The Particle as a Statistical Ensemble of Events in Stueckelberg-Horwitz-Piron Electrodynamics

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Stueckelberg Covariant Mechanics

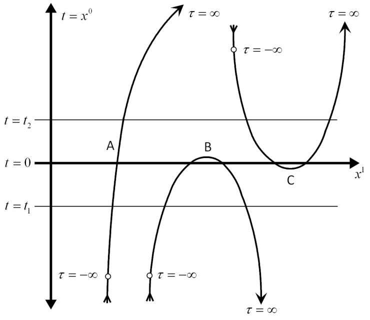
Worldline Theory of Particles and Antiparticles (1941)

Dynamical theory of spacetime events

- Equations of motion for event $x^{\mu}(\tau)$
- Evolution of $x^{\mu}(\tau)$ traces worldline
- Coordinate time $t = x^0$ may increase or decrease under evolution
- Single worldline describes pair annihilation and creation

Requires new evolution parameter τ

- Monotonic replacement for $t = x^0$
- Poincaré invariant
- Independent of spacetime coordinates
- Distinguishes chronological time τ and coordinate time x^0



Worldlines

Type A. Usual type, with one solution to $t(\tau) = x^0$ for each x^0

Type B. Annihilation type, with two solutions to $t(\tau) = x^0$ for $x^0 \ll 0$ and no solutions for $x^0 \gg 0$

Type C. Creation type, with two solutions to $t(\tau) = x^0$ for $x^0 \gg 0$ and no solutions for $x^0 \ll 0$

Translated from: E. C. G. Stueckelberg, *Helv. Phys. Acta* **14** (1941) 322.

Covariant Canonical Mechanics

Upgrade nonrelativistic classical and quantum mechanics

$$\left.\begin{array}{c}\text{Newtonian time } t \\ + \\ \text{Galilean symmetry}\end{array}\right\} \longrightarrow \left\{\begin{array}{c}\text{Evolution parameter } \tau \\ + \\ \text{Poincaré symmetry}\end{array}\right.$$

Inherit methods of nonrelativistic classical and quantum mechanics

$$K = \frac{1}{2M} p^\mu p_\mu \quad \longrightarrow \quad \left\{\begin{array}{l} \dot{x}^\mu = \dfrac{\partial K}{\partial p_\mu} = \dfrac{p^\mu}{M} \qquad \dot{p}^\mu = -\dfrac{\partial K}{\partial x_\mu} = 0 \\ \hline \left.\begin{array}{l} \dot{x}^0 = \dfrac{p^0}{M} \\ \dot{\mathbf{x}} = \dfrac{\mathbf{p}}{M} \end{array}\right\} \longrightarrow \quad \dfrac{d\mathbf{x}}{dt} = \dfrac{\mathbf{p}}{p^0} \end{array}\right.$$

Free particle permits reparameterization $\tau \longrightarrow t$

Covariant Mechanics with Interactions

Two-body Hamiltonian — Horwitz and Piron (1973)

$$K = \frac{p_{1\mu} p_1^\mu}{2M_1} + \frac{p_{2\mu} p_2^\mu}{2M_2} + V(x_1, x_2)$$

Generalize classical central force problems

$$V(x_1, x_2) = V(\rho) \qquad \text{where} \qquad \rho = \sqrt{(\mathbf{x}_1 - \mathbf{x}_2)^2 - (t_1 - t_2)^2}$$

Separation of center of mass and relative motion

$$K = \frac{P^\mu P_\mu}{2M} + \frac{p^\mu p_\mu}{2m} + V(\rho) = \frac{P^\mu P_\mu}{2M} + K_{rel}$$

where

$$P^\mu = p_1^\mu + p_2^\mu \qquad p^\mu = \frac{M_2 p_1^\mu - M_1 p_2^\mu}{M} \qquad M = M_1 + M_2 \qquad m = \frac{M_1 M_2}{M}$$

Relativistic bound states and scattering solutions

Selection rules, radiative transitions, perturbation theory, Zeeman and Stark effects, bound state decay

