Entropy fluctuations reveal microscopic structures

George Ruppeiner

New College of Florida

18-30 November 2019
5'th International Electronic Conference on Entropy and Its Applications

Here is my talk outline

Entropy fluctuations

↘

Thermodynamic information geometry

↘

Thermodynamic Ricci curvature scalar R

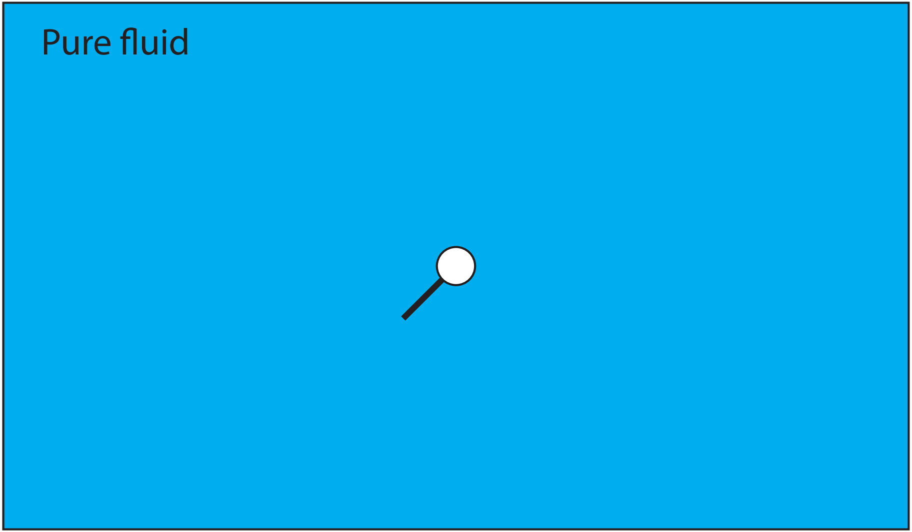
↘

R and interactions at the mesoscale

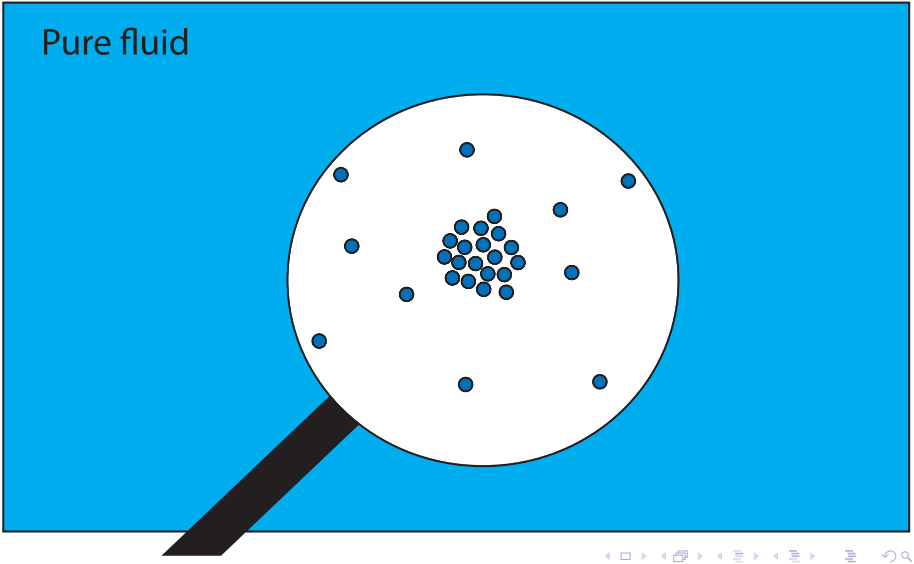
↘

Fluids, spins, and black holes

Uniformity prevails at the macroscopic level

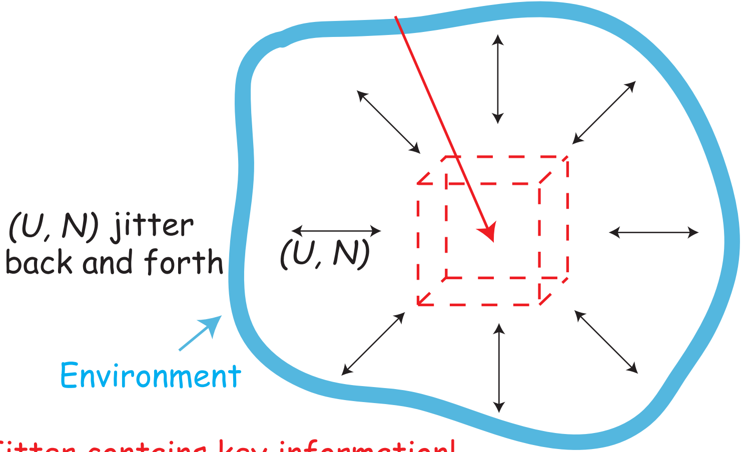


Structure emerges at mesoscopic length scales



The basic structure is well known

open fluid volume V, energy U, particle number N



Jitter contains key information!

Thermodynamic fluctuation theory gives the probability

- Einstein (1904) $(k_B = 1)$

$$\textit{probability} \propto \exp\left(S_{universe}\right).$$

- Expand entropy $S_{universe}$ about its maximum:

$$\textit{probability} \propto \exp\left(-\tfrac{1}{2} g_{\mu\nu} \Delta x^\mu \Delta x^\nu\right),$$

where $(x^1, x^2) = (U, N)$,

$$g_{\mu\nu} = -\frac{\partial^2 S}{\partial x^\mu \partial x^\nu}, \text{ heat capacities, etc.}$$

and S is the thermodynamic entropy.

A thermodynamic information metric results

- $\Delta\ell^2 = g_{\mu\nu}\Delta x^\mu \Delta x^\nu$ is a probability "distance."

- *Greater distance has a less probable fluctuation.*

- This is the entropy metric.

  Weinhold (1975), Ruppeiner (1979)

- Related to Fisher-Rao metric (1945).

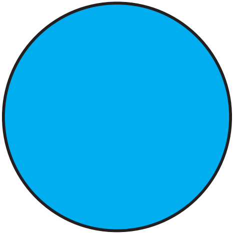
  Brody, Diósi, Dolan, Ingarden, Janyszek, Johnston, Mrugała, Salamon

The Ricci curvature scalar R follows

