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Fast Biexciton Preparation in a Quantum Dot Using On-Off Pulses †

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Abstract: We study the efficient creation of biexciton state in a quantum dot using laser pulses, when starting from the ground state, and show that a simple on-off-on pulse-sequence can prepare the target state faster than the frequently used constant and hyperbolic secant pulse profiles. The pulse durations in the sequence are obtained from the solution of a transcendental equation. We also use numerical optimization to show that the suggested pulse-sequence creates the biexciton state at the quantum speed limit of the system.

Keywords: semiconductor quantum dots; biexciton; quantum control; coherent control; quantum speed limit

1. Introduction

Controlling with lasers pulses the exciton and biexciton states in semiconductor quantum dots is an important research field which has attracted considerable attention, since these systems offer an advantageous solid state platform for the implementation of quantum information technologies [1]. A basic problem in this area is the efficient generation of the biexciton state, when starting from the ground state of the quantum dot [2–6]. Usually, this transition is achieved by applying a linearly-polarized laser pulse with constant or hyperbolic secant profile [2].

Here we present results which suggest that a simple on-off-on modulation of the pulse shape [7], where the durations of the on-off segments can be found by solving a transcendental equation, can efficiently generate the biexciton state faster than the frequently used constant and hyperbolic secant pulses. Additionally, numerical optimization confirms that the suggested pulse-sequence generates the biexciton state at the quantum speed limit (minimum possible time) of the system, at least for the broad range of maximum pulse amplitude values used in our simulations.

The structure of the paper is as follows. In the next section we briefly describe the basic methodology followed in this research, while in Section 3 we present results which support our statement about the fast preparation of biexciton state. Section 4 concludes this work.

2. Methods

The Hamiltonian of the biexciton system in the dipole approximation is

$$H_B(t) = E|1\rangle\langle 1| + (2E + E_B)|2\rangle\langle 2| - \mu\mathcal{E}_{\mathcal{SQD}}(t)(|0\rangle\langle 1| + |1\rangle\langle 2| + H.c.), \tag{1}$$

where $|0\rangle, |1\rangle$, and $|2\rangle$ denote the ground, exciton, and biexciton states, respectively, E is the exciton energy (the ground state energy is set to zero), E_B is the biexciton energy shift, μ is the dipole moment of the semiconductor quantum dot for both the ground-exciton and exciton-biexciton transitions, and $\mathcal{E}_{SQD}$ is the electric field inside the semiconductor quantum dot

$$\mathcal{E}_{SQD}(t) = \frac{\hbar}{\mu}\left[\frac{\Omega(t)}{2}e^{-i\omega t} + H.c.\right]. \tag{2}$$

The time-dependent Rabi frequency $\Omega(t)$ is the control function used to create the biexciton state starting form $|0\rangle$ and without loss of generality is take to be real, while ω is the angular frequency of the laser.

Using the transformed probability amplitudes $b_0 = c_0, b_1 = c_1 e^{i\omega t}, b_2 = c_2 e^{2i\omega t}$, fixing the laser frequency at the two-photon resonance value $\hbar\omega = E + E_B/2$ and performing the rotating wave approximation, we find the transformed Hamiltonian

$$\tilde{H}_B(t) = \hbar \begin{pmatrix} 0 & -\frac{\Omega*}{2} & 0 \\ -\frac{\Omega}{2} & -\frac{E_B}{2\hbar} & -\frac{\Omega*}{2} \\ 0 & -\frac{\Omega}{2} & 0 \end{pmatrix}. \tag{3}$$

A further transformation

$$a_0 = \frac{1}{\sqrt{2}}(b_2 + b_0), \quad a_1 = b_1, \quad a_2 = \frac{1}{\sqrt{2}}(b_2 - b_0), \tag{4}$$

leads to $\dot{a}_2 = 0$, while a_0, a_1 satisfy the two-level system

$$i\begin{pmatrix} \dot{a}_0 \\ \dot{a}_1 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{\Omega}{\sqrt{2}} \\ -\frac{\Omega}{\sqrt{2}} & -\frac{E_B}{2\hbar} \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix}. \tag{5}$$

