



Investigation of guided wave interaction with discontinuities in the axisymmetric damped waveguide

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Introduction:

- ✓ SHM and NDE techniques are utilized to monitor wire breakage, and one of the prominent methods is to practice guided ultrasonic wave propagation and acoustic emission (AE) monitoring.
- Use to pitting corrosion and other breakages, the load-carrying capacity of bridge cables is affected. It is of paramount importance for the bridge engineers and stakeholders to monitor the existing infrastructures.
- The axisymmetric semi-analytical finite element (SAFE) method is used to study wave properties of a cylindrical waveguide, especially, a high strength steel wire.
- ✓ This paper discusses the hybrid standard 3D FE and SAFE method.
- In this paper, the numerical modeling of wave scattering by a structural discontinuity (cable damage) in axisymmetric steel wire - viscoelastic waveguide medium for analyzing the wave interaction by inhomogeneity.

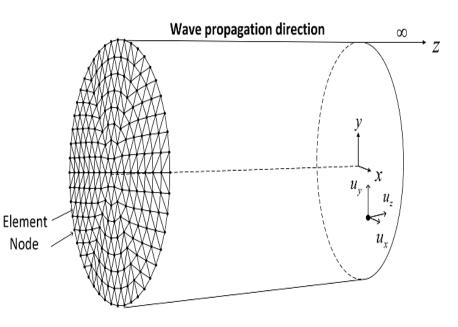
Simulation of scattering and reflection of AE signals

• The mathematical framework for an infinitely long, axisymmetric waveguide immersed in a vacuum is represented using the semi-analytical method.

(Hayashi et al., 2003; Bartoli et al., 2006; Marzani et al., 2008)

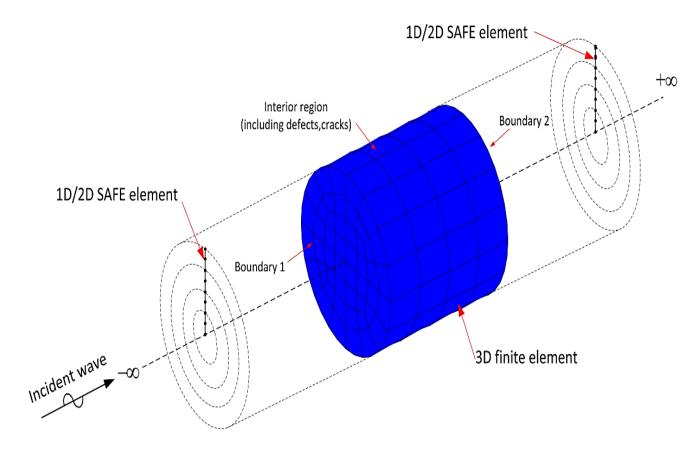
- The method is established in a rectangular Cartesian coordinate system, and the cross-section of the waveguide medium lies in the x-y plane.
- Figure shows a SAFE model representing fluctuations in the temporal and spatial domain along the wave propagation axis z with wavenumber, k, and frequency, ω.

SAFE model



Mathematical framework for the Hybrid 3D FE-SAFE method

- For the area simulated by the SAFE method, either the general section method or the axisymmetric section semi-analytical method can be used.
- An example diagram of the hybrid FE-SAFE, in which composite parts such as defects and cracks need to be included in the 3D-FE area.
- Consider the steady-state response of the 3D-FE area unit under external excitation at a particular frequency.



• Basic structural dynamic formula used, K, M are the general structural stiffness matrix and mass matrix $(K - \omega^2 M)U^{\omega} = F^{\omega}$ U^{ω}, F^{ω} are steady-state structural response & external excitation vectors