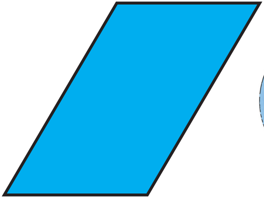
- Metric leads to the curvature scalar R.
- Thermodynamic R has units of volume.
- R is always a feature of a Fisher-Rao metric.
- Physical interpretation requires additional theory.

Ruppeiner (1983), Diósi and Lukáks(1985)

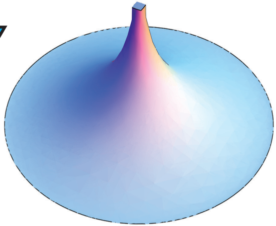
R is a signed quantity



$R < 0$



$R = 0$



$R > 0$

R can be negative, zero, or positive.

I use Weinberg's (1972) sign convention.

R has been calculated in many models

Model	n	d	R sign	$\vert R\vert$ divergence
Ideal Bose gas	2	3	$-$	$T \to 0$
Ising ferromagnet	2	1	$-$	$T \to 0$
Critical regime	2	$\cdots$	$-$	critical point
Mean-field theory	2	$\cdots$	$-$	critical point
van der Waals (critical regime)	2	3	$-$	critical point
Spherical model	2	3	$-$	critical point
Ising on Bethe lattice	2	$\cdots$	$-$	critical point
Ising on random graph	2	2	$-$	critical point
q-deformed bosons	2	3	$-$	critical line
Tonks gas	2	1	$-$	$\vert R\vert$ small
Ising antiferromagnet	2	1	$-$	$\vert R\vert$ small
Ideal paramagnet	2	$\cdots$	0	$\vert R\vert$ small
Ideal gas	2	3	0	$\vert R\vert$ small
Multicomponent ideal gas	> 2	3	$+$	$\vert R\vert$ small
Ideal gas paramagnet	3	3	$+$	$\vert R\vert$ small
Kagome Ising lattice	2	2	$\pm$	critical line
Takahashi gas	2	1	$\pm$	$T \to 0$
Gentile's statistics	2	3	$\pm$	$T \to 0$
M-statistics	2	$2, 3$	$\pm$	$T \to 0$
Anyons	2	2	$\pm$	$T \to 0$
Potts model ($q > 2$)	2	1	$\pm$	$T \to 0$
Finite Ising ferromagnet	2	1	$\pm$	$T \to 0$
Ising-Heisenberg	2	1	$\pm$	$T \to 0$
q-deformed fermions	2	3	$+$	$T \to 0$
Ideal Fermi gas	2	$2, 3$	$+$	$T \to 0$
Ideal gas Fermi paramagnet	3	3	$+$	$T \to 0$
Unitary thermodynamics	2	3	$+$	$T \to 0$

