

New perspectives for multifrequency GW astronomy: strong gravitational lensing of GW

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Abstract: Direct detection of gravitational waves was for a long time a holy grail of observational astronomy. The situation changed in 2015 with the first ever registration of gravitational wave signal (GW150914) by laboratory interferometers on the Earth. Now, successful operating runs of LIGO/Virgo gravitational wave detectors, resulting with numerous observations of gravitational wave signals from coalescing double compact objects (mainly binary black hole mergers) with the first evidence of coalescing binary neutron star system, elevated multimessenger astronomy to the unprecedented stage. Double compact objects (binary black hole systems, mixed black hole-neutron star systems and double neutron star systems) are the main targets of future ground based and space-borne gravitational wave detectors opening the possibility for multifrequency gravitational wave studies and yielding very rich statistics of such sources. This, in turn, makes possible that certain, non-negligible amount of double compact objects will have a chance of being strongly lensed. In this paper we will discuss new perspectives for future detections of gravitational wave signals in the case of strong gravitational lensing. First, the expected rates of lensed gravitational wave signals will be presented. Multifrequency detections of lensed gravitational wave events will demand different treatment at different frequencies, i.e. wave approach vs. geometric optics approach. New possibilities emerging from such multifrequency detections will be also discussed.

Keywords: gravitational lensing; gravitational waves: sources; gravitational waves: experiments

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1. Introduction

Successful operating runs of LIGO/Virgo gravitational wave (GW) interferometers brought us recently numerous (more than 50 events already observed) direct detections of GW signals from coalescing double compact objects (DCOs). These are overwhelmingly binary black hole (BH-BH) mergers [1] (including GW150914 - the first ever laboratory registration of GW signal [2]) but the first evidence of coalescing binary neutron star (NS-NS) system [3] and probably the first mixed black hole-neutron star (BH-NS) merger [4] has also been recorded. In consequence, multimessenger astronomy has been elevated to the unprecedented stage which is particularly visible in the case of NS-NS coalescence (GW170817) when the electromagnetic (EM) counterpart to GW signal was identified at different EM wavelengths [5,6]. In this light, possibility that data collected by LIGO/Virgo detectors (and recently started KAGRA Observatory) would be in the future supported by new generation of ground-based GW interferometers like Einstein Telescope (ET) [7] and space-borne GW detectors seems to be extremely promising. In fact,

we expect that increased sensitivity of such planned GW detectors will bring us rich statistics and thus big catalog of DCO events up to cosmological distances. Apart of an improved sensitivity, future space missions like LISA [8] and DECihertz Interferometer Gravitational wave Observatory (DECIGO [9,10] and its smaller scale version B-DECIGO [11]) are aimed to probe GW spectrum to frequencies lower than 1 Hz which is inaccessible for ground-based detectors due to irremovable seismic noise. This would allow to observe DCOs in the inspiralling phase for weeks up to years before they will be observable from the ground as entering merger stage. In particular, DECIGO will cover mHz to hHz GW frequencies providing thereby an intersection with low-frequency LISA window and hHz range of ground-based detectors which opens possibility for multifrequency gravitational wave studies. On the other hand, reach statistics of GW events observed by ET or DECIGO allow us to expect that certain, non-negligible amount of DCOs will have a chance of being gravitationally lensed. In this paper we explore this possibility more deeply. In particular, predictions concerning expected detection rates of lensed GW signals from DCO systems in the case of ET and DECIGO/B-DECIGO detectors will be discussed in more details.

2. Method

In order to calculate expected detection rates for unlensed and lensed DCO systems in the case of planned ET and DECIGO/B-DECIGO missions it is necessary to investigate firstly two main components underlying our studies: DCO merger rates and detector sensitivity. At this point we would like to emphasize that our calculations are based on the procedure which will be only briefly summarized here. More informations concerning the method presented in this section can be found in our previous papers [12,13,14].

