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EFFICIENCY OF AN ARRANGEMENT IN SERIES OF IRREVERSIBLE THERMAL ENGINES WORKING AT MAXIMUM POWER

Luis E. Vidal-Miranda[1], J. C. Chimal-Eguía[2], J. C. Pacheco-Paez[3,*], R. T. Páez-Hernández[4]
[1,3,4] Área de Física de Procesos Irreversibles, Departamento de Ciencias Básicas, Universidad Autónoma Metropolitana, U-Azcapotzalco. A. San Pablo 180, Col. Reynosa, Ciudad de México CP 02200, Mexico
[2] Centro de Investigaci´on en Computación del Instituto Polit´ecnico Nacional, Av. Miguel Othón de Mendizábal s/n. Col. La Escalera, Ciudad de M´exico, CP 07738, Mexico.

ABSTRACT: Within the context of finite-time thermodynamics several regimes of performance have been used to study the well known Curzon-Ahlborn (CA) heat engine model [1-5]. Also the optimal performance and the effects on environment are studied to find the best approximation with real heat engines.

In this work we present a model of an arrangement in series of irreversible Carnot heat engines, which consist of *k* reservoirs connected in series, this heat engine model is working under three different regime of performance: maximum power output, maximum ecological function [6] and maximum efficient power [7]. At first we used three reservoirs, and we calculated its efficiency. For the case of maximum power output we calculated the efficiency for the case of the generalizing of k reservoirs, and we get an efficiency expression similar to the one of Curzon-Ahlborn, the irreversibilities are taken into account by irreversibility parameter R. Finally we present the comparison of the efficiencies obtained under three differents regimes of performance.

I. Introduction:

Thermodynamic efficiency is known as one of the more popular criteria after Carnot [1] to analyze the performance of thermal engines. Carnot found that any engine extracting heat from a reservoir at temperature T_1 has to deliver some heat to a reservoir at lower temperature T_2 while work is doing. Moreover, Carnot showed that maximum efficiency in the cyclic process is

$$\eta_C = 1 - T_2/T_1 \tag{1}$$

which is known as Carnot efficiency. The limitations of Classical Equilibrium Thermodynamics (CET) to formulate useful criteria describing the performance of real engines motivated the development of a new field, known as *Finite Time Thermodynamics* (FTT) [2,3], which keeping the formalism as close as possible of equilibrium thermodynamics while introduces simple modifications to take into account the main sources of irreversibility observed in real engines. A paradigmatic model in FTT is due to Curzon- Ahlborn (CA) [4]. Assuming that the heat transfers obey a Newton aw, they found that the engine working at maximum power has the efficiency given by,

$$\eta_{CA} = 1 - \sqrt{T_2/T_1} \tag{2}$$

The endoreversible engine is mainly based in the idea that, for many processes, it is possible to conceive the internal relaxation times as being negligibly short compared with the duration of the full processes. Previous research has recently proposed a manner to include the internal contributions to the global entropy production by means of the Clausius inequality. If any internal irreversibility is considered, then the Clausius inequality gives,

$$\Delta S_{1W} + \Delta S_{2W} < 0 \tag{3}$$

Expression (3) becomes an equality by means of

$$\Delta S_{1W} + R\Delta S_{2W} = 0 \tag{4}$$

Where is called the non-endoreversibility parameter, given by

$$R = \Delta S_{1W}/|\Delta S_{2W}| \tag{5}$$

II. Non-endoreversible engine working at maximum power:

For case of a non-endoreversible CA engine, as shown in Fig. 1

$$Q_1/T_{1W} = RQ_2/T_{2W} \quad (6) \quad or \quad Q_2/Q_1 = T_{2W}/RT_{1W} \tag{7}$$

The efficiency inside of the CA engine is

$$\eta_{ne} = 1 - (T_{2W}/RT_{1W}) \tag{8}$$

If we calculate the work per unit time, the power output, but now using (8) instead of $\eta = 1 - (T_{2W}/T_{1W})$, we have

$$W(\eta_{ne}, R) = \alpha\beta\eta_{ne}\left[\frac{RT_1(1-\eta_{ne}) - T_2}{(\alpha + \beta R)(1-\eta_{ne})}\right] \tag{9}$$

The point η^*_{MP} where the function of (9) reaches its maximum value is obtained by means of $(\partial W/\partial \eta_e)|_{\eta^*_{MP}} = 0$, and this condition gives

$$\eta^2_{ne} - 2\eta_{ne} + [1 - T_2/RT_1] = 0 \tag{10}$$

that is,

$$\eta^*_{MP} = 1 - \sqrt{T_2/RT_1} \tag{11}$$

which is a kind of non-endoreversible CA efficiency. The power output for endoreversible CAN engines is zero at $\eta_{ne}=0$ and when $\eta = \eta_C$ [11], and for the non-endoreversible case $W(\eta_{ne}, R)$ has zeros at η_{ne}=0 and we have

$$\eta_{ne} = \eta_C = 1 - T_2/RT_1 \tag{12}$$

that is, at a kind of non-endoreversible Carnot efficiency.