4-vector and scalar potentials required to reproduce well-known phenomenology

Stueckelberg-Horwitz-Piron (SHP) Canonical Mechanics

- Irreducible chronological time τ — determines temporal ordering of events
- Order of physical occurrence may differ from order of observed coordinate times x^0 as events appear in measuring apparatus
- Event *occurrence* $x^\mu(\tau_1)$ at τ_1 is irreversible — unchanged by subsequent $(\tau_2 > \tau_1)$ event at same spacetime coordinates $x^\mu(\tau_2) = x^\mu(\tau_1)$
- Resolves grandfather paradoxes
- No closed timelike curves — return trip to past coordinate time x^0 takes place while chronological time τ continues to increase
- In SHP QED, particle propagator $G(x_2 - x_1, \tau_2 - \tau_1)$ vanishes unless $\tau_2 > \tau_1$
- Super-renormalizable QED with no matter loops
- τ-retarded causality equivalent to Feynman contour for propagators — follows from vacuum expectation value of τ-ordered operator products
- Covariant Hamiltonian generates evolution of 4D block universe defined at τ to infinitesimally close 4D block universe defined at $\tau + d\tau$
- Standard Maxwell electrodynamics = equilibrium limit
  Dynamic system $\rightarrow$ τ-independent and static 4D block universe

Stueckelberg-Horwitz-Piron (SHP) Electrodynamics

Unified gauge theory — Saad, Horwitz, and Arshansky (1989)

Generalized Stueckelberg-Schrodinger equation

$$i\hbar\partial_\tau\psi\left(x,\tau\right)=K\psi\left(x,\tau\right)=\left[\frac{1}{2M}\left(p^\mu-\frac{e}{c}a^\mu\right)\left(p_\mu-\frac{e}{c}a_\mu\right)-\frac{e}{c}\phi\right]\psi\left(x,\tau\right)$$

Invariant under local gauge transformations

$$\psi(x,\tau) \quad\rightarrow\quad e^{\frac{ie}{\hbar c}\Lambda(x,\tau)}\,\psi(x,\tau)$$

Vector potential $\quad a_\mu(x,\tau) \quad\rightarrow\quad a_\mu(x,\tau)+\partial_\mu\Lambda(x,\tau)$

Scalar potential $\quad \phi(x,\tau) \quad\rightarrow\quad \phi(x,\tau)+\partial_\tau\Lambda(x,\tau)$

Global gauge invariance $\longrightarrow$ conserved current $\qquad \partial_\mu j^\mu+\partial_\tau\rho=0$

$$j^\mu=-\frac{i}{2M}\left\{\psi^*(\partial^\mu-\frac{ie}{c}a^\mu)\psi-\psi(\partial^\mu+\frac{ie}{c}a^\mu)\psi^*\right\} \qquad \rho=\left|\psi(x,\tau)\right|^2$$

5D Notations and Conventions

Formal designations in analogy with $x^0 = ct$

$$x^5 = c_5 \tau \qquad \text{and} \qquad \partial_5 = \frac{1}{c_5}\partial_\tau$$

Five explicitly τ-dependent gauge fields $a_\mu(x,\tau)$ and $a_5(x,\tau) = \dfrac{1}{c_5}\phi(x,\tau)$

Index conventions

$$\lambda, \mu, \nu = 0,1,2,3 \qquad \text{and} \qquad \alpha, \beta, \gamma = 0,1,2,3,5$$

$$g_{\alpha\beta} = \text{diag}(-1,1,1,1,\pm 1)$$

Gauge transformations

$$a_\alpha(x,\tau) \rightarrow a_\alpha(x,\tau) + \partial_\alpha \Lambda(x,\tau)$$

Conserved current

$$\partial_\alpha j^\alpha = 0$$

Classical Lagrangian Mechanics

Lagrangian

$$L = \dot{x}^\mu p_\mu - K = L = \frac{1}{2} M \dot{x}^\mu \dot{x}_\mu + \frac{e}{c} \dot{x}^\alpha a_\alpha$$

Lorentz force

$$\frac{d}{d\tau} \frac{\partial L}{\partial \dot{x}_\mu} - \frac{\partial L}{\partial x_\mu} = 0 \quad \longrightarrow \quad \frac{d}{d\tau} \left[M \dot{x}^\mu + \frac{e}{c} a^\mu(x, \tau) \right] = \frac{e}{c} \dot{x}^\alpha \partial^\mu a_\alpha(x, \tau)$$

$$M \ddot{x}^\mu = \frac{e}{c} \left[\dot{x}^\alpha \partial^\mu a_\alpha - (\dot{x}^\nu \partial_\nu + \partial_\tau) a^\mu \right] = \frac{e}{c} f^\mu{}_\alpha(x, \tau) \dot{x}^\alpha$$

where

$$f^\mu{}_\alpha = \partial^\mu a_\alpha - \partial_\alpha a^\mu \qquad \dot{x}^5 = c_5 \dot{\tau} = c_5$$

Particles and fields may exchange mass

$$\frac{d}{d\tau} (-\tfrac{1}{2} M \dot{x}^2) = g_{55} \frac{e c_5}{c} \, f^{5\mu} \dot{x}_\mu$$