2.1. DCO merger rate

Intrinsic DCO merger rate (i.e. intrinsic inspiral rates as a function of source redshift) can be calculated on the basis of the source evolution modeled according to the analytical formula (valid only for NS-NS systems; see [12] and references therein). We decided however, for a different approach in which we take values of intrinsic inspiral rates as forecasted with using StarTrack code [15] (data available at <https://www.syntheticuniverse.org>) for each of DCO type and for a given redshift slice from $z = 0.04$ to $z = 17$. This detailed population synthesis calculations are based on several thoroughly motivated assumptions concerning star formation rates, galaxy mass distributions, stellar populations and metallicities as well as on two galaxy metallicities evolution scenarios with redshift (assigned in [15] as "high-end" and "low-end") reflecting varied chemical composition of the Universe. DCO formation depends on several uncertain to some degree processes like physics of common envelope phase of binary system evolution or supernova explosion mechanism. Thus, StarTrack population synthesis calculations has been performed for four scenarios: standard and three of its modifications (optimistic common envelope, delayed supernova explosion and high BH natal kicks; more details can be found in [13,14,15]).

2.2. Detector sensitivity

Second question which must be taken into account prior to calculations concerning DCO detection rates is sensitivity of a particular detector for

which analysis is performed. ET and DECIGO design is planned to be based on the triangular geometry of interferometers. ET is proposed as three nested detectors forming equilateral triangle with arm lengths of 10 km within underground infrastructure [16]. Contrary, DECIGO will be composed of four clusters putted in the heliocentric orbit where each cluster will consist of three drag-free spacecrafts in equilateral triangle configuration with arm lengths of 1000 km [17]. Even modest project B-DECIGO (in comparison to the original design of DECIGO) still makes a spectacular impression: it will consist of three satellites forming 100 km equilateral triangle [11,18]. An increased size of interferometer length arms and planned implementation of several new technologies will translate into a greatly improved sensitivity of ET or DECIGO also according to property (see [17]) that a single triangular detector unit is equivalent for two standard L-shaped interferometers rotated by 45°. Detector noise power spectrum allows to estimate the so-called characteristic distance parameter r_0 of a given detector - a distance which can be probed by a given detector with a given sensitivity. Polynomial approximation to the ET noise curve in the initial configuration gives $r_0 = 1527$ Mpc (and may be increased to $r_0 = 1918$ Mpc for advanced "xylophone" configuration) [16]. On the other hand, DECIGO and B-DECIGO noise spectrum [17,18,19] gives estimated values of characteristic distance parameters respectively: $r_0 = 6709$ Mpc and $r_0 = 535$ Mpc, which means that DECIGO in full original design would be able to probe about 64 times larger volume than ET (see discussion in [14]).

2.3. Detection rates for unlensed events

The main goal of our studies is to make predictions concerning rates at which ET and DECIGO/B-DECIGO would be able to observe gravitationally lensed DCO sources. As a first step we need to estimate yearly detection rates of unlensed GW sources originating at redshift z_s and producing signal with S/N ratio exceeding detector's threshold $r_0=8$ [12,13,17,19]. For this we consider matched filtering S/N ratio for a single detector according to [20] which allows to take into account in our calculations usually non-optimal random orientation of DCO system with respect to the detector. Another problem is that the confusion noise from unresolved binaries may severely deteriorate ability of a given detector to see GW signals from DCO systems. Our calculations concerning this issue (see detailed analysis supported by Figure 1 in [14]) show that DECIGO sensitivity will be significantly affected mainly by unresolved BH-BH systems (B-DECIGO will be affected much less) and thus we have to modify DECIGO (and B-DECIGO) noise spectrum such that it will include confusion noise from unresolved systems separately for each kind of DCOs and for each binary system evolution scenario considered in this work according to StarTrack simulations. This procedure leads to the effective change of characteristic distance parameter r_0 values for DECIGO and B-DECIGO (differently for each DCO type and evolutionary scenario). It should be noticed here that all our estimations described in this and the next section have been made

within flat Λ CDM cosmology with $H_0 = 70 \text{ kms}^{-1}\text{Mpc}^{-1}$ and $\Omega_m = 0.3$, which is compatible with the assumptions underlying StarTrack population synthesis simulation.