Implementation of Hybrid 3D FE-SAFE method

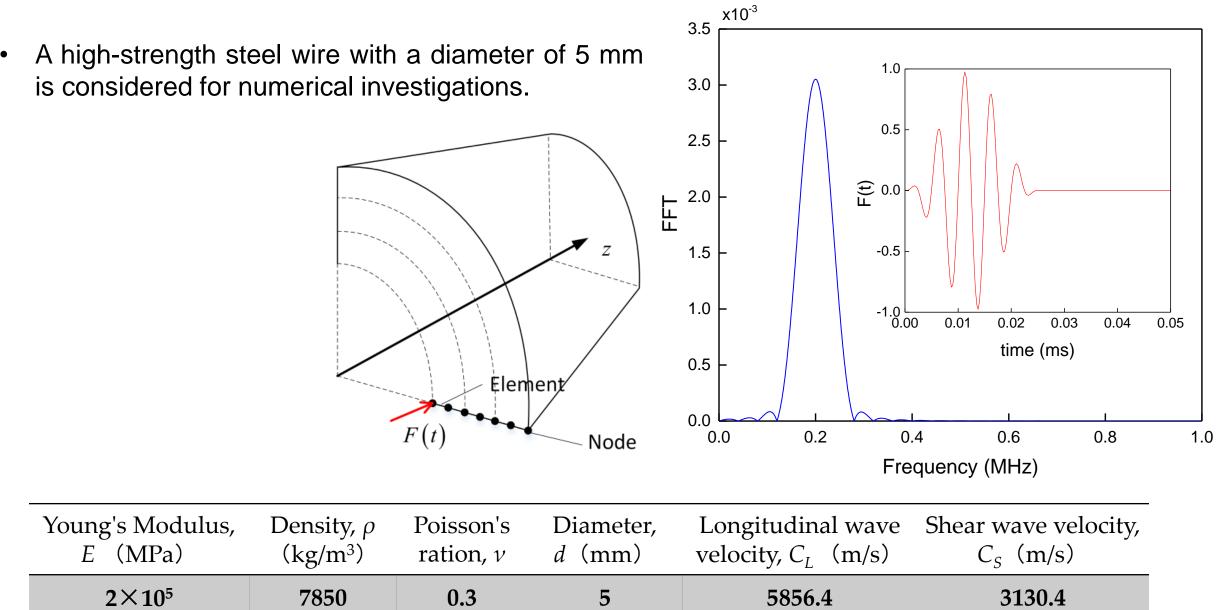
- Theoretically, the modal superposition method needs to be applied to the -∞ to B1 or B2 to +∞ cross-section as per the number of available DOF.
- It is computationally expensive, and most evanescent wave components will attenuate sharply to near zero within the 3D-FE region.
- For the 3D-FE region, the energy carried by the incident wave (input from the B1 section) should be equal to the sum of the energy carried by the reflected wave (output from the B1 section) and the transmitting wave (output from the B2 section).
- The measure of energy can still use the z-direction Poynting vector component. The total energy ratio is defined as,

$$R_{eg} = \frac{\operatorname{Re}(P_{refl}^{z}) + \operatorname{Re}(P_{trans}^{z})}{\operatorname{Re}(P_{inc}^{z})}$$

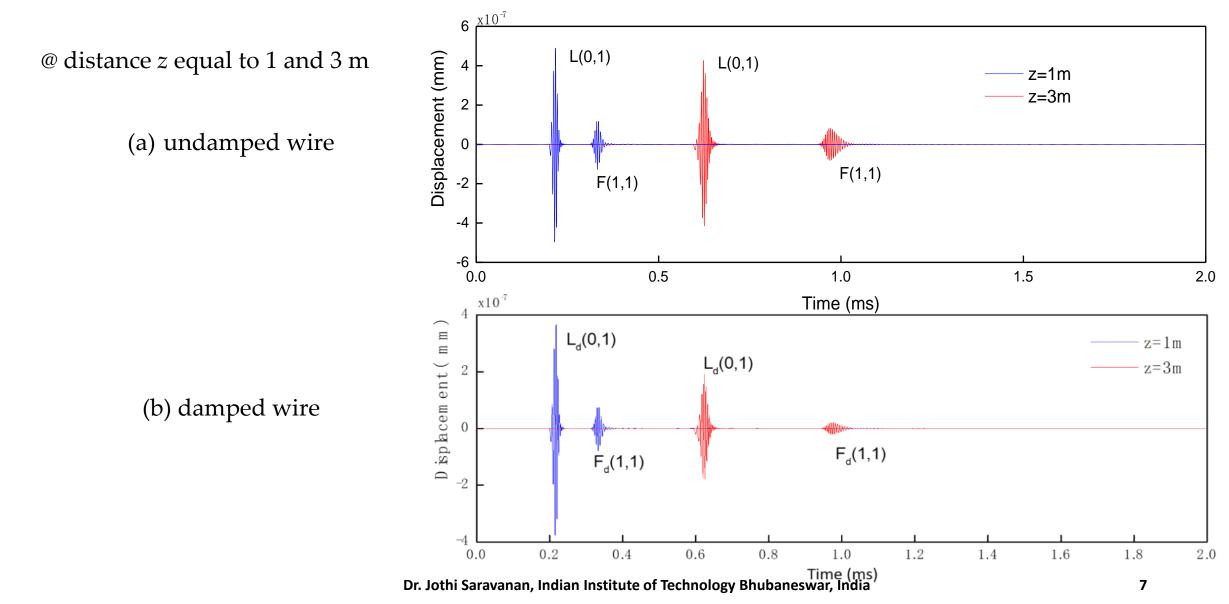
 P_{refl} , P_{trans} , and P_{inc} are the sum of the *z* components of the various modal Poynting vectors. The subscripts *refl, trans,* and *inc* represent reflected, transmitting, and incident wave, respectively.