n = number of independent thermodynamic variables, and d = spatial dimension

A number of authors made model calculations . . .

S. Bellucci
J. Chance
B. P. Dolan
D. W. Hook
H. Janyszek
K. Kaviani
R. P. K. C. Malmini
H.-O. May
H. Mohammadzadeh
J. Nulton
H. Oshima
N. Rivier
A. Sahay
T. Sarkar
Z. Talaei

D. Brody
A. Dalafi-Rezaie
H. Hara
W. Janke
D. A. Johnston
R. Kenna
P. Mausbach
B. Mirza
R. Mrugała
T. Obata
A. Ritz
G. Ruppeiner
P. Salamon
G. Sengupta
M. R. Ubriaco

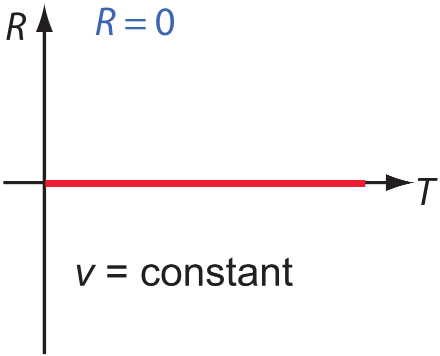
The sign of R characterizes interactions ...

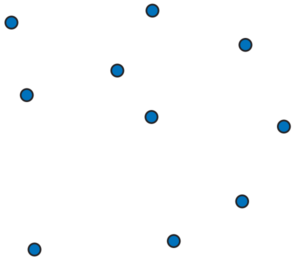
- $R < 0$ for attractive interactions.
- $R > 0$ for repulsive interactions.
- $R = 0$ for the ideal gas (noninteracting).

. . . and $|R|$ measures mesoscopic cluster size

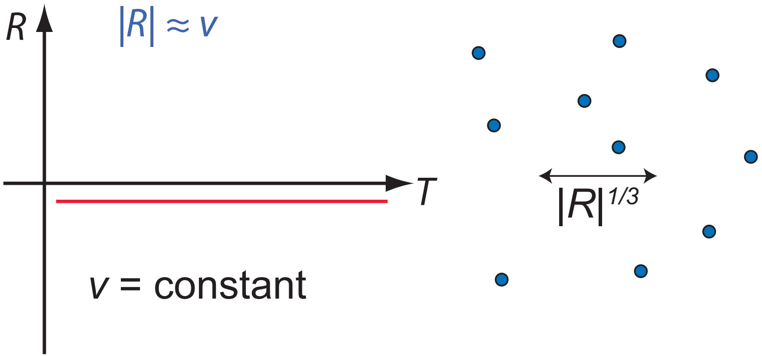
- R diverges at critical points ($R \to -\infty$).
- $|R| \propto \xi^d$, with correlation length ξ.
- $R = -2\,\xi^d$, asymptotically.