Kinetic Term for Field

Standard considerations

Velocity-potential $\rightarrow$ current-potential integral

$$\dot{X}^{\alpha} a_{\alpha} \rightarrow \int d^4x \; \dot{X}^{\alpha}(\tau)\delta^4\left(x - X(\tau)\right) a_{\alpha}(x,\tau) = \frac{1}{c}\int d^4x \; j^{\alpha}(x,\tau) a_{\alpha}(x,\tau)$$

$$j^{\alpha}(x,\tau) = c\dot{X}^{\alpha}(\tau)\delta^4\left(x - X(\tau)\right)$$

Kinetic action term for field

- Not imposed by physical foundations
- Most obvious candidate $\quad \mathcal{L}_{\text{kinetic}} = \frac{1}{4c} f^{\alpha\beta}(x,\tau) f_{\alpha\beta}(x,\tau)$
- Lorentz and gauge invariant
- Contains only first order derivatives
- Produces Maxwell-like field equations
- Admits wave equation and Green's function
- Coulomb scattering $\rightarrow$ wrong dynamics $\quad a^0(x,\tau) = \frac{e}{4\pi|\mathbf{x}|} \, \delta\left(\tau - \left(t - \frac{|\mathbf{x}|}{c}\right)\right)$

Kinetic Term for Field

Higher-order derivative term $\quad f^{\alpha\beta}f_{\alpha\beta} \;\to\; f^{\alpha\beta}f_{\alpha\beta} + (\lambda/2)^2\left(\partial_\tau f^{\alpha\beta}\right)\left(\partial_\tau f_{\alpha\beta}\right)$

Electromagnetic action

$$S_{\text{em}} = \int d^4x d\tau \left\{ \frac{e}{c^2}\, j^\alpha(x,\tau) a_\alpha(x,\tau) - \int \frac{ds}{\lambda}\, \frac{1}{4c} \left[f^{\alpha\beta}(x,\tau)\Phi(\tau-s) f_{\alpha\beta}(x,s) \right] \right\}$$

Field interaction kernel

$$\Phi(\tau) = \delta(\tau) - \left(\frac{\lambda}{2}\right)^2 \delta''(\tau) = \int \frac{d\kappa}{2\pi} \left[1 + \left(\frac{\lambda\kappa}{2}\right)^2 \right] e^{-i\kappa\tau}$$

Inverse function

$$\int \frac{ds}{\lambda}\, \varphi(\tau - s)\, \Phi(s) = \delta(\tau) \to \varphi(\tau) = \lambda \int \frac{d\kappa}{2\pi}\, \frac{e^{i\kappa\tau}}{1 + (\lambda\kappa/2)^2} = e^{-2|\tau|/\lambda}$$

Field equations

Variation wrt a_α $\qquad \partial_\beta f_\Phi^{\alpha\beta}(x,\tau) = \partial_\beta \int ds\; \Phi(\tau - s) f^{\alpha\beta}(x,s) = \frac{e}{c} j^\alpha(x,\tau)$

Invert with φ $\qquad \partial_\beta f^{\alpha\beta}(x,\tau) = \frac{e}{c} \int ds\; \varphi(\tau - s)\, j^\alpha(x,s) = \frac{e}{c}\, j_\varphi^\alpha(x,\tau)$

Ensemble of Events

Shift integral in current

$$j_\varphi^\alpha(x,\tau) = \int ds\ \varphi(\tau - s)\, j^\alpha(x,s) = \int ds\ e^{-2|s|/\lambda}\ j^\alpha(x,\tau - s)$$

$j_\varphi^\alpha(x,\tau) =$ weighted superposition of instantaneous currents $j^\alpha(x,\tau - s)$

Originate at events $X^\mu(\tau - s)$ displaced from $X^\mu(\tau)$ by s on worldline

Regard j_φ^α as current produced by ensemble of events in neighborhood of $X^\mu(\tau)$

Independent random events with constant average rate $= 1/\lambda$ events per second

Poisson distribution of events

Average time between events $= \lambda$

Probability $e^{-s/\lambda}/\lambda$ at τ that next event will occur following time interval $s > 0$

Extend displacement to positive and negative values: $e^{-2|s|/\lambda}$

Construct ensemble of events $\varphi(s)X^\mu(\tau - s)$ along worldline

Weight $\varphi(s) =$ probability of event delayed from τ by interval at least $|s|$

Green's function selects from ensemble unique event at lightlike separation

Field Equations

5D pre-Maxwell equations

$$\partial_\beta f^{\alpha\beta}(x,\tau) = \frac{e}{c}\, j_\varphi^\alpha(x,\tau) \qquad \epsilon^{\alpha\beta\gamma\delta\epsilon}\partial_\alpha f_{\beta\gamma} = 0$$