The ground state initial conditions $c_0(0) = 1, c_1(0) = c_2(0) = 0$, same for b_i, give $a_0(0) = 1/\sqrt{2}, a_1(0) = 0, a_2(0) = -1/\sqrt{2}$. But $a_2(t) = a_2(0) = -1/\sqrt{2}$ is constant, and from Equation (4) we see that if the control $\Omega(t)$ is selected such that at the final time $t = T$ it is $a_0(T) = -1/\sqrt{2}$, then $b_2(T) = -1 \Rightarrow |c_2(T)| = 1$ and the biexciton state is perfectly prepared. The two-level state $(a_0\ a_1)^T$ is normalized with $1/\sqrt{2}$ instead of the usual 1, thus $a_1(T) = 0$. It becomes obvious that the control $\Omega(t)$ should be chosen such that the initial and final states of the two-level system (5) differ by a π-phase, and this imposes the following condition on the propagator U of the system:

$$U = \begin{pmatrix} -1 & 0 \\ 0 & \text{indif.} \end{pmatrix}. \tag{6}$$

For a constant pulse $\Omega(t) = \Omega_0$ with duration T, propagator U can be easily found to be

$$U = e^{i\omega_B T}\begin{pmatrix} \cos\omega T - in_z \sin\omega T & -in_x \sin\omega T \\ -in_x \sin\omega T & \cos\omega T + in_z \sin\omega T \end{pmatrix}, \tag{7}$$

where

$$\omega = \sqrt{\omega_B^2 + \frac{\Omega_0^2}{2}}, \quad n_x = -\frac{1}{\sqrt{2}}\frac{\Omega_0}{\omega}, \quad n_z = \frac{\omega_B}{\omega}. \tag{8}$$

Using condition (6) we can find the duration and amplitude of the fastest constant pulse which completely generates the biexciton state.

$$T = \frac{\pi}{\omega_B}, \quad \Omega_0^{min} = \omega_B\sqrt{6} \approx 2.45\omega_B. \tag{9}$$

This pulse and the corresponding time-evolution of the biexciton population $|c_2(t)|^2$ are depicted in Figures 1a,b, respectively.

For the hyperbolic secant pulse profile, $\Omega(t) = \Omega_0 \operatorname{sech}(t/t_p)$, with t_p corresponding to the pulse width, we apply the Rosen-Zener method [8] to the two-level system (5), which is also described in Ref. [2]. Following this approach, one can show that the width and amplitude of the shortest hyperbolic secant pulse which achieves complete biexciton preparation are

$$t_p = \frac{\sqrt{3}}{2}\frac{1}{\omega_B}, \quad \Omega_0 = 4\sqrt{\frac{2}{3}}\omega_B. \tag{10}$$

The corresponding pulse is displayed in Figure 1c, and the time evolution of $|c_2(t)|^2$ in Figure 1d.