(a) the ideal gas shows zero R

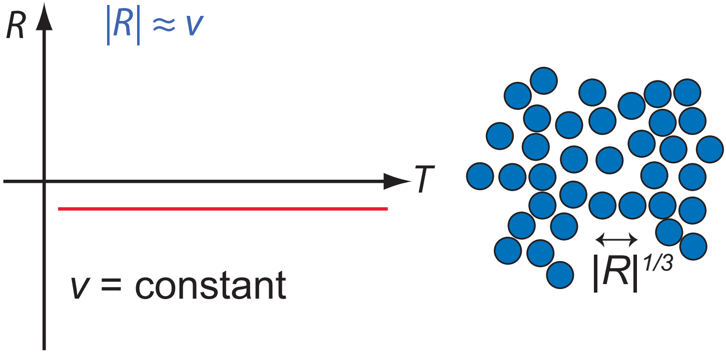




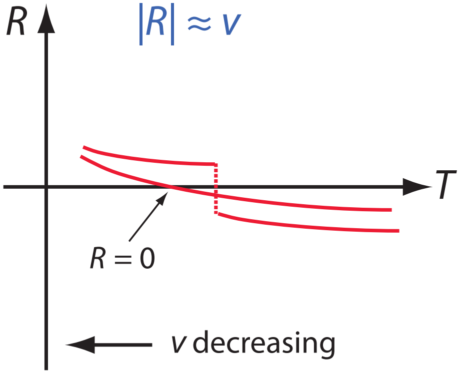
(b) the rare-field gas shows small negative R