4D component form

$$\partial_\nu\, f^{\mu\nu} - \frac{1}{c_5}\partial_\tau\, f^{5\mu} = \frac{e}{c}\, j_\varphi^\mu \qquad \partial_\mu\, f^{5\mu} = \frac{e}{c}\, j_\varphi^5 = \frac{c_5}{c}\, e\rho_\varphi$$

$$\partial_\mu f_{\nu\rho} + \partial_\nu f_{\rho\mu} + \partial_\rho f_{\mu\nu} = 0 \qquad \partial_\nu f_{5\mu} - \partial_\mu f_{5\nu} + \frac{1}{c_5}\partial_\tau f_{\mu\nu} = 0$$

Analog of 3-vector Maxwell equations

$$\nabla\times\mathbf{B} - \frac{1}{c}\partial_t\mathbf{E} = \frac{e}{c}\,\mathbf{J} \qquad \nabla\cdot\mathbf{E} = \frac{e}{c}\, J^0$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \nabla\times\mathbf{E} + \frac{1}{c}\partial_t\mathbf{B} = 0$$

Mass-Energy-Momentum Tensor

Noether symmetry $\sim$ translation invariance

$$T^{\alpha\beta} = -\left(g^{\alpha\beta} f_{\Phi}^{\delta\gamma} f_{\delta\gamma} - f^{\alpha}{}_{\gamma} f_{\Phi}^{\beta\gamma}\right) \qquad \partial_\beta T^{\alpha\beta} = \frac{e}{c} f^{\alpha\beta} j_\alpha$$

Classical conservation law

$$\int d^4z\ \partial_\beta T^{\alpha\beta} = \int d^4z\ \partial_\mu T^{\alpha\mu} + \int d^4z\ \partial_5 T^{\alpha 5} = \frac{d}{d\tau} \int d^4z\ T^{5\alpha}$$

$$\begin{aligned} \frac{e}{c} \int d^4z\ f^{\alpha\beta}(z,\tau)\, j_\alpha(z,\tau) &= \frac{e}{c} \int d^4z\ f^{\alpha\beta}(z,\tau)\, \dot{x}_\alpha(\tau)\, \delta^4(z-x) \\ &= \frac{e}{c}\ f^{\alpha\beta}(x,\tau)\, \dot{x}_\alpha(\tau) \end{aligned}$$

$$\frac{d}{d\tau}\left(\int d^4z\ T^{5\mu} + M\dot{x}^\mu\right) = 0 \qquad \frac{d}{d\tau}\left(\int d^4z\ T^{55} - \frac{1}{2} M\dot{x}^2\right) = 0$$

Total mass-energy-momentum of particle + field conserved

Wave Equation and Greens Function

Wave equation

$$\partial_\beta \partial^\beta a^\alpha = \left(\partial_\mu \partial^\mu + \left(g_{55}/c_5^2\right)\ \partial_\tau^2\right) a^\alpha = -\frac{e}{c}\ j_\varphi^\alpha\,(x,\tau)$$

Greens function

$$\partial_\beta \partial^\beta a^\alpha = \left(\partial_\mu \partial^\mu + \left(g_{55}/c_5^2\right)\ \partial_\tau^2\right) G(x,\tau) = -\delta^4(x,\tau)$$

Principal part solution

$$\begin{aligned} G_P(x,\tau) &= -\frac{1}{2\pi}\delta(x^2)\delta(\tau) - \frac{c_5}{2\pi^2}\frac{\partial}{\partial x^2}\frac{\theta(-g_{55}g_{\alpha\beta}x^\alpha x^\beta)}{\sqrt{-g_{55}g_{\alpha\beta}x^\alpha x^\beta}} \\ &= G_{Maxwell} + G_{Correlation} \end{aligned}$$

Contribution from $G_{Correlation}$

- Smaller than $G_{Maxwell}$ by c_5/c and drops off as $1/\left|\mathbf{x}\right|^2$
- Neglected at low energy

Liénard-Wiechert potential

Arbitrary event $X^{\mu}(\tau)$ produces current

$$j_{\varphi}^{\alpha}(x,\tau) = \int ds\ \varphi(\tau - s)\,\dot{X}^{\alpha}(s)\,\delta^{4}[x - X(s)]$$

Green's function selects unique event from ensemble at lightlike separation

$$\int d\tau f(\tau)\,\delta[g(\tau)] = \frac{f(\tau_R)}{|g'(\tau_R)|} \quad , \quad \tau_R = g^{-1}(0)$$