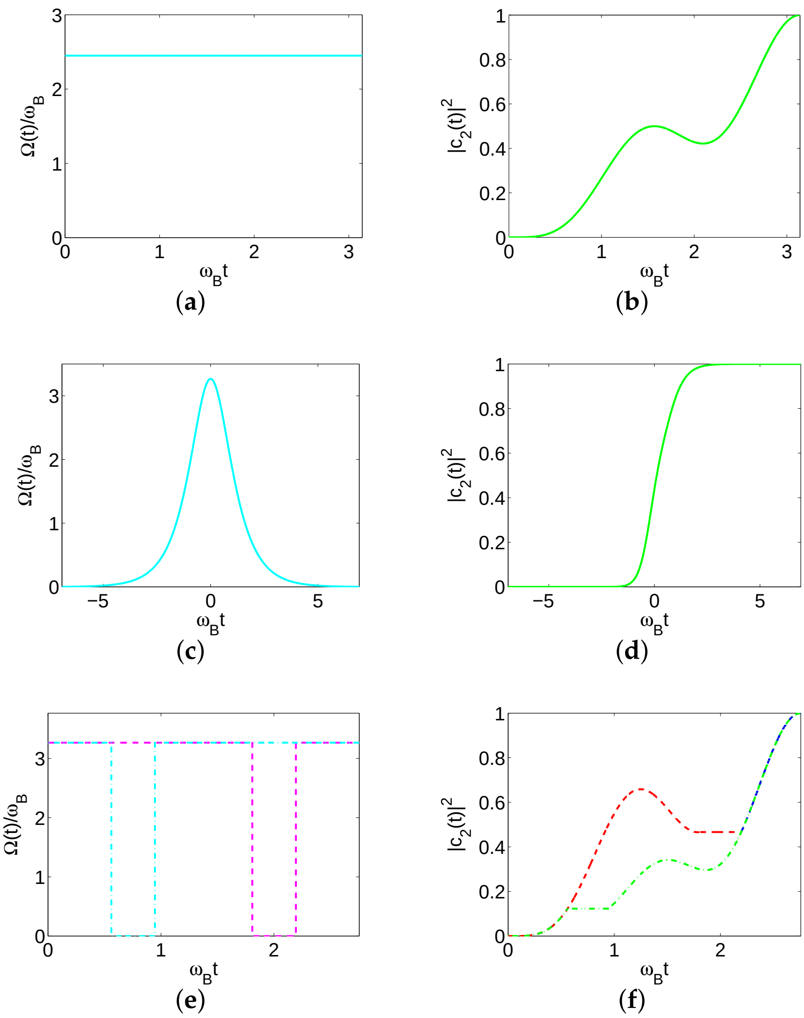

Figure 1. (**a**,**b**) Minimum-duration constant pulse, preparing the biexciton state in time $T = \pi/\omega_B$, and corresponding evolution of population $|c_2(t)|^2$. (**c**,**d**) Shortest-width hyperbolic secant pulse achieving complete biexciton preparation, and corresponding evolution of population $|c_2(t)|^2$. (**e**,**f**) Minimum-time on-off-on pulse-sequences, for the same maximum amplitude as in the hyperbolic secant pulse, and corresponding evolution of population $|c_2(t)|^2$. We plot both pulse-sequences, with interchanged the first and last on-pulses, which accomplish biexciton preparation in the same duration.

3. Results and Discussion

As we previously explained, the two-level system should comes back to its starting state having obtained a π-phase. It turns out that a pulse-sequence of the form on-off-on is the simplest one to achieve this, since the first on-pulse removes the Bloch vector from the north pole, the intermediate off-pulse rotates it parallel to the equator, and the final on-pulse brings it back to the north pole. Let t_i, $i = 1, 2, 3$ be the durations of these pulses. In order to find the minimum possible total duration $T = t_1 + t_2 + t_3$ such that the π-phase condition is satisfied, we follow a new methodology along the lines of Ref. [7], which exploits the fact that in each time interval the Hamiltonian of the two-level system is constant. This way we find that the duration t_2 of the middle off-pulse should satisfy the transcendental equation

$$\tan\left[\omega(\frac{\pi}{2\omega_B} - t_2)\right] = -n_z \tan\omega_B t_2, \tag{11}$$

the durations of the initial and final on-pulses are given in terms of t_2 by the following expressions

$$t_1 = \frac{\pi}{2}\left(\frac{1}{\omega_B} \mp \frac{1}{\omega}\right) - t_2, \quad t_3 = \frac{\pi}{2}\left(\frac{1}{\omega_B} \pm \frac{1}{\omega}\right) - t_2, \tag{12}$$

while the total duration is

$$T = t_1 + t_2 + t_3 = \frac{\pi}{\omega_B} - t_2. \tag{13}$$

In the above relations, ω, n_z are given in Equation (8).

