



Motivation

The neuron densely wired each other on both structure and functional connectivity level to drive information processing in the brain. In this study, I only describe functional connectivity with Functional Magnetic Resonance Imaging (fMRI) data. On the one hand, since the relationships between the variables are often non-linear and of unknown functional type, and the number of such variables is large, traditional tools are ineffective, e.g., fMRI data exist significant dimensional problems or “curse dimensionality” problem [1]. On the other hand, how to quantitative measure information share or separate among functional brain regions. However, the information-theoretic technique, Total Correlation [2, 3, 4] can perfectly address the above issues.

Methods

Total Correlation is one of several generalizations of mutual information, and the multivariate constraint, or multi-information constraint, is another name for it. It determines how redundant or dependent a set of n random variables is.

Entropy $H(X)$ indicates how surprising it is, on average, to get a symbol x from a random variable X that can take the possible symbols x_1, x_2, x_n each with probability $p(x_i)$:

$$H(X) = - \sum_{i=1}^n p(x_i) \log p(x_i) \quad (1)$$

The amount of information that is contained in two variables X, Y is given by the joint entropy,

$$H(X, Y) = - \sum_{i=1}^n \sum_{j=1}^m p(x_i, y_j) \log p(x_i, y_j) \quad (2)$$

The concept of total correlation, $TC(X_1, X_2, \dots, X_N)$, provides a direct and effective way of assessing the dependency among multiple variables:

$$TC(X_1, X_2, \dots, X_N) = \sum_{i=1}^N H(X_i) - H(X_1, X_2, \dots, X_N) \quad (3)$$

It can be noted that for the case of $N = 2$ (Bivariate Case), total correlation is equivalent to the well-known mutual information Eq. 2. However, we are investigating variables normally $N > 2$ (N-variate Case) in the biological neural network, e.g., the human brain. In N-variate Case situation, we used Correlation Explanation clustering method which is an information-theoretic theory for learning maximally informative abstract representations of data [5].

Results

Datasets

The data came from a resting-state fMRI experiment in which a single participant kept alert wakefulness while watching across but did not perform any other behavioral activity. The data was preprocessed, and time series from various brain regions of interest (ROIs) were collected. The ROIs are listed as follows,

Cau, **Caudate**; Pau, **Paudate**; Thal, **Thalamus**; Fpol, **Frontal pole**; Ang, **Angular gyrus**; SupraM, **Supramarginal Gyrus**; MTG, **Middle Temporal Gyrus**; Hip, **Hippocampus**; PostPHG, **Posterior Parahippocampal gyrus**; APHG, **Anterior parahippocampal gyrus**; Amy, **Amygdala**; ParaCing, **Paracingulate gyrus**; PCC, **Posterior cingulate cortex**; Prec, **Precuneus**.

Results

1. Bivariate Case

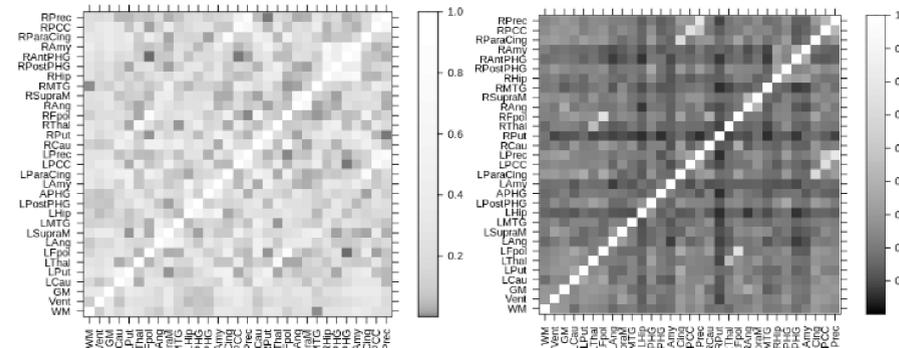


Figure 1. Absolute value of pairwise linear correlation (left) and total correlation (right).