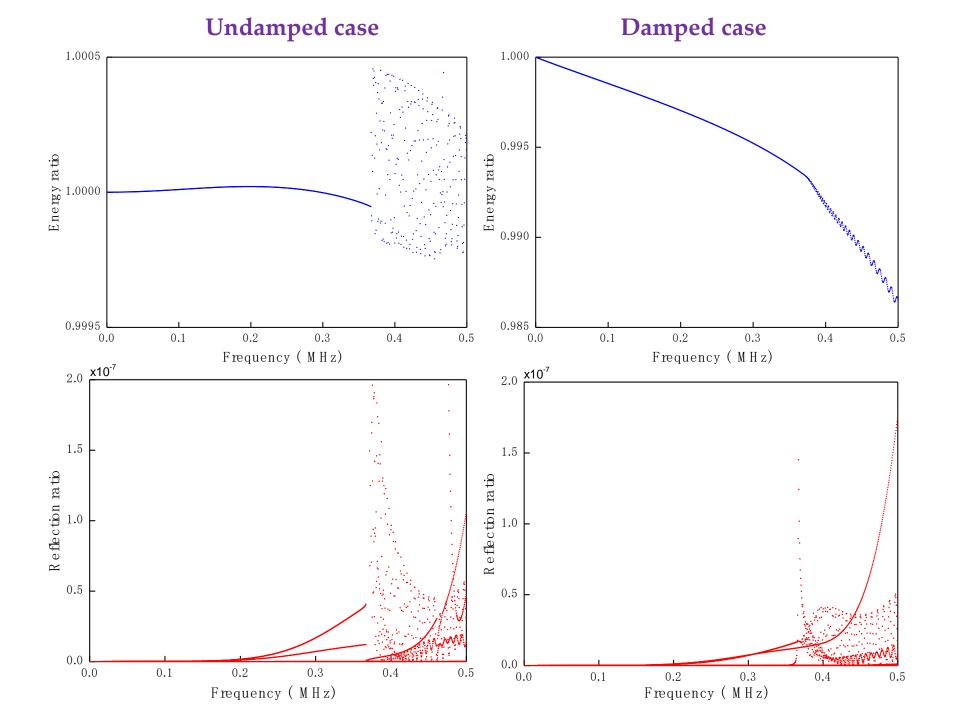
Numerical investigations: Analysis of wave propagation





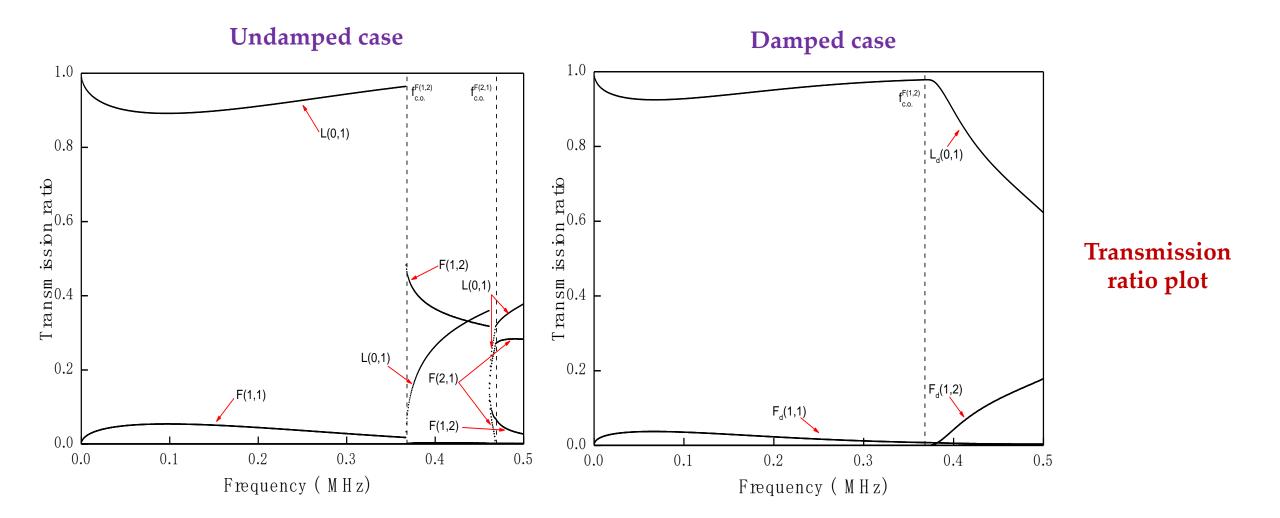
Longitudinal and flexural modes in an undamped and damped steel wire representing axial displacement response