2.4. Detection rates for lensed events

Similarity of mathematical description between GWs and EM waves (i.e. Einstein field equations in terms of metric perturbations within weak field regime vs EM field equations in the Lorentz gauge) leads to resemblance in their properties, which in turn allows us to expect that GWs should experience the same geometric-optics effects as EM waves. In particular, GWs also travels along null geodesics and thus we expect that they should undergo gravitational lensing phenomenon. Strong gravitational lensing occurs when the light emitted from distant source encounters massive object (galaxy or clusters of galaxies) on its way to an observer on the Earth. Consequently we observe multiple, magnified and distorted images of the source. In addition, non-zero time delay between images occurs as a simple result of two phenomena: geometrical difference between light paths from different images and the influence of gravitational potential of the lens on travel time of the light known as Shapiro time delay [21,22]. In the case of GW strong gravitational lensing, assuming that the mass distribution in the lensing galaxy follows singular isothermal sphere (SIS) model strongly supported by galaxy lensing studies according to which population of lenses comprise massive elliptical galaxies [23], we expect to observe two time delayed waveforms with different amplitudes but of similar temporal structure (i.e. frequency drift) [12,13,14]. While the ET will register two different merger signals, separated by lensing time delay, DECIGO will likely detect the interference pattern between unresolved images [24] (argumentation can be found in the next section). This highlights the importance of multifrequency observations. In our analysis we follow SIS assumption for mass distribution in lensing galaxies. We also consider the case when both images: brighter I+ and fainter one I- produce signals visible by detector (i.e. detected SNR: r_+ and r_- exceed detector's threshold $r_0=8$). Then we calculate total optical depth for lensing on the basis of velocity dispersion distribution of stars in lensing galaxies according to modified Schechter function (see discussion e.g. in [14]) with parameters representing the population of elliptical galaxies. Since DECIGO/B-DECIGO mission are planned for 4 years continuous operation time we should also correct our results accordingly: finite duration time of observations may be relevant for those signals which come near the beginning or the end of the survey (they may be missed if we register only one image due to time delay and thus we would not be able to judge properly if this signal was lensed).

3. Results and Discussion

According to analysis described in previous section expected yearly detection rates of resolvable inspiraling DCO systems producing GW signals above detectors threshold are of order of: $10^4 - 10^6$ for ET, $10^2 - 10^5$ for DECIGO and $10 - 10^5$ for B-DECIGO depending on a particular type of DCO system, population synthesis scenario (standard,

optimistic CE, delayed SN, high BH kicks) and galaxy metallicity evolution model (“high-end” or “low-end”). Results concerning DCO detection rates are collected for ET in Table 1 and Table 2 of [13] and for DECIGO/B-DECIGO in Table 2 of [14]. Expected DCO strong gravitational lensing rates has been assessed under assumption that GW source would be detectable without lensing (both images: I+ and I- produce signals with S/N ratio exceeding detector’s threshold of $\rho=8$) and with taking into account planned finite duration time of the survey: first 1, 5 and 10 years of ET operation and 4 years of DECIGO nominal duration. Results, as in the case of unlensed detection rates described above, depend on the type of DCO system, binary evolution scenario and galaxy metallicities. Consequently, we obtain that ET would be able to detect 50 – 100 lensed GW events per year (mainly BH-BH systems) which is consistent with corresponding results obtained if one takes into account the Earth rotation effect on DCO lensing rates [25]. On the other hand, DECIGO and B-DECIGO yearly detection rates of lensed GW signals from DCO inspirals are significantly reduced in comparison to ET predictions. Due to contamination of unresolved systems DECIGO and B-DECIGO could register about 50 lensed GW events per year but it would be only signals from BH-BH inspiraling systems – the chance that DECIGO or B-DECIGO will detect lensed NS-NS or mixed BH-NS inspirals is negligible [14]. These results are reported in details in Table 3 and Table 4 of [13] for ET and Table 3 of [14] for DECIGO/B-DECIGO.