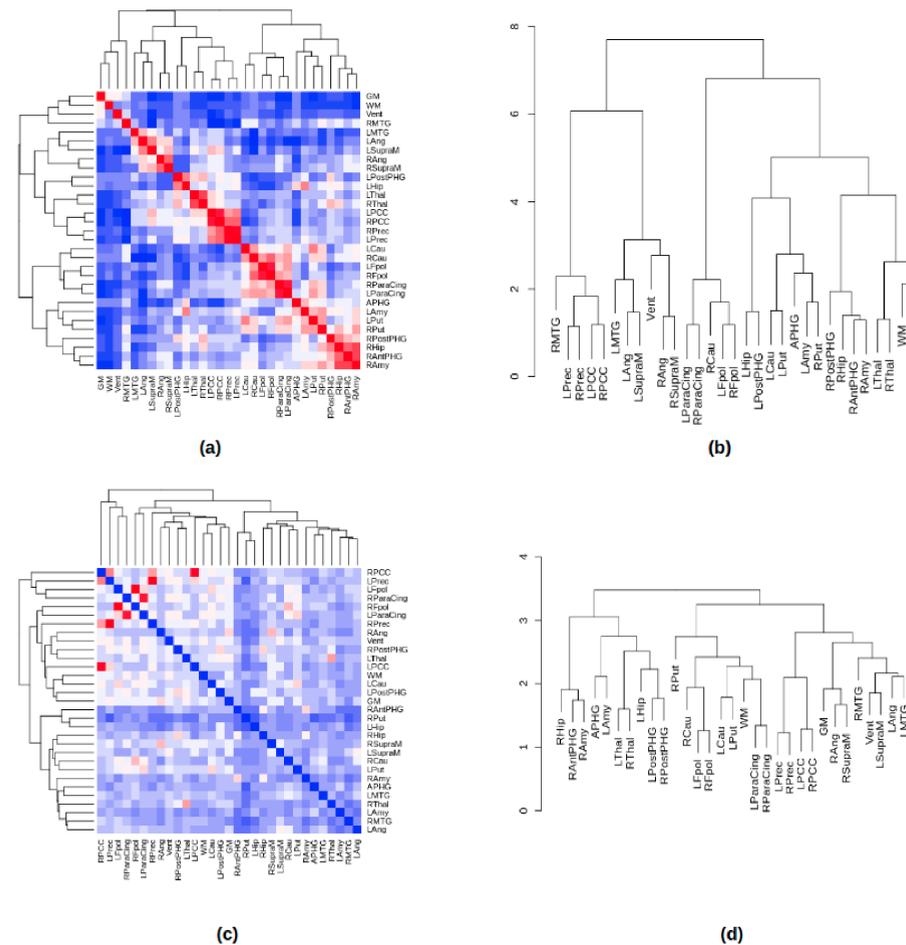


Figure 2. Dendrograms for the hierarchical clustering of variables using linear correlation coefficient (a,b) and total correlation(c,d).

2. N-variate Case

Parameters We fit a CorEx model consisting of three layers with 10, 3, and 1 units. The hidden factor at each layer takes 3 discrete values.

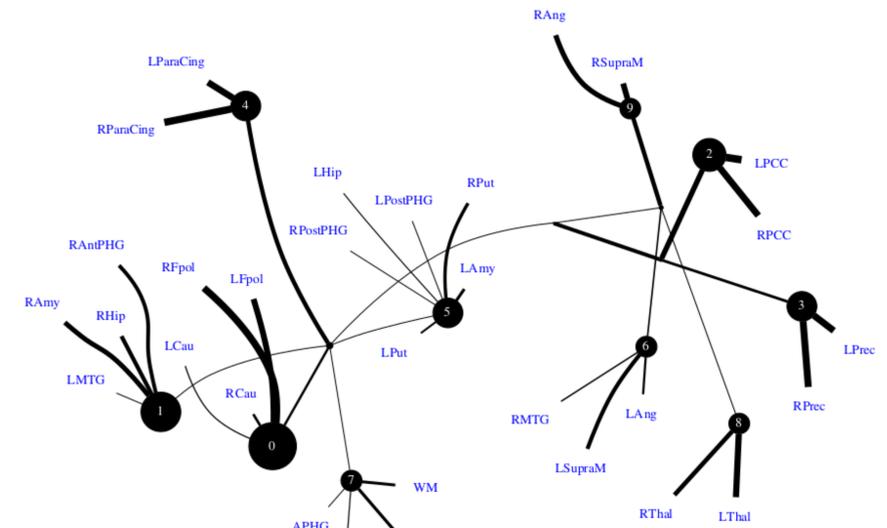


Figure 3. The diagram presented graphs with thresholded weights at 0.19 for display. The edge thickness is proportional to mutual information and node size reflective of total correlation among child nodes.

Conclusions and Discussion

In this study, two things are measured with the information-theoretic technique - Total Correlation. Firstly, quantitative measures intra-cortex regions dependent or independent from others from the information-theoretic views. Secondly, Total correlation is an efficient way to assess functional connectivity of human brain, according to the findings. Thirdly, total correlation is a powerful clustering method with a graph to find multivariate independence, and it has the potential to apply to other field investigations in the future.

Due to the limited space on the page, we only showed data on resting-state functional connectivity and did not show its application to mental illnesses. However, we will have a related paper soon, and we will share it with you when it comes out. Please feel free to provide me with suggestions.

References

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