Fisher information and thermodynamic cost of near-equilibrium computation

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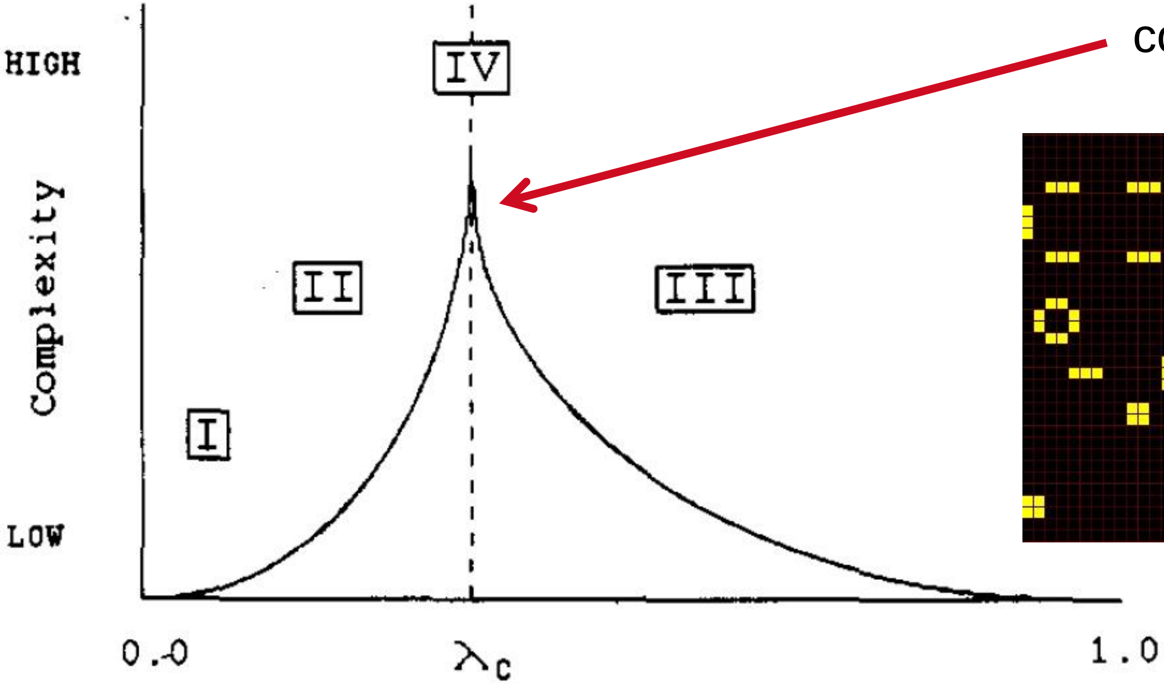
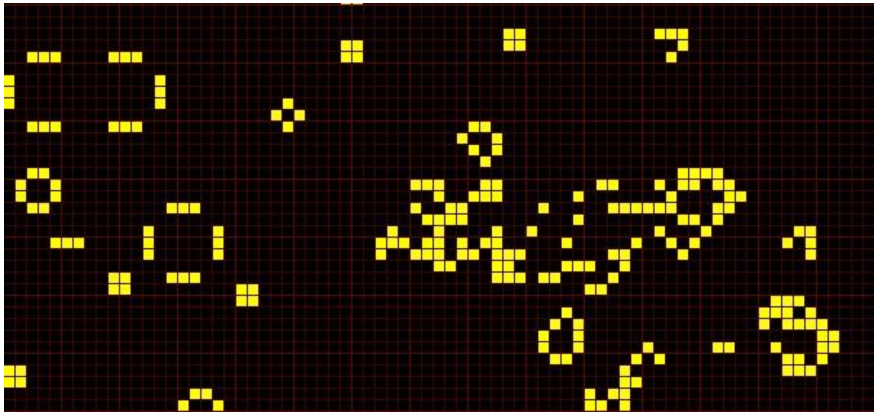
Motivation…

Chris Langton, "Computation at the edge of chaos: Phase transitions and emergent computation" (1991):

- how can emergence of computation be explained in a dynamic setting?
- how is it related to complexity of the system in point?