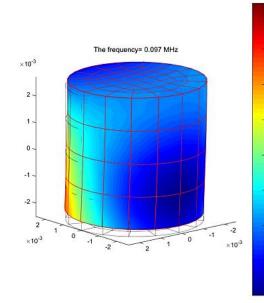
Total energy ratio plot





The curve becomes continuous, and the energy-carrying mode decreases where only L(0,1) and F(1,1) modes appear in the area above 0.37MHz. By considering the material damping, the modes that are initially shown in the undamped case around the cut-off frequency have a huge attenuation factor.

3D-FE area axial-displacement distribution plot at various frequency



At f = 0.097 MHz

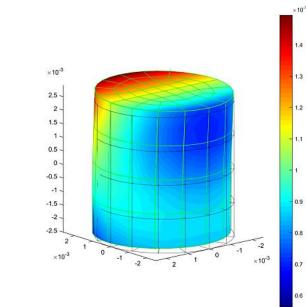
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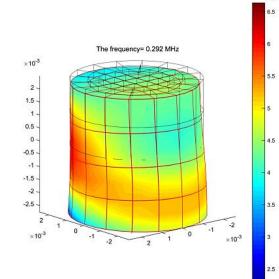
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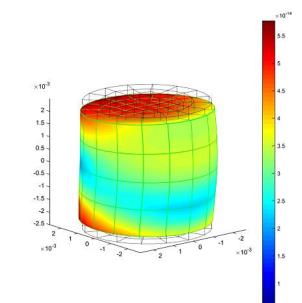
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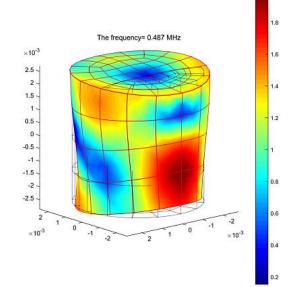
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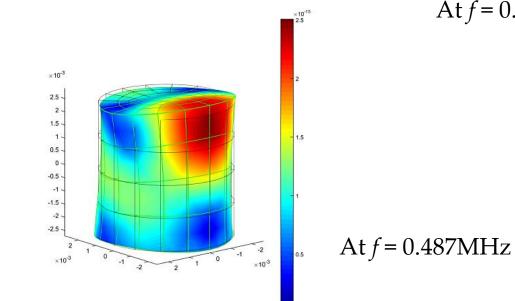




