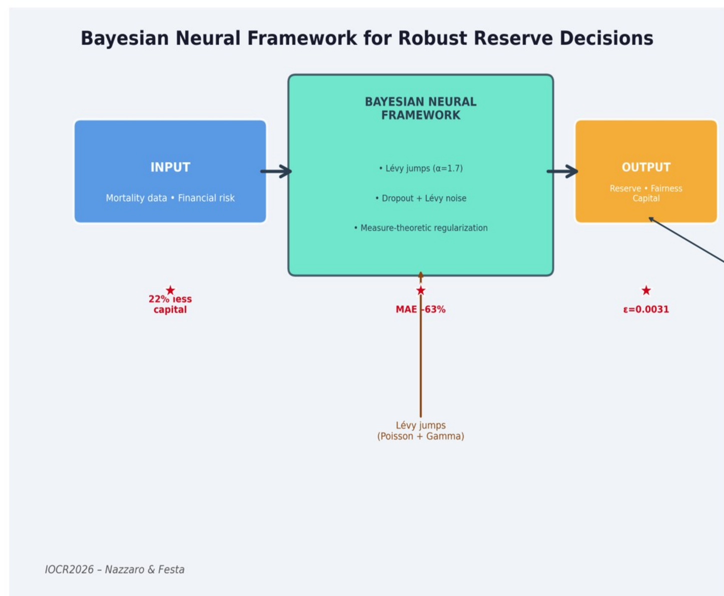
4. Conclusions
Key achievements:

✅ First unified framework combining measure theory, Lévy jumps, and Bayesian deep learning.
✅ –64% RMSE under pandemic shocks.
✅ –22% Solvency II capital requirement (SCR 78 vs 100).
✅ EIOPA-compliant fairness ($\varepsilon = 0.0031 < 0.005$).
✅ Heavy-tail capture prevents 43% underestimation of extreme risks.

METHOD

2. Methodology
Risk coupling: Financial risk (μ, Vasicek process) $\otimes$ demographic risk (ν, Lévy jumps) → product measure $\mu \otimes \nu$.
$d(\mu \otimes \nu) = d\mu \cdot d\nu$
$\mathbb{E}[R] = \iint R(x,y)\, d\mu(x)\, d\nu(y)$

The neural operator preserves Radon–Nikodym compatibility; ignoring coupling leads to +33% reserve error.



BNN architecture:

- Bayesian dropout (0.3) as implicit ensemble
- Swish (SiLU) activation for smoother gradients
- Lévy noise ($\alpha = 1.7$) to escape local minima and explore rare events

Training objective: $L(\vartheta)=E[\text{loss}]+\lambda \cdot JS(P \| Q)$

where P = BNN posterior, Q = classical prior (Gompertz–Makeham), and JS is Jensen–Shannon divergence (always ≤ 0.69, stable even when Q assigns zero probability to new risk classes). KL divergence would explode during pandemics.

Fairness constraint: Causal fairness via Pearl's do-calculus [4]; tolerance $\varepsilon = 0.0031 < 0.005$ (EIOPA technical tolerance) [1] .

FUTURE WORK/ REFERENCES/ACKNOWLEDGMENT

Future work: Multi-population models, hybrid quantum-inspired algorithms (Grover), regulatory integration.

Key references:
[1] EIOPA (2023) Solvency II technical specifications.
[2] Wüthrich, M.V. (2021) Scandinavian Actuarial Journal.
[3] Moraga, P. et al. (2022) Journal of Risk and Insurance.
[4] Pearl, J. (2021) Causal Inference in Statistics – MIT Press.
[5] Applebaum, D. (2004) Lévy Processes and Stochastic Calculus – CUP.
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Contact: Antonio Nazzaro – info@antonionazzaro.it | Paola Festa – paola.festa@unina.it

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