The expressions with the $\pm$ signs in Equation (12) indicate that the durations of the initial and final on-pulses can be interchanged. For the transcendental equation (11) to have at least one solution, the pulse-sequence amplitude should be larger than the threshold value $\Omega_0^{min} = \omega_b\sqrt{6}$, which is found by setting $t_2 = 0$. We focus our attention in the range $\Omega_0 > \Omega_0^{min}$, since such values can be easily obtained in experiments and also lead to durations $T < \pi/\omega_B$, the shorter duration achieved with a constant pulse. Note that for larger Ω_0, Equation (11) can have more solutions, in which case we pick the larger t_2, corresponding to the shorter total duration $T = \pi/\omega_B - t_2$. For very large values of Ω_0, the shortest duration tends to the limiting value $\pi/(2\omega_B)$.

We present a specific example where $\Omega_0 = 4\sqrt{2/3}\omega_B$, the maximum amplitude of the hyperbolic secant pulse profile that was used in the previous section. The corresponding solution of transcendental Equation (11) is $t_2 = 0.3861/\omega_B$, resulting in a total duration $T = \pi/\omega_B - t_2 \approx 2.7555/\omega_B$. In Figure 1e we show the two symmetric pulse-sequences corresponding to the values of t_1, t_3 from Equation (12), with cyan dashed-dotted line for the upper sign (smaller t_1) and magenta dashed line for the lower sign (larger t_1). In Figure 1f we plot the time dependence of biexciton population $|c_2(t)|^2$ for the two cases. Now the biexciton state is prepared in much shorter duration compared to the hyperbolic secant case, shown in Figure 1d, requiring a time of about $4/\omega_B$.

In Figure 2 we show with blue solid line the duration T of the minimum-time on-off-on pulse-sequence for maximum amplitude values in the interval $\sqrt{6} \leq \Omega_0/\omega_B \leq 45$. When $\Omega_0 = \omega_B\sqrt{6}$ then $T = \pi/\omega_b$, while for large Ω_0 the duration tends to $\pi/(2\omega_b)$. We next use the numerical optimal control solver BOCOP [9], to find the quantum speed limit [10] (minimum necessary time) for complete preparation of the biexciton state, for specific representative values of the maximum amplitude Ω_0. In each run the control is bounded as $0 \leq \Omega(t) \leq \Omega_0$. For each value of Ω_0 used, the corresponding quantum speed limit obtained numerically is plotted also in Figure 2 with a red square. From this plot becomes obvious that, at least for amplitudes Ω_0 in the considered range, the speed limit coincides with the duration of the corresponding minimum-time on-off-on pulse-sequence.

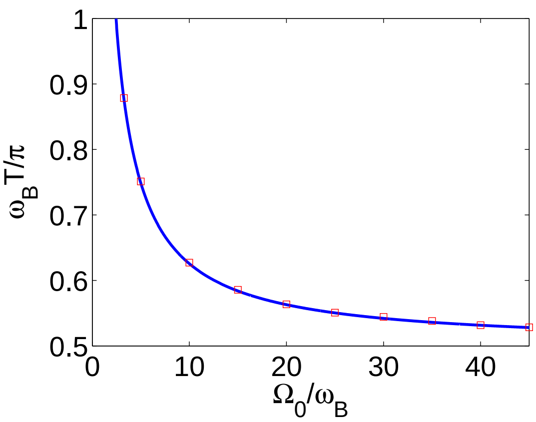

Figure 2. The blue solid line represents the duration of the minimum-time on-off-on pulse-sequence, as a function of maximum amplitude Ω_0. The red squares correspond to the quantum speed limit obtained with numerical optimal control, for various values of Ω_0.

4. Conclusions

We studied the problem of pulsed biexciton state preparation in a quantum dot and showed that, when using a on-off-on pulse-sequence with carefully selected pulse durations, the desired state can be reached faster than with the frequently used constant and hyperbolic secant pulses. Additionally, using numerical optimization, we showed that the suggested pulse-sequence prepares the target state at the quantum speed limit (minimum time needed) for a wide range of the maximum pulse amplitude.

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