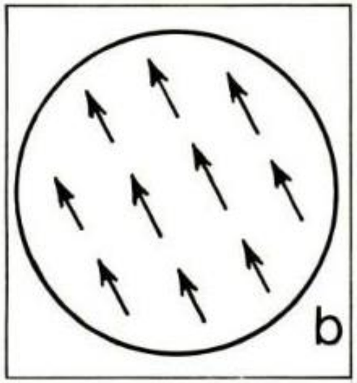
complex high-level structures

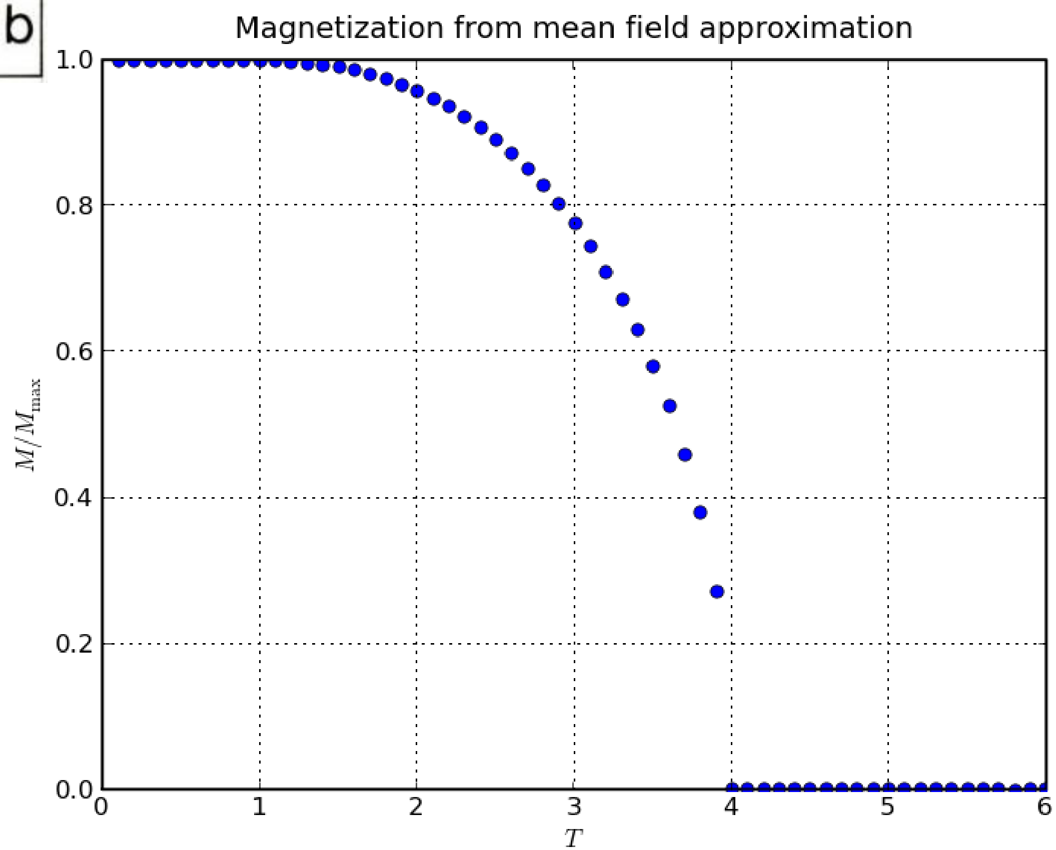


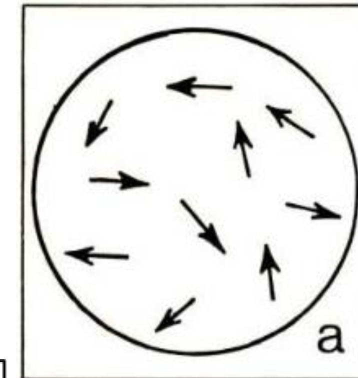
Outline (...back to thermodynamics)

- Edge of chaos, criticality and phase transitions
- Sensitivity of computation (Fisher information)
- Uncertainty of computation (entropy curvature)
- Example: collective motion