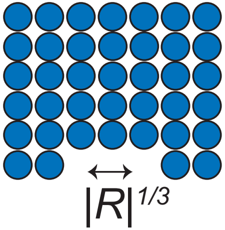


(c) the liquid shows small negative R

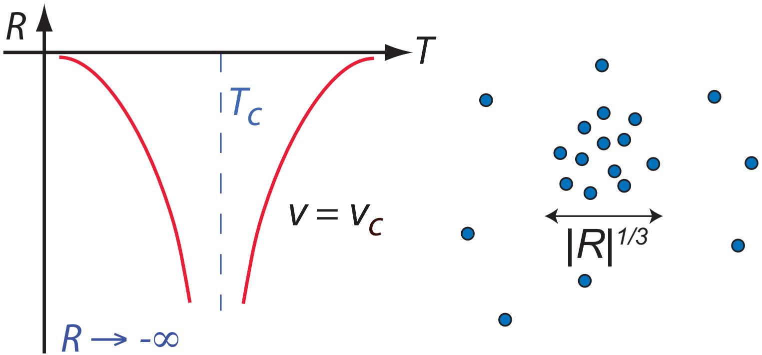


(d) the solid phase shows small positive R

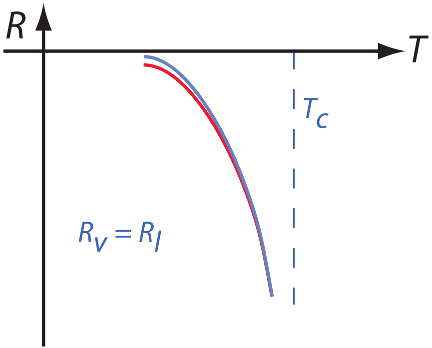


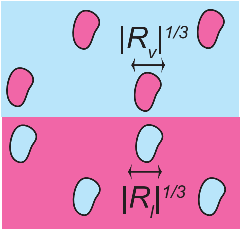


(e) the critical point shows $R \to -\infty$

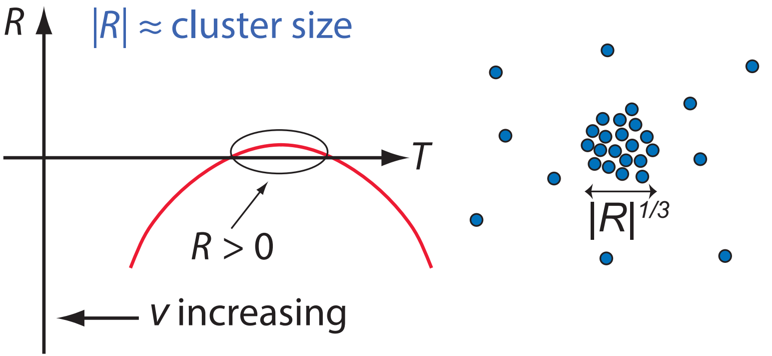


(f) the coexistence curve has equal R's in the phases

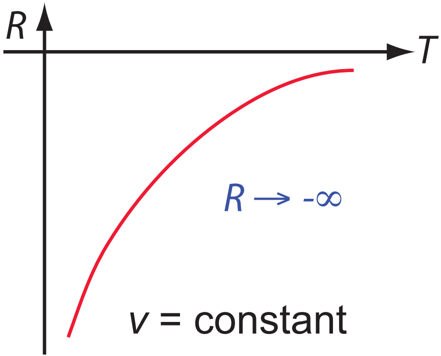


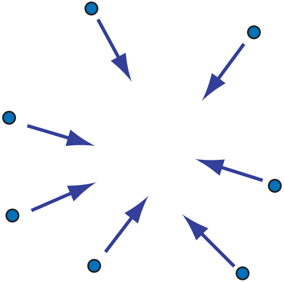


(g) the repulsive cluster, with $R > 0$, is logical

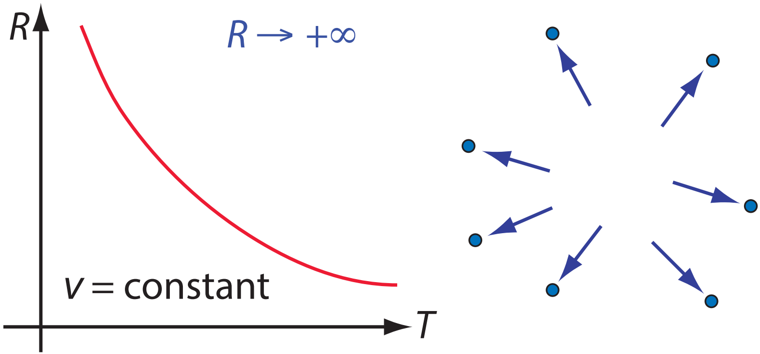


(h) the ideal Bose gas attracts

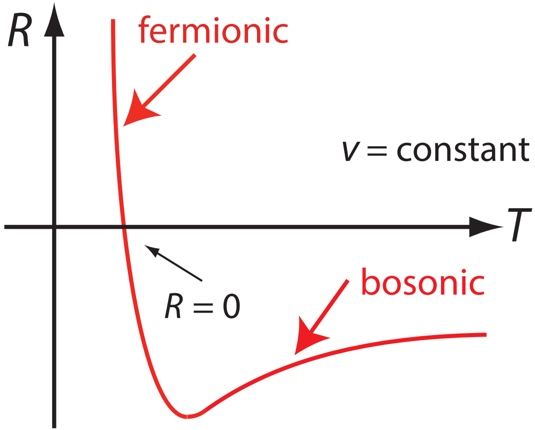




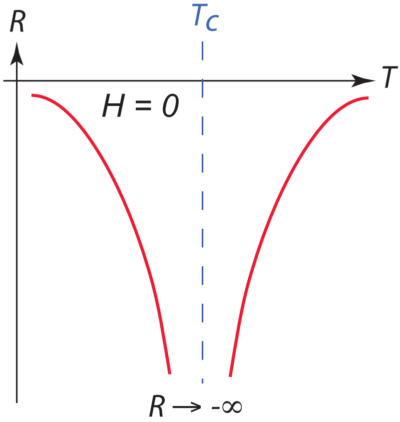
(i) the ideal Fermi gas repels

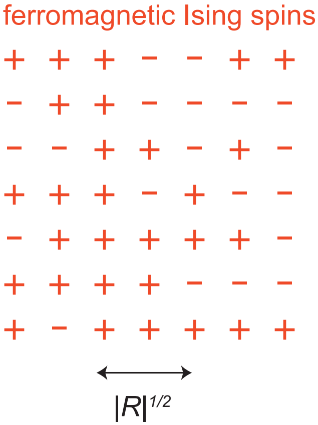