At f = 0.292MHz



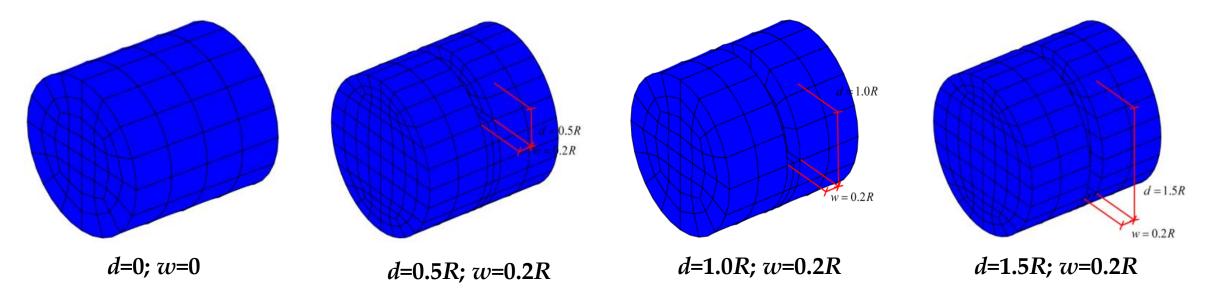
× 10



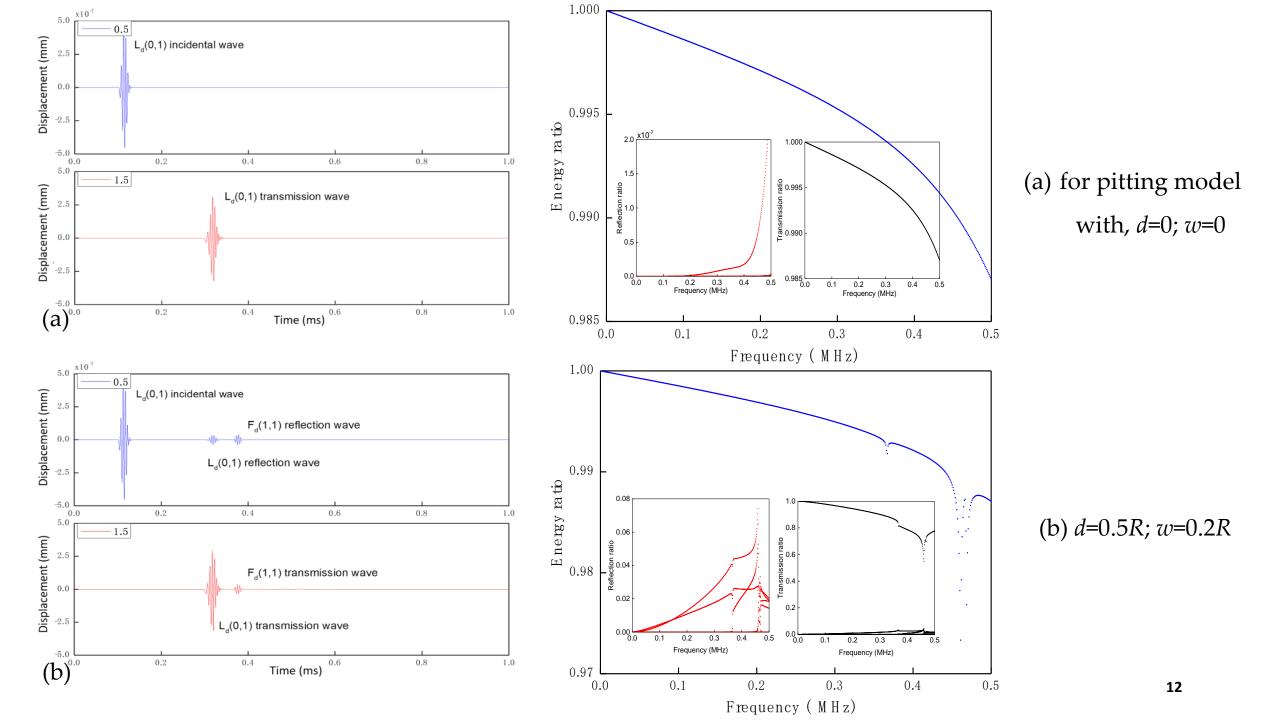