Phase transitions and order parameters

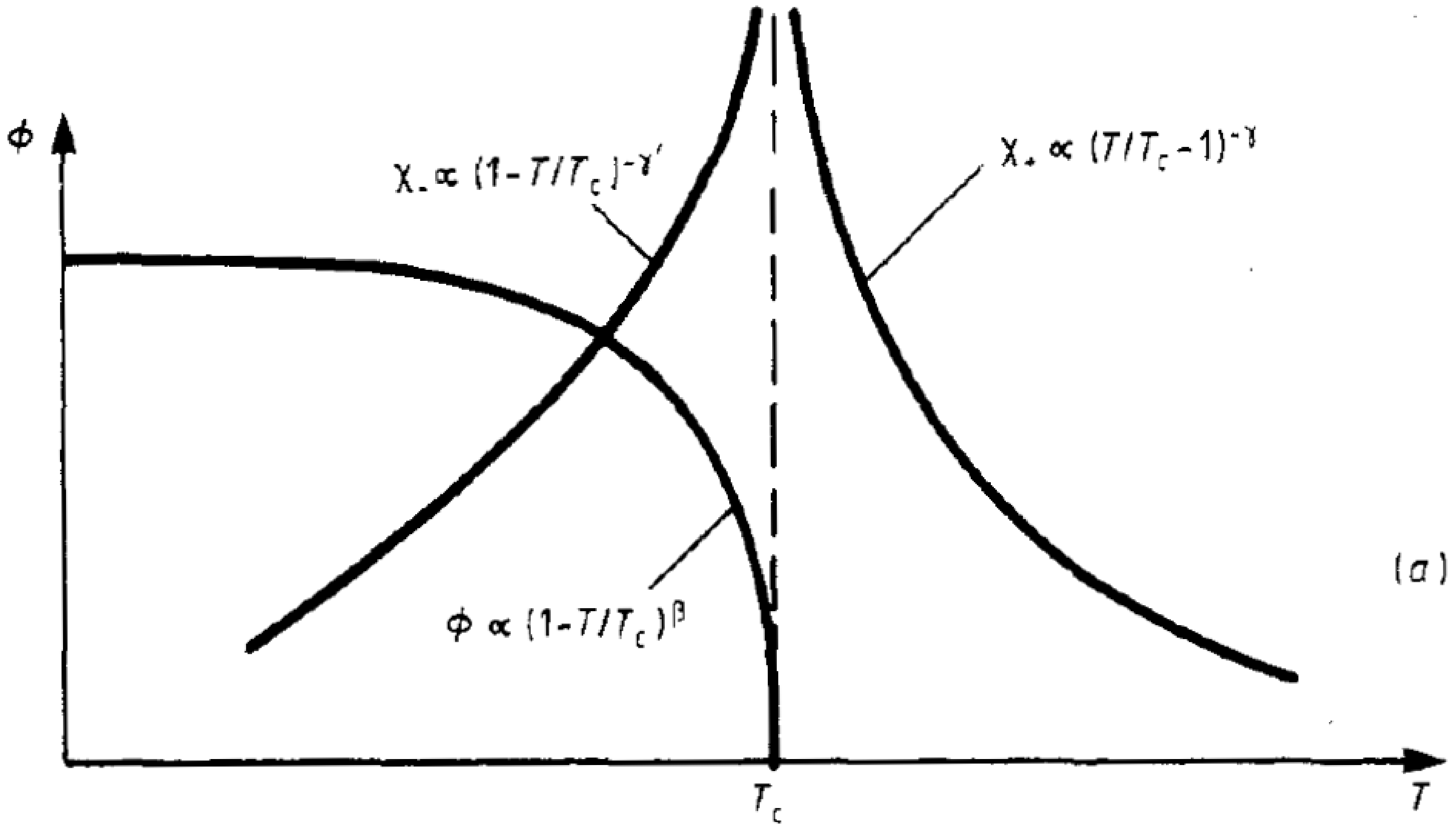




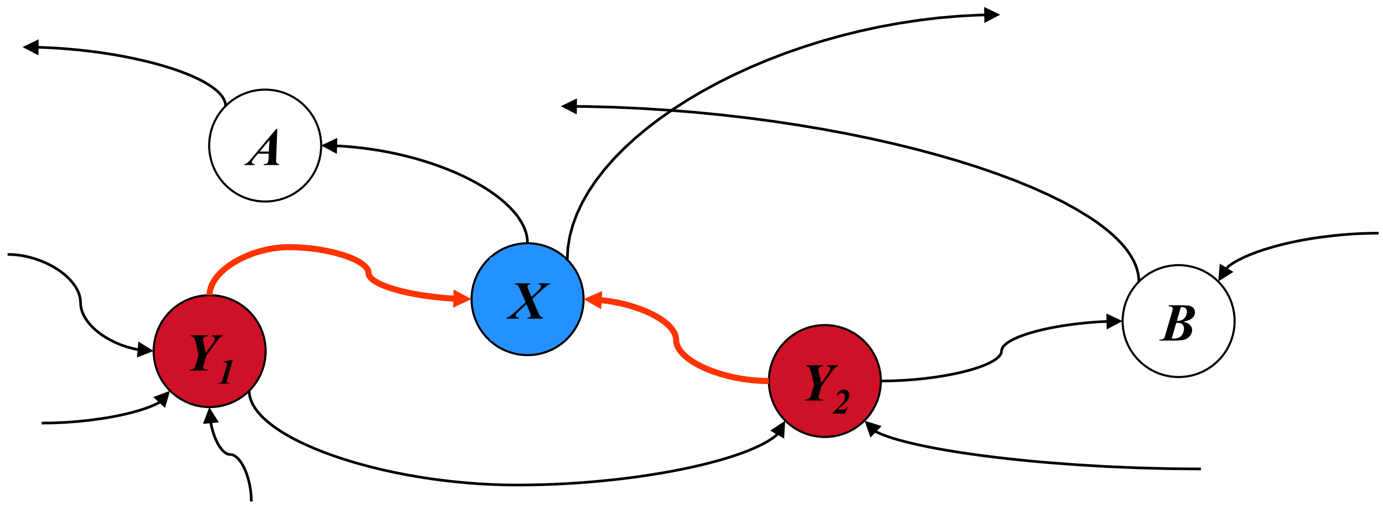


Derivative of order parameter (divergence)

K Binder (1987)



An example: Random Boolean Networks (RBNs)



RBNs have:

- N nodes in a directed structure
- which is determined at random from an average in-degree

$$\overline{K}$$

Each node has:

- Boolean states updated synchronously in discrete time
- update table determined at random, with some bias r

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Random Boolean Networks – phases of dynamics

Ordered

- Low connectivity (small *K*) or activity (*r* close to 0 or 1)
- High regularity of states and strong convergence of similar global states in state space

Chaotic

- High connectivity and activity
- Low regularity of states and divergence of similar global states

Critical

- The "edge of chaos", separating ordered and chaotic phases
- Change at a node in the network spreads marginally
- Compromise between "stability" and "evolvability"
- Given bias *r*, can calculate *K*

$$K_c = \frac{1}{2r(1-r)}$$

Fisher Information: sensitivity

› A way of measuring the amount of information that an observable random variable X has about an unknown parameter θ

$$F(\theta) = \int_x \left(\frac{\partial \ln(p(x|\theta))}{\partial \theta} \right)^2 p(x|\theta) dx$$

• Fisher information is not a function of a particular observation, since the random variable X is averaged out