Retarded time τ_R satisfies $(x - X(\tau_R))^2 = 0$ and $x^0 - X^0(\tau_R) > 0$

Potential

$$a^{\alpha}(x,\tau) = \frac{e}{2\pi}\int ds\ \varphi(\tau - s)\,\dot{X}^{\alpha}(s)\ \delta\left[(x - X^{\alpha}(s))^2\right]$$

$$= \frac{e}{4\pi}\varphi(\tau - \tau_R)\,\frac{\dot{X}^{\alpha}(\tau_R)}{\left|(x^{\mu} - X^{\mu}(\tau_R))\,\dot{X}_{\mu}(\tau_R)\right|}$$

Standard Liénard-Wiechert potential $\times$ factor φ containing τ-dependence

Low Energy Limit
Nonrelativistic Coulomb problem

'Static' event evolving along x^0-axis

$$x(\tau) = (c\tau, 0, 0, 0)$$

Potential

$$a^0(x,\tau) = \frac{e}{4\pi|\mathbf{x}|}\varphi\left(\tau - \left(t - \frac{|\mathbf{x}|}{c}\right)\right) \qquad a^5(x,\tau) = \frac{c_5}{c}a^0(x,\tau)$$

Test event on parallel trajectory at

$$x(\tau) = (c\tau, \mathbf{x})$$

Yukawa-type potential

$$a^0(x,\tau) = \frac{e}{4\pi|\mathbf{x}|}e^{-|\mathbf{x}|/\lambda c}$$

Photon mass $m_\gamma \sim \hbar/\lambda c^2$

$m_\gamma \sim$ experimental error on photon mass $= 10^{-18} eV/c^2 \;\Rightarrow\; \lambda > 10^{-2}$ sec

Concatenation

Conserved Maxwell current — Stueckelberg's argument

$$\partial_\alpha j^\alpha = \partial_\mu j^\mu + \partial_\tau \rho = 0 \quad \longrightarrow \quad \partial_\mu J^\mu(x) = \partial_\mu \int_{-\infty}^{\infty} d\tau \; j^\mu(x,\tau) = 0$$

Following Stueckelberg

$$\left. \begin{array}{c} \partial_\beta f^{\alpha\beta}(x,\tau) = \dfrac{e}{c} j^\alpha(x,\tau) \\ \\ \partial_{[\alpha} f_{\beta\gamma]} = 0 \end{array} \right\} \xrightarrow[\int d\tau]{} \left\{ \begin{array}{c} \partial_\nu F^{\mu\nu}(x) = \dfrac{e}{c} J^\mu(x) \\ \\ \partial_{[\mu} F_{\nu\rho]} = 0 \end{array} \right.$$

where

$$F^{\mu\nu}(x) = \int_{-\infty}^{\infty} d\tau \; f^{\mu\nu}(x,\tau) \qquad \text{and} \qquad A^\mu(x) = \int_{-\infty}^{\infty} d\tau \; a^\mu(x,\tau)$$

Aggregation of all events occurring at spacetime point x over all τ

SHP theory $\longrightarrow$ on-shell Maxwell theory $\sim$ equilibrium limit

Static Limit

Two 'speeds of light' c and c_5 in SHP

Elastic particle-particle and particle-antiparticle scattering depends on $(1 \pm c_5/c)$

Empirical requirement: $0 < c_5/c \ll 1$

Static equilibrium — freeze microscopic system

Slow τ-evolution to zero by taking $c_5/c \to 0$

Homogeneous pre-Maxwell equation imposes condition

$$c_5 \left(\partial_\nu f_{5\mu} - \partial_\mu f_{5\nu}\right) + \partial_\tau f_{\mu\nu} = 0 \quad \xrightarrow[c_5 \to 0]{} \quad \partial_\tau f_{\mu\nu} = 0$$

τ-dependence resides in $\varphi(\tau - \tau_R) \implies \lambda \to \infty \implies \varphi(\tau) \to 1 \, , \, m_\gamma \to 0$

τ-independent fields $\implies$ pre-Maxwell equations $\longrightarrow$ Maxwell equations

Currents concatenate

$$j_\varphi^\alpha (x,\tau) = \int ds \; 1 \cdot j^\alpha (x,s) = J^\alpha (x) \quad \Rightarrow \quad j_\varphi^\mu (x,\tau) = J^\mu (x) \qquad \partial_\tau j_\varphi^5 = 0$$

$$\partial_\mu j_\varphi^\mu (x,\tau) + \frac{1}{c_5} \partial_\tau j_\varphi^5 (x,\tau) = \partial_\mu J^\mu (x) = 0$$