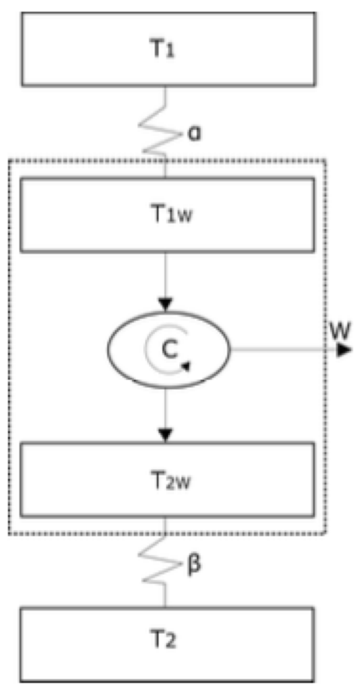
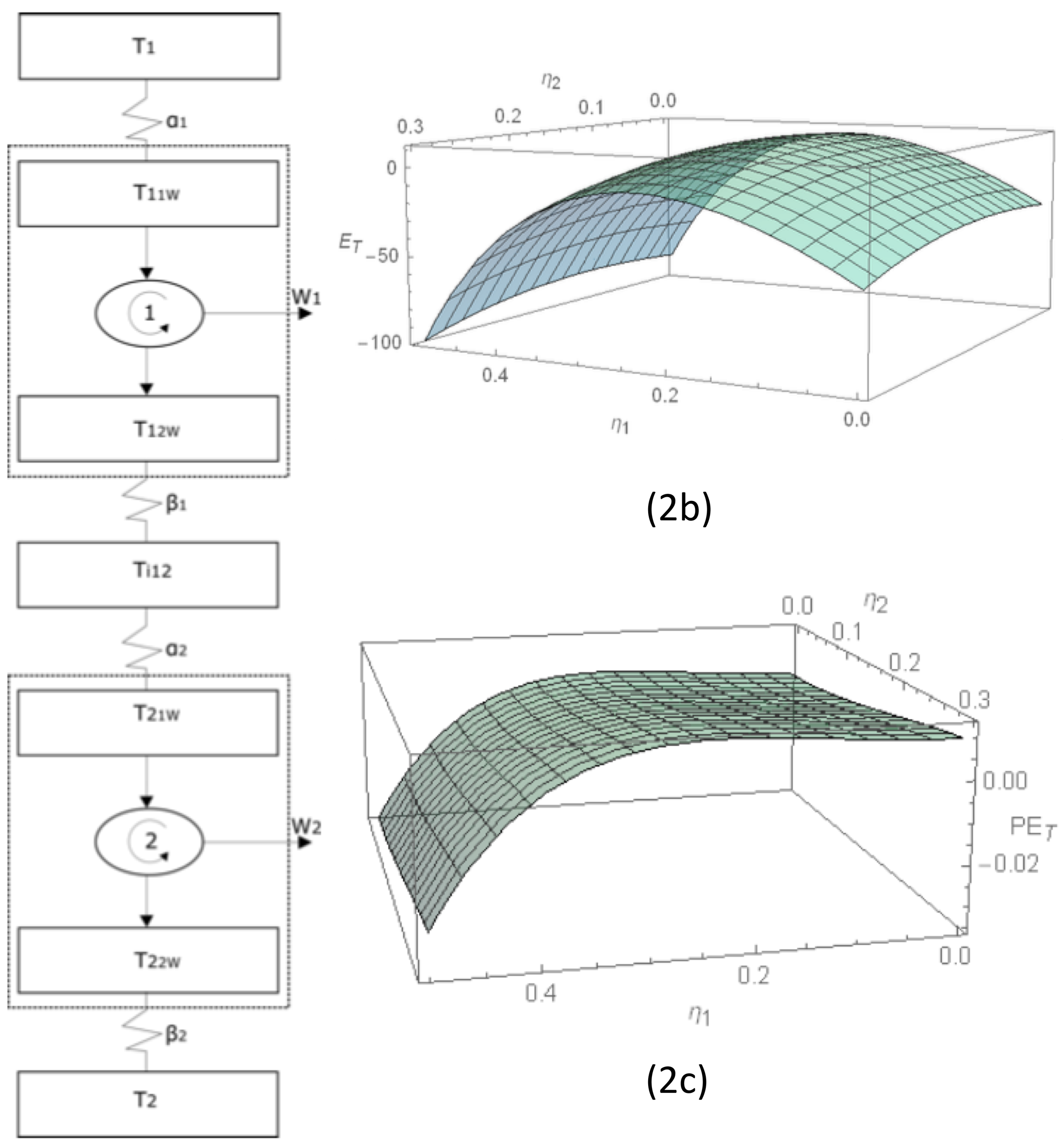


Fig.1

III. Series of irreversible thermal engines at maximum power:

Heat engines with several heat sources are common for many real-world applications such as industrial heat-recovery systems and solar energy installations.