Fisher Information and order parameters

$$G(T,\theta_i) = U(S,\phi_i) - TS - \phi_i\theta_i$$

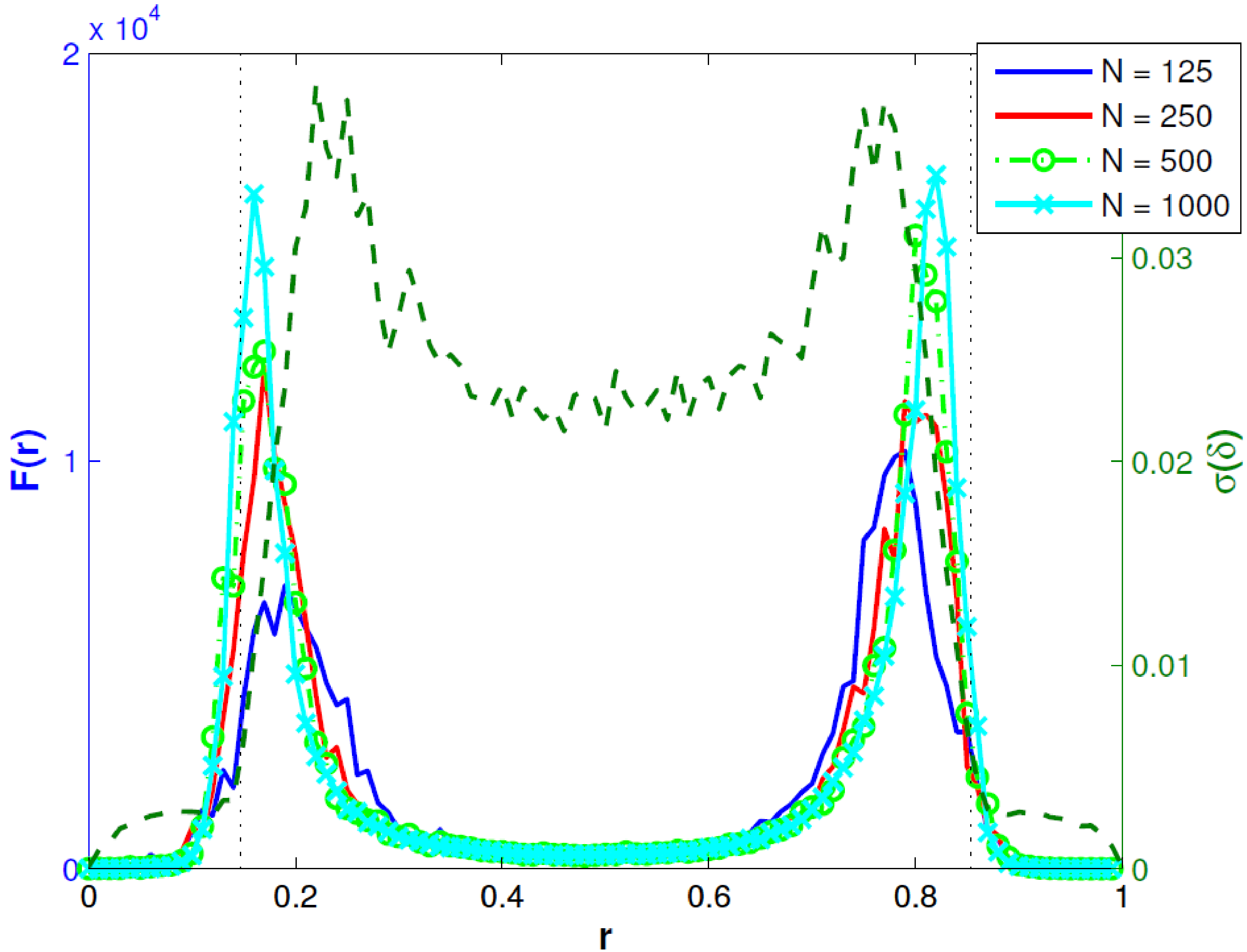
$$F_{ij}(\theta) = \beta \frac{\partial \phi_i}{\partial \theta_j}$$

Fisher information matrix

Rate of change of the order parameter

M. Prokopenko, J. T. Lizier, O. Obst, and X. R. Wang, Relating Fisher information to order parameters, *Physical Review E*, 84, 041116, 2011.