(j) the anyon transition from Bose to Fermi

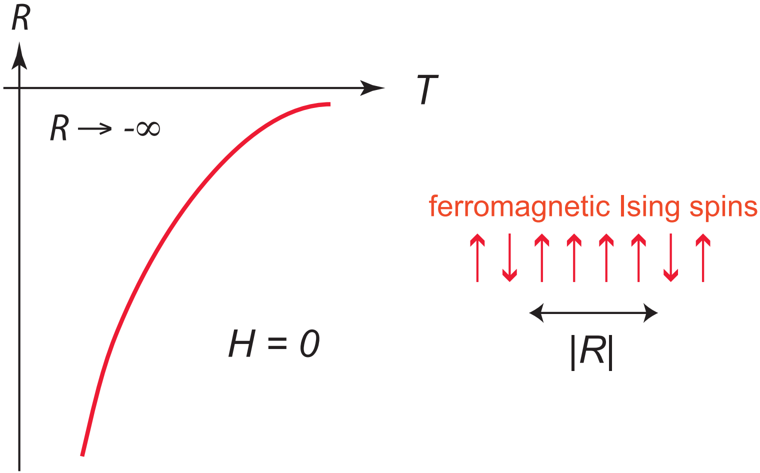


(k) the 2D Ising critical point shows $R \to -\infty$

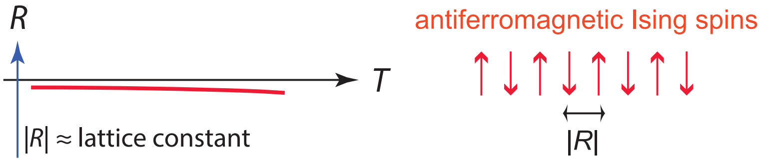




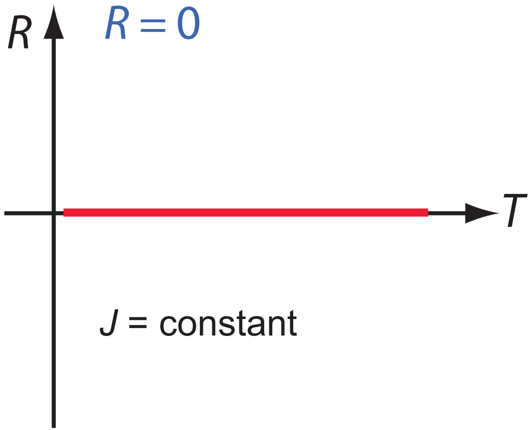
(I) the 1D Ising critical point shifts to $T \to 0$



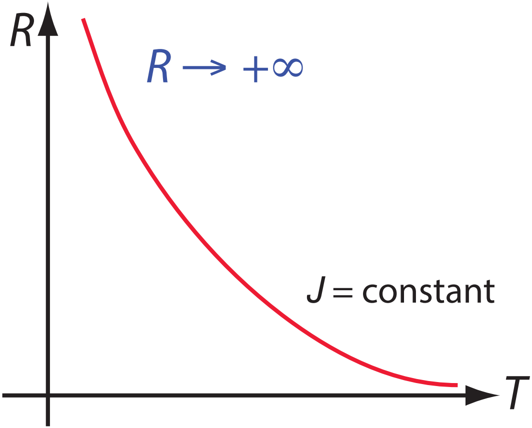
(m) the 1D Ising antiferromagnet looks liquid-like



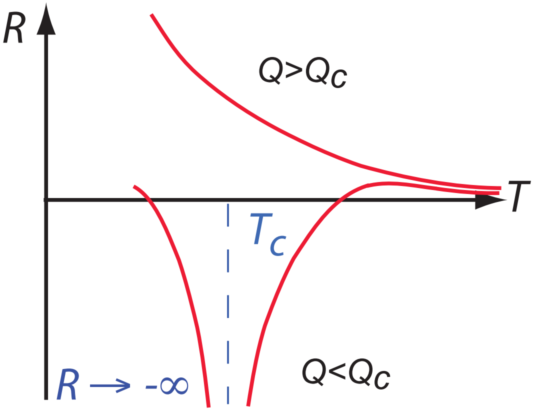
(n) the BTZ black hole looks like an ideal gas



(o) the Kerr black hole resembles Fermi gas as $T \to 0$



(p) the RN-AdS black hole has a critical point



Conclusion: calculate R whenever you can!

- R measures mesoscopic structures naturally.
- Other thermodynamic functions can be useful, but which “are right”?
- R is invariant and universal.
- R is always available!