(2a) (2b) (2c)

Fig.2

A schematic diagram of a thermal engine system is shown in Fig. (2a). Two irreversible Carnot heat engine cycles in series form a single cycle operating between reservoirs at temperatures T_1 and T_2 ($T_1 > T_2$). Waste heat from the first cycle is used totally as the heat source of the second cycle. The working fluids in each cycle system flow continuously so that combined cycle operates in steady state. According to figure 2, we can get

$$\eta = \eta_1 + \eta_2(1 - \eta_1) \tag{13}$$

Using the first law of thermodynamics in Fig. (2a), we get

$$\dot{W}_2 = \dot{Q}_2 - \dot{Q}_2 \tag{14}$$

$$\dot{W}_2 = \dot{Q}_3 - \dot{Q}_4 \tag{15}$$

$$\dot{W} = \dot{W}_1 + \dot{W}_2 \tag{16}$$

$$\dot{Q}_2 = \dot{Q}_3 \tag{17}$$

$$\dot{W} = \dot{W}_1 + \dot{W}_2 \tag{18}$$

From equations (13) and (17) we obtain,

$$\eta = 1 - \left[\frac{T_2}{R^2 T_1}\right] \tag{20}$$

For the case o k cycles, we get

$$\eta = 1 - \left[\frac{T_2}{R^k T_1}\right] \tag{21}$$

IV. Ecologica function criteria:

In 1991 Angulo-Brown[5] introduced a new optimization criteria, the so called ecological function, tha is given by

$$E = W - T_2\sigma \tag{22}$$

Where W is the power output and σ is the entropy production of the engine, for the case of the Fig.(2a), we calculated the total power by means of W_T = W_1+W_2, the total entropy production as $\sigma_T = \sigma_1 + \sigma_2$ and the total ecological function as $E_T = E_1 + E_2$.

In Fig. (2b) we can observe the behavior of the total ecological function for the engine.

IV. Efficient power criteria:

In 2006 Yilmaz[6] proposed a criteria given by the product of the power and the efficiency of the engine, this criteria is know as efficient power and can be written as

$$PE = \eta W \tag{22}$$

As similar as ecological function the total power of the engine is calculated as, W_T = W_1+W_2. In Fig. (2c) is presented the behavior of the total efficient power for theengine.

$$W_T = \alpha_1\beta_1\eta_1\frac{R_1T_1(1-\eta_1) - T_{i12}}{(\alpha_1 + \beta_1 R_1)(1-\eta_1)} + \alpha_2\beta_2\eta_2\frac{R_2T_{i12}(1-\eta_2) - T_2}{(\alpha_2 + \beta_2 R_2)(1-\eta_2)} \tag{22}$$

$$\sigma_T = Q_{11}\left(\frac{1}{T_{i12}} + \frac{1}{T_{11W}}(1 - \frac{1}{R_1}) - \frac{1}{T_1}\right) - \frac{W_1}{T_{i12}} + Q_{21}\left(\frac{1}{T_2} + \frac{1}{T_{21W}}(1 - \frac{1}{R_2}) - \frac{1}{T_{i12}}\right) - \frac{W_2}{T_2} \tag{23}$$

$$E_{m1} = \frac{\alpha_1\beta_1(T_{i12} + R_1T_1(\eta_1 - 1))(T_1T_{i12} + R_1(T_{11W}T_{i12} - T_1(T_{11W} + T_{i12}) + 2T_1T_{11W}\eta_1))}{R_1T_1T_{11W}(\alpha_1 + R_1\beta_1)(\eta_1 - 1)} \tag{24}$$

$$PE_T = \alpha_1\beta_1{\eta_1}^2\frac{R_1T_1(1-\eta_1) - T_{i12}}{(\alpha_1 + \beta_1 R_1)(1-\eta_1)} + \alpha_2\beta_2{\eta_2}^2\frac{R_2T_{i12}(1-\eta_2) - T_2}{(\alpha_2 + \beta_2 R_2)(1-\eta_2)} \tag{25}$$

Conclusions:

The interest of this work is of teaching and as an immediate application of FTT. Some results are taken from the literature, nevertheless other they are novelty obtained of the model. Here is shown that real models can be approximate as a series of cycles working at maximum power output. The results could be considered for an irreversible engine. Also is important to remark that (21) is reduced to (12) when only there is one cycle. Also other criteria of performance were considered and we get new results.

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