Fisher Information – finite-size RBNs



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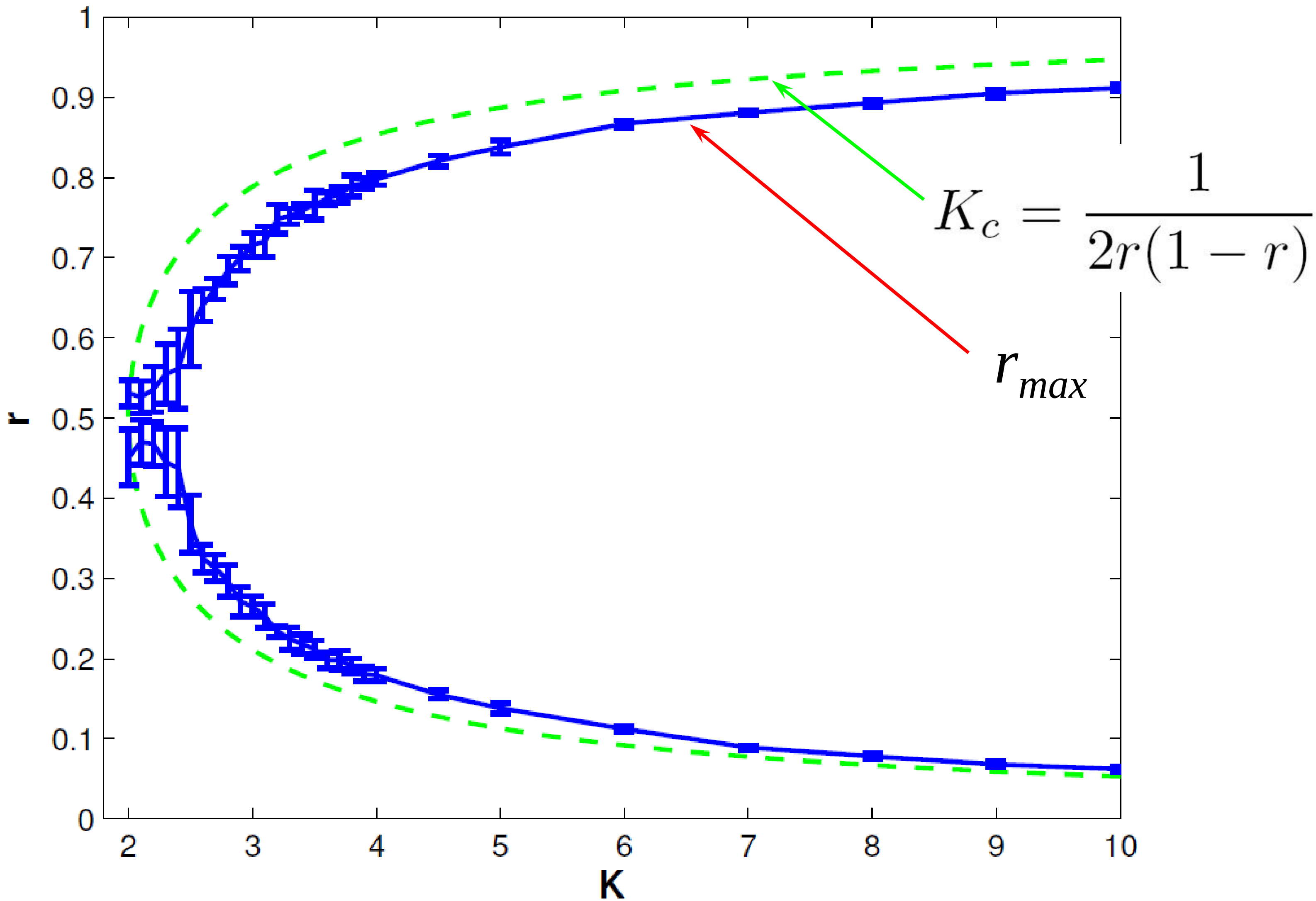
Phase diagram – via Fisher information



A thermodynamic connection between Fisher information and exported entropy

$$G(T,\theta_i) = U(S,\phi_i) - TS - \phi_i\theta_i$$

$$F_{ij}(\theta) = \beta \frac{\partial \phi_i}{\partial \theta_j}$$

$$\Delta\sigma_{\mathrm{exp}} = -d\left(\frac{U(S,\phi_i) - \phi_i\theta_i}{T}\right)$$

$$\frac{\partial \Delta\sigma_{exp}}{\partial \theta_i} = \left(\frac{\partial^2 S}{\partial \theta_i^2} - F(\theta_i)\right) d\theta_i$$

Prokopenko, M., Einav, I. Information thermodynamics of near-equilibrium computation, *Physical Review E*, 91(6), 1-8, 2015.

Difference between two curvatures

$$\frac{\partial \Delta \sigma_{exp}}{\partial \theta_i} = \left(\frac{\partial^2 S}{\partial \theta_i^2} - F(\theta_i) \right) d\theta_i$$

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Generic difference between two curvatures

$$\frac{d^2\mathbb{S}}{d^2\theta} = \frac{d^2 S}{d^2\theta} - F(\theta)$$

$$\frac{\partial \Delta\sigma_{exp}}{\partial\theta_i} = \left(\frac{\partial^2 S}{\partial\theta_i^2} - F(\theta_i)\right) d\theta_i$$

Generic difference between two curvatures

$$\frac{d^2\mathbb{S}}{d^2\theta} = \frac{d^2 S}{d^2\theta} - F(\theta)$$

$\mathbb{S}$ ➤ ?

Generic difference between two curvatures

$$\frac{d^2\mathbb{S}}{d^2\theta} = \frac{d^2S}{d^2\theta} - F(\theta)$$

$$\frac{d^2\langle\beta U_{gen}\rangle}{d\theta^2} = \frac{d^2S}{d\theta^2} - F(\theta)$$

E. Crosato, R. Spinney, R. Nigmatullin, J. T. Lizier, M. Prokopenko, Thermodynamics of collective motion near criticality, 2017.