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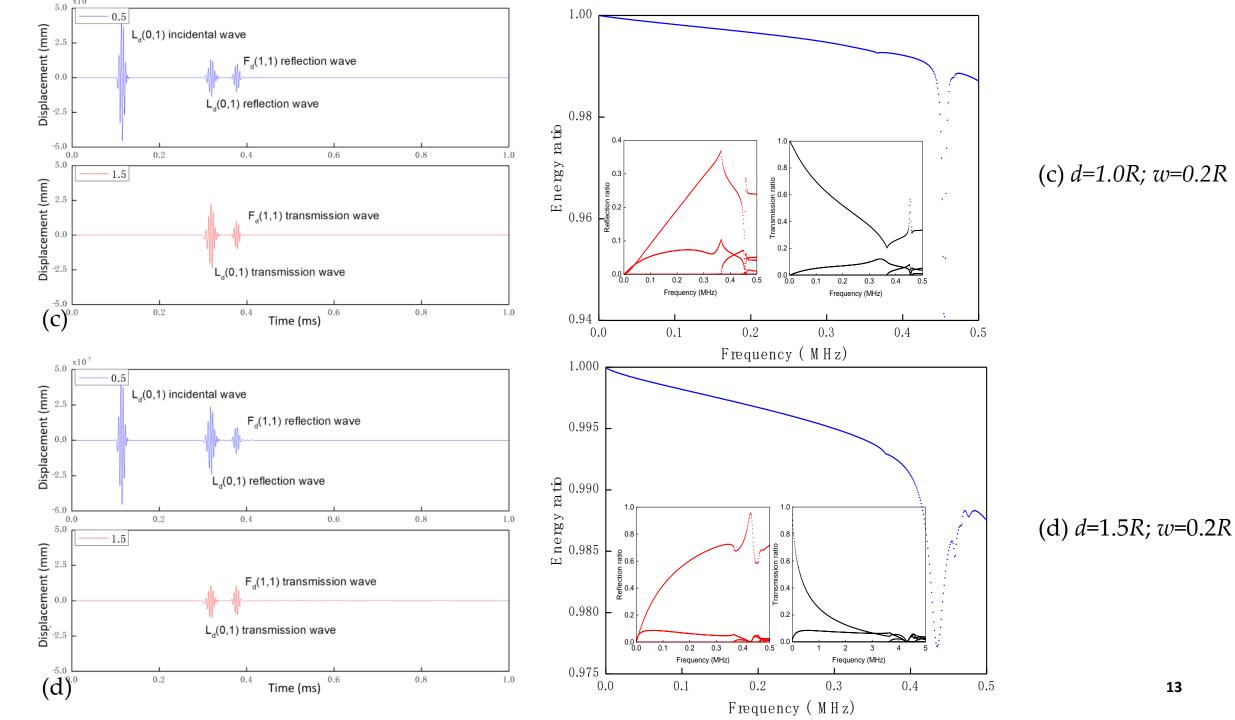
Analysis of wave scattering due to structural discontinuity