It should be noted, however, that in the case of gravitational lensing description made with light ray formalism may be not appropriate. In fact lens acts like a diffraction barrier with a narrow slit (actually strong lensing phenomena can be contemplated as an equivalent to double slit interference of EM waves) and thus there exist some conditions when the wave nature of GWs becomes relevant. In general, if GWs wavelength λ_{GW} is much shorter than the lens mass scale (e.g. Schwarzschild radius of the lens mass) the wave nature of GWs may be ignored and gravitational lensing may be safely analysed within geometric optics limit as like in the case of the light. Contrary, if λ_{GW} is much longer than the Schwarzschild radius of the lens mass, then GW wave nature becomes relevant and the wave optics limit should be used instead of geometric optics [24,26]. Because GW signal from DCO systems are coherent diffraction and interference could be embedded into amplification of the wave intensity calculated in the case of monochromatic waves propagating in the presence of weak gravitational potential according to Helmholtz equation [21,26,27,28], influencing our predictions concerning strong GW lensing. It can be shown [26] that in the situation when diffraction becomes important, it makes lensing inefficient in the sense that the magnification starts to be small. This problem is especially significant in the era of multifrequency detections of lensed GWs. In the case of space-borne interferometers like LISA or DECIGO $\lambda_{GW} \sim 1\text{AU}$ indicating that different formalisms (geometric optics vs wave optics) should be used for lensed GW events observed with different detectors operating in different frequency domains. In particular, in the case of strong GW lensing observed by DECIGO detector, interference effects between unresolved multiple images may be observable as a beat pattern in time domain which seems to be very promising for lens mass and time delay distance measurements [24].

4. Conclusions

In this paper we have discussed new perspectives for multifrequency GW astronomy in the case of GW strong gravitational lensing. New generation of ground-based detectors like ET or space-borne missions like DECIGO/B-DECIGO will considerably enrich GWs statistics making highly probable that some GW signals from DCO inspiral systems will undergo strong gravitational lensing. Our robust prediction is that ET would be able to register up to a hundred strongly lensed GW events per year. On the other hand, we predicted that DECIGO would be highly contaminated due to stochastic noise from unresolved DCO systems influencing detection rates of lensed GW signals seen by this detector. In particular DECIGO/B-DECIGO will not be able to see any lensed NS-NS or BH-NS

systems but they could be able to detect yearly up to $O(10)$ strongly lensed BH-BH inspirals. Space interferometers will be able to significantly complement global network composed of ground-based GW detectors opening new possibilities for multifrequency observations of GW events. In particular DECIGO mission dedicated to explore decihertz GW frequency range would be able to observe DCO inspiral events for days to months before they enter frequency band observable by GW detectors on the Earth. But, in the case of GW observations carried via space interferometers wave phenomena like diffraction and interference of GWs from lensed images should be taken into account.

Supplementary Materials: The following are available online at www.mdpi.com/xxx/s1, Figure S1: title, Table S1: title, Video S1: title.

Author Contributions: For research articles with several authors, a short paragraph specifying their individual contributions must be provided. The following statements should be used “Conceptualization, X.X. and Y.Y.; methodology, X.X.; software, X.X.; validation, X.X., Y.Y. and Z.Z.; formal analysis, X.X.; investigation, X.X.; resources, X.X.; data curation, X.X.; writing—original draft preparation, X.X.; writing—review and editing, X.X.; visualization, X.X.; supervision, X.X.; project administration, X.X.; funding acquisition, Y.Y. All authors have read and agreed to the published version of the manuscript.” Please turn to the CRediT taxonomy for the term explanation. Authorship must be limited to those who have contributed substantially to the work reported.

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