Revisiting our motivating questions…

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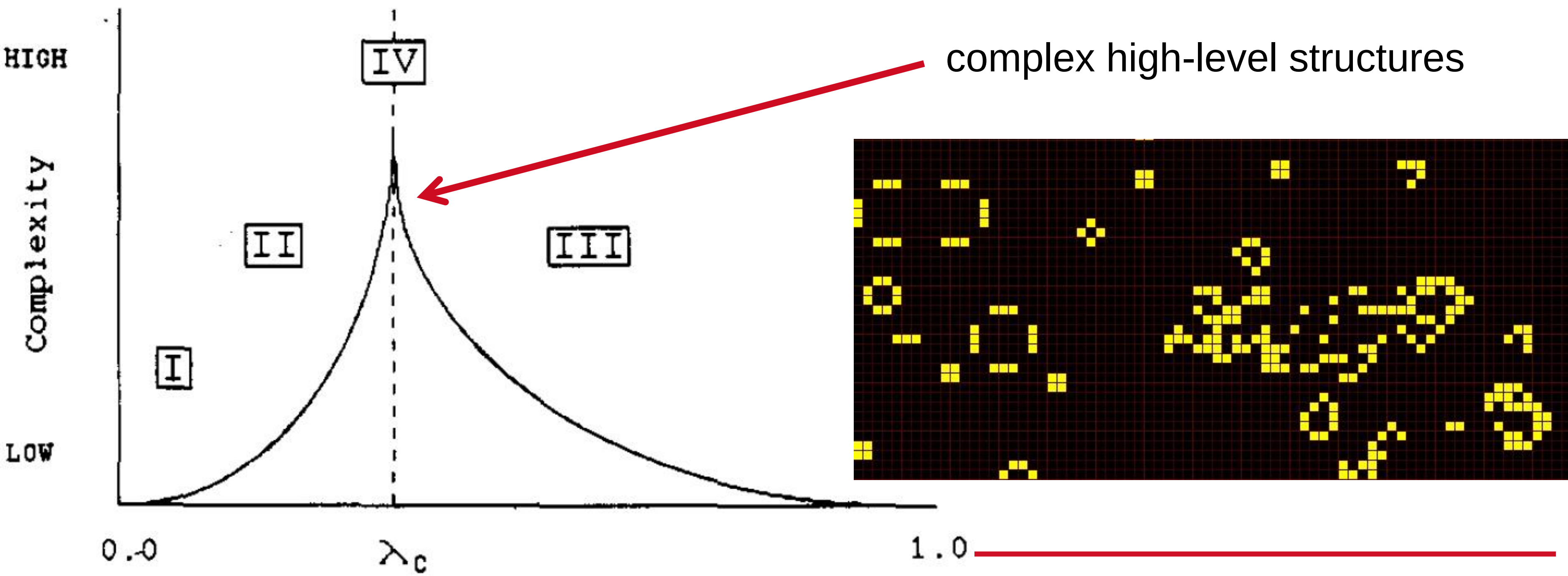
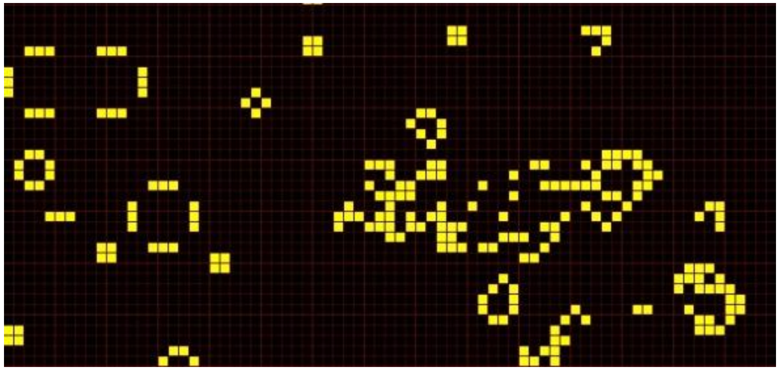
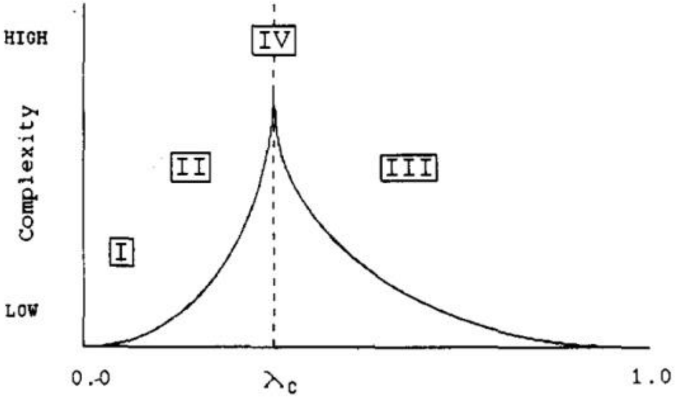