• Pitting corrosion models of steel wire for various depths

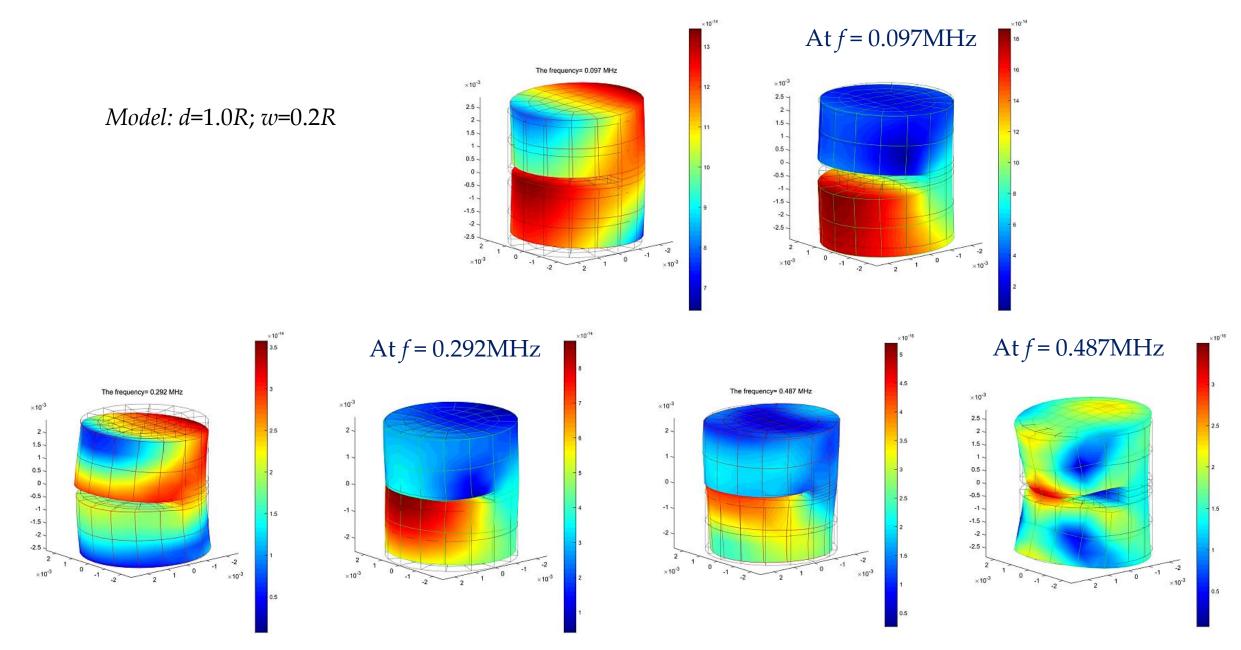


- Axial displacement response at 0.5m and 1m representing incidental reflected and transmitted wave
- Energy ratio plot with power reflection and transmission coefficient





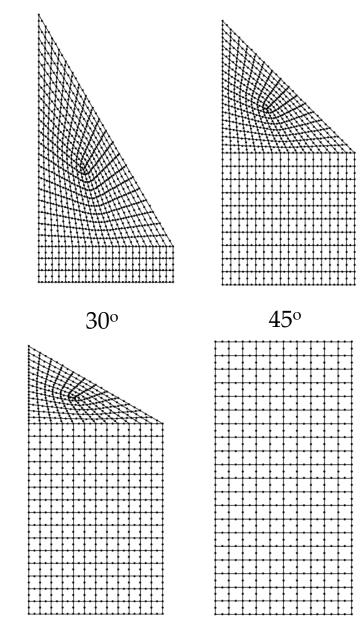
3D FE region displacement response distribution for pitting corrosion model



Analysis of wave reflection

- There are two primary forms of steel wire breakage, namely, (a) due to fatigue fracture, where the broken wire is usually flat; (b) due to corrosion, the cross-section of the wire is gradually reduced.
- If the section is axisymmetric, 3D units can effectively transform the problem to 2D, reducing the calculation cost and improving efficiency and accuracy.
- Therefore, the broken wire sections are all simplified by the axisymmetric tapered section
- The 90° taper angle is the flattened condition of the steel wire caused by fatigue fracture.

