On Spacetime Duality and Bounce Cosmology of a Dual Universe

The 1st Electronic Conference on Universe
22-28 February 2021

Dr Mohammed B. Al-Fadhli
University of Lincoln, UK.
OUTLINE

- Extended Field Equations for a Closed Universe
- Bounce from a Closed Early Universe
- Spiral Galaxy Rotation under External Fields
- Early Universe Boundary Contribution
- Conclusions and Future Works
Extended Field Equations

- Bounce cosmology provides an alternative perception of the Universe in which our Universe expanded from a hot and very dense state of a previously collapsed Universe.

- The recent Planck Legacy (PL18) release revealed a closed and positively curved early Universe.

- In realising a singularity-free paradigm, a minimal Universe’s radius can be sought by considering the boundary contribution of the early Universe plasma.

To consider the pre-existing/background curvature $\mathcal{R}$ according the PL18 release while complying with the energy conservation law, a modulus of spacetime deformation $E_D$ is utilised to extend the Einstein–Hilbert action as

$$ S = \int \left[ \frac{E_D}{2} \frac{R}{\mathcal{R}} + L \right] \sqrt{-g} \, d^4x $$

where $R$ is the Ricci scalar curvature and $g$ is the determinant of the metric tensor $g_{\mu\nu}$ and $L$ is the Lagrangian density. The extended field equations are

$$ R_{\mu\nu} - \frac{1}{2} R \hat{g}_{\mu\nu} + \frac{R - \mathcal{R}}{\mathcal{R}} (K_{\mu\nu} - \frac{1}{2} K \hat{g}_{\mu\nu}) = \frac{8\pi G}{c^4} T_{\mu\nu} $$

The new boundary term/tensor is only significant at high-energy limits such as with black holes and early Universe where the difference between the induced $R$ and pre-existing $\mathcal{R}$ curvatures is significant.
Bounce from a Closed Early Universe

- A closed and positively curved early Universe model is considered with referenced to the early Universe radius of curvature upon the CMB emissions and the referenced early Universe scale factor.

- A new dimensionless scale factor for this metric is defined as \( \frac{a}{a_p} \)

**Figure 1.** A referenced FLRW metric. \( r_p \) is the reference radius upon the emission of the CMB and \( a_p \) is the reference scale factor.

The referenced metric is

\[
[g_{\mu\nu}(x)] = \text{diag} \left( -c^2 \frac{a^2(t)}{a_p^2} / \left( 1 - \frac{r^2}{r_p^2} \right), \frac{a^2(t)}{a_p^2} r^2 \frac{a^2(t)}{a_p^2} r^2 \sin^2 \theta \right)
\]

The resulted referenced Friedmann equations are

\[
H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G \rho}{3} - \frac{c^2}{a^2} \frac{a_p^2}{r_p^2}
\]

and

\[
\dot{H} \equiv \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + 3 \frac{3P}{c^2} \right)
\]
The solved wave function of the Universe’s evolution is

$$\overline{\psi_L(\eta)} = \mp \frac{E}{6E_D} \left( \left( 1 - \cos \frac{c}{r_p} \eta \right)^2 + \frac{c^2}{H_\eta^2 a_p^2} \left( \eta - \sin \frac{c}{r_p} \eta \right)^2 \right)^{1/2} e^{i \cot \frac{H_\eta a_p \left( 1 - \cos \frac{c}{r_p} \eta \right)}{c \left( \eta - \sin \frac{c}{r_p} \eta \right)}}$$

Figure 2. Evolution of the wave function of matter coupled with radiation of one side of the Universe (orange curve) and radiation only wave function (blue curve) in addition to the straight line of light worldlines in red (diagram is not to scale).

The positive and negative solutions of the wave function imply that matter and antimatter in the plasma evolved in opposite directions.

Matter and antimatter of early Universe plasma could be separated upon the emission of the CMB and evolved in opposite directions as result of the phenomenon of plasma drift.

Cosmic evolution of radiation coupled with matter/antimatter is predicted to experience three distinct phases.
At the first phase, the matter and antimatter sides expanded in opposite directions away from the early Universe plasma, possibly corresponding the CMB dipole anisotropy.

At the second phase, both sides entering a state of free fall towards each other under gravitational acceleration.