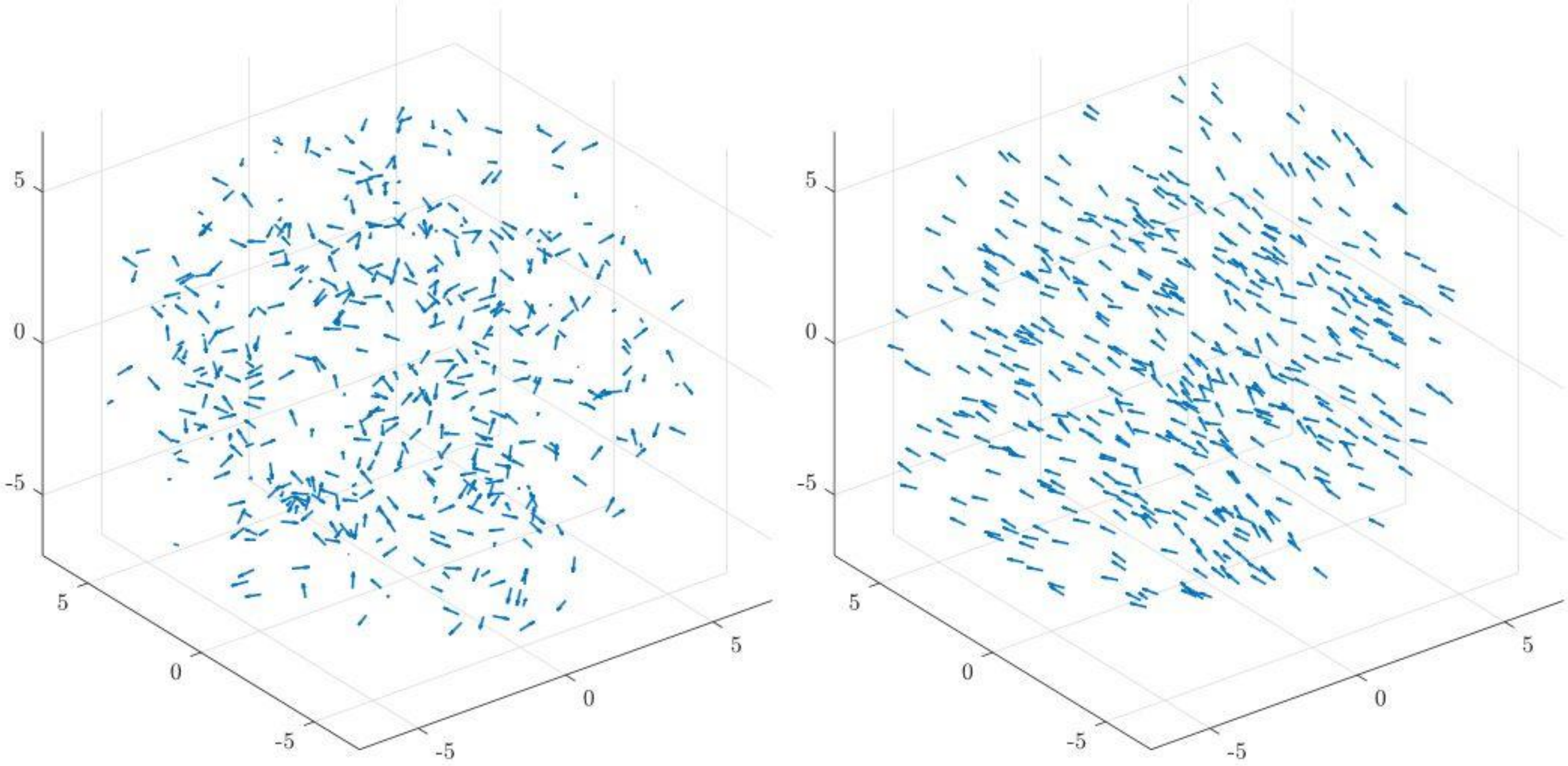
Computation: sensitivity vs uncertainty

$$\frac{d^2\langle \beta U_{gen}\rangle}{d\theta^2} = \frac{d^2 S}{d\theta^2} - F(\theta)$$





Collective motion



(a) $J = 0.001,\ n_c = 20$

(b) $J = 0.2,\ n_c = 20$

E. Crosato, R. Spinney, R. Nigmatullin, J. T. Lizier, M. Prokopenko, Thermodynamics of collective motion near criticality, 2017.