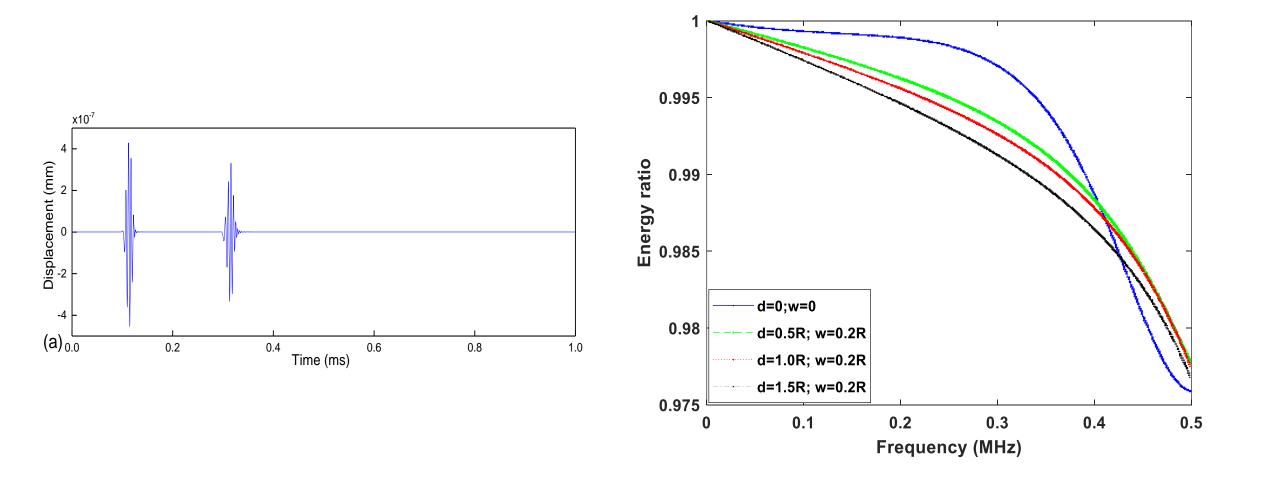
Tapered steel wire model





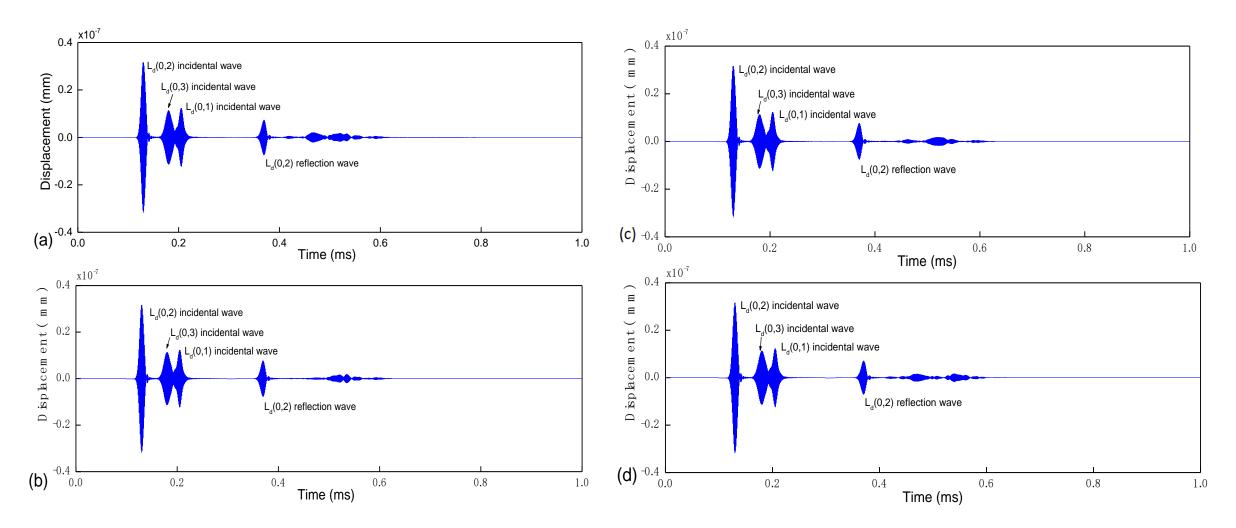


Axial displacement at 0.5m with excitation center frequency at 0.2MHz

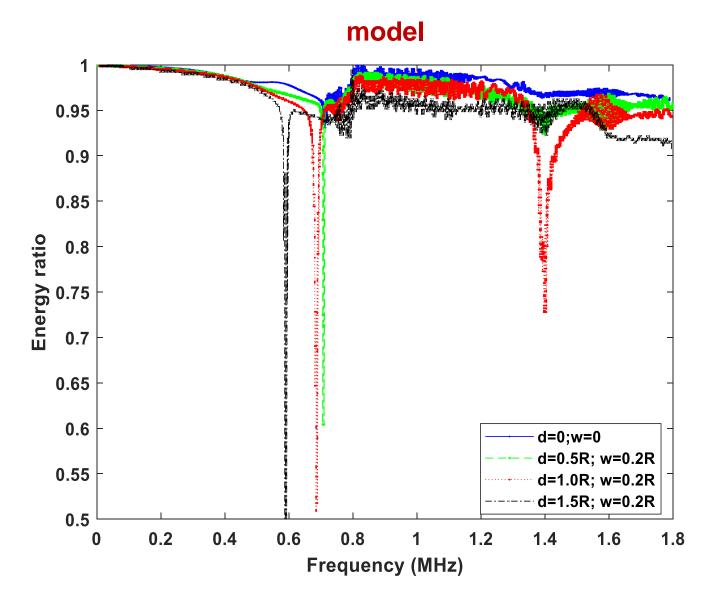


Axial displacements at 0.5m with excitation center frequency,

1MHz for four pitting models (a-d)



Reflected energy ratio results for respective axial displacement for pitting



Conclusions

- ✓ The scattering response calculation relies on the hybrid FE-SAFE method, and frequency domain analysis is used.
- ✓ The basic idea is to express the frequency response of the cross-section at the two ends of the 3D finite element part as a modal superposition calculated by the semianalytical method.
- ✓ It can be observed from the numerical investigations on pitting corrosion and free end reflection that, with considering the material damping, the non-homogeneous part will lead to the transition between the propagation modes and cause additional energy dissipation.
- ✓ For the research of wave reflection and scattering based on the hybrid element method, the algorithm using internal DOF condensation and modal acceleration method can effectively improve the calculation efficiency. It is suitable for models with a large number of DOF in the 3D-FE area.

Thank you for your kind attention