At the final phase, spatial contraction is predicted where the Universe experiences a contraction leading to a big Crunch.
The Hubble parameter evolution function is

\[ H_\eta = H_a \left( \frac{1}{5} \cot^5 \frac{c}{2r_p} \eta + \frac{2}{3} \cot^3 \frac{c}{2r_p} \eta + \cot \frac{c}{2r_p} \eta \right) + H_p \]

**Figure 4.** The predicted Hubble parameter \( H \) and its rate \( \dot{H} \) along with a rectified Hubble parameter reflecting the reverse evolution direction.

- The Hubble parameter starts at a hyperbolic rate then it decreases which can be due to gravity between the two sides, until it reaches its minimal value. Then, it starts to increase at the second phase in reverse direction.
- According to mechanics, the minus sign of Hubble parameter at the second phase indicates an opposite direction.
- The opposite signs of the acceleration and the expansion speed in the first phase indicate a slowing down while the matching signs in the second phase indicate the expansion speed is increasing.
The model predicted that the spacetime curvature increases with the highest degree occurring at the phase transition as was shown in Figure 3.

It can be inferred the fast-orbital speeds observed for outer stars are a result of the variation of spacetime curvature.

A simulation of a spiral galaxy as a forced vortex was created through incrementally flatter worldline curvature as shown in Figure 5.

**Figure 5.** (a) External fields exerted on a galaxy because of the divergence of spatial curvature through the imaginary time dimension. Green curves represent the curvature of spacetime worldlines. Blue curves represent the simulated spacetime continuum flux. (b) Spiral galaxy rotation. Blue represents the slowest tangential speeds, and red represents the fastest speeds.
Early Universe Boundary Contribution

• At high energy limits, gravitational contributions of early Universe plasma boundary can be obtained using the boundary term/tensor.

• The reference radius of curvature \( r_p > 0 \) because any reduction in the volume causes an increase in the pressure, which can realise a singularity-free paradigm.

The boundary term/tensor is

\[
\frac{R - \mathcal{R}}{\mathcal{R}} \left( K_{\mu\nu} - \frac{1}{2} K q_{\mu\nu} \right) = \frac{8\pi G}{c^4} T_{\mu\nu}
\]

The induced metric is

\[
[q_{\mu\nu}(x)] = \text{diag} \left( -c^2, \frac{a^2(t)}{a_p^2} R^2, \frac{a^2(t)}{a_p^2} R^2 \sin^2 \theta \right)
\]

The solved extrinsic curvature tensor is

\[
[K_{\mu\nu}(x)] = \text{diag} \left( 0, -\frac{a^2(t)}{a_p^2} R, -\frac{a^2(t)}{a_p^2} R \sin^2 \theta \right)
\]

The trace of the extrinsic curvature and the pre-existing curvature at reference time are

\[
K = K_{\mu\nu} q^{\mu\nu} = 2/R \quad \text{and} \quad \mathcal{R}_p = 1/r_p^2
\]

Ricci scalar curvature in terms of the difference between energy densities as

\[
R_p = \frac{6G}{c^2} \left( \frac{4\pi P_p}{c^2} - \frac{4\pi \rho_p}{3} \right)
\]

The reference radius of curvature is

\[
r_p = \frac{4G P_p V_p}{c^4} = \sqrt[3]{\frac{E_p}{2\pi E_D}}
\]
Conclusions and Future Works

1. A closed early Universe model was considered. The evolution of the Universe from early plasma was modelled utilising quantised spacetime continuum worldlines.

2. The positive and negative solutions of the wave function imply that matter and antimatter in the early Universe plasma evolved in opposite directions.

3. Both sides expand away from the early plasma during the first phase. Then, during the second phase, they reverse their directions and fall towards each other under gravitational accelerating. This could explain the effects attributed to dark energy as well as the observed dark flow.

4. The model predicted a nascent hyperbolic expansion is followed by a phase of decelerating spatial expansion during the first $\sim 10$ Gyr, followed by a second phase of accelerating expansion, theoretically resolving the tension in Hubble parameter measurements.

5. The simulated external fields on galaxies could explain the effects attributed to dark matter.

6. Finally, this theoretical work should be tested against observational data in the future works.
THANK YOU

Mohammed Al-Fadhli
Malfadhli@lincol.ac.uk
@DrMoFadhli