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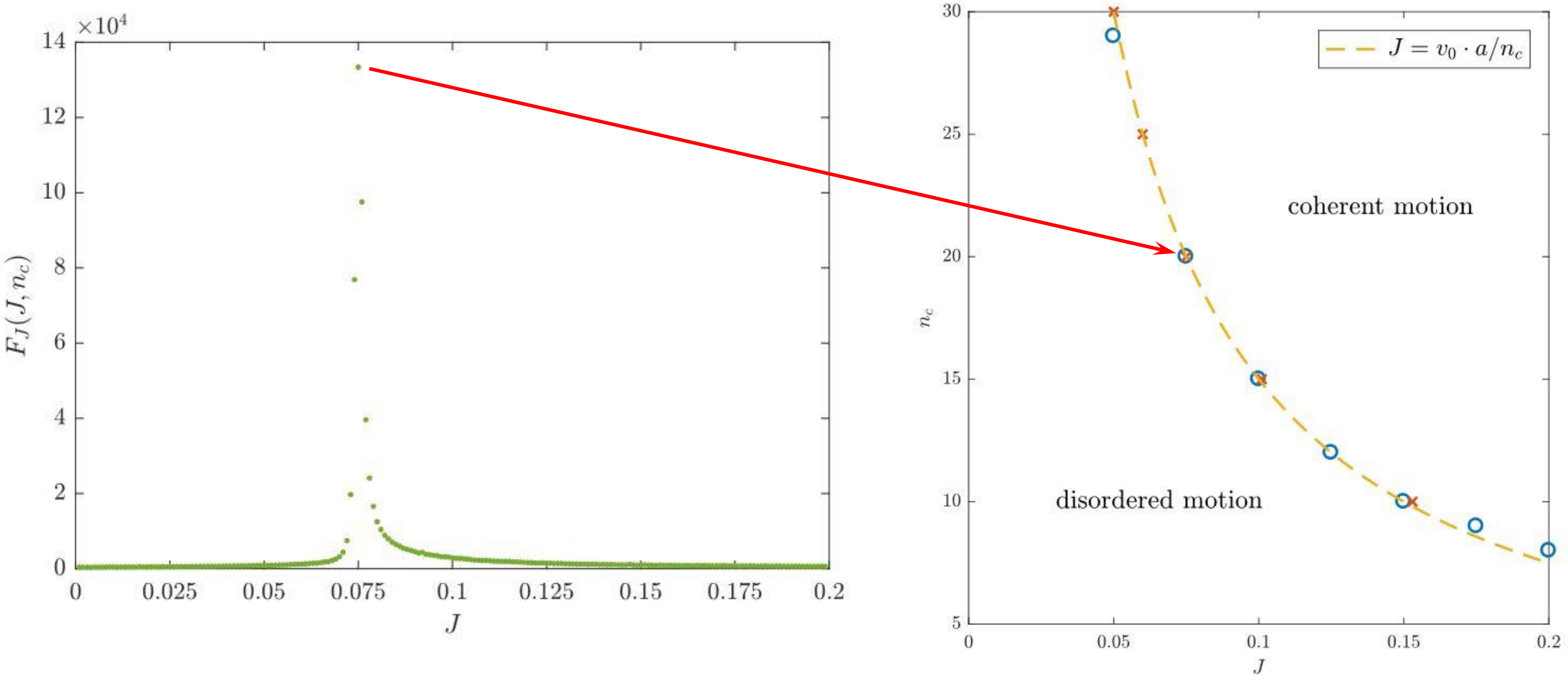
Fisher information in collective motion



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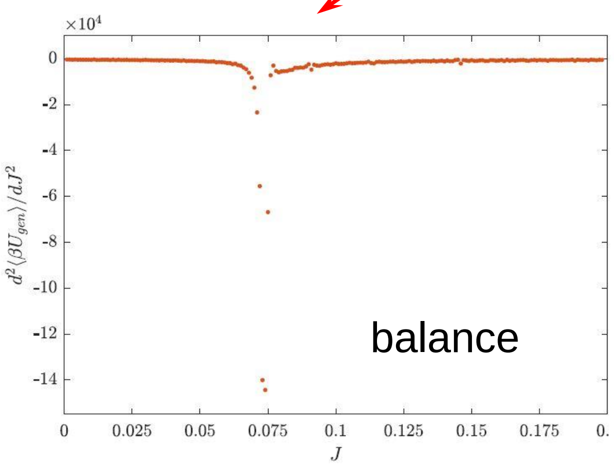
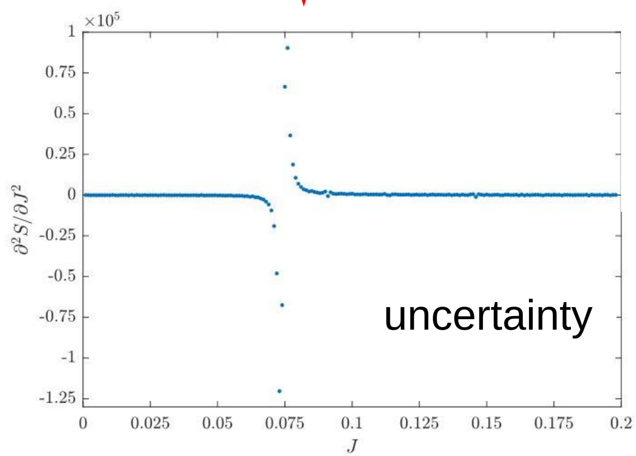
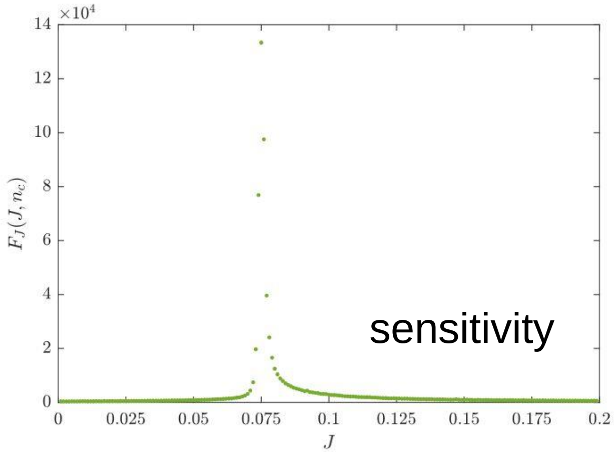
Balance between sensitivity and uncertainty

$$\frac{d^2\langle\beta U_{gen}\rangle}{d\theta^2} = \frac{d^2 S}{d\theta^2} - F(\theta)$$







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Conclusions

- Edge of chaos: balance between order and chaos
- Sensitivity of computation: Fisher information
- Uncertainty of computation: entropy curvature
- Balance: uncertainty vs sensitivity

Thank you! ...MCXS

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