# Theory and Modeling of Eddy Current Type Inductive Conductivity Sensors [†]

## Julius Harms [1],[*], T. A. Kern [1]

[1]   Institute for Mechatronics in Mechanics, Hamburg University of Technology; imek@tuhh.de

[*]   Correspondence: Julius.harms@tuhh.de

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**Abstract:** While transformer-type conductivity sensors are the usual type of inductive sensors, this paper describes the theory behind less used eddy current sensors. This type of sensor measures conductivity of a liquid by inducing eddy currents and observing the effect on the sensor coil, which allows a simpler sensor design and promises a cost advantage in implementation. A novel model description is derived from the Maxwell equations and implemented by an equivalent RLC circuit. The designed model is validated by comparisons with experimental observations and FEM simulations. The result leads to a better understanding of the physical effects of the sensor and the influencing parameters for future sensor developments. The aim is to provide starting points for further sensor development of low-cost inductive conductivity sensors.

## 1. Introduction

Inductive conductivity sensors are widely used for salinity measurements from standard industrial processes to highly accurate deep ocean measurements. Due to their property of not being in contact with the measuring medium, inductive sensors have a clear advantage over conductive measuring methods. Unwanted effects of biofouling and polarization can be reduced by protecting housings, resulting in a durable and low maintenance sensor. Despite the increasing demand for low maintenance in situ monitoring sensors, the costs for inductive sensors are still higher than for conductive ones. Due to their more complex design, the production of accurate inductive sensors is more expansive than simple 4-electrode contacting conductivity cells. Motivated by the great advantages of measurements with high space-resolution through large sensor quantities in the ocean, the authors of this paper see a need of accurate but low cost inductive conductivity sensors which can be used for long time measurements. The established and mostly used technology for inductive conductivity measurements is the transformer type sensor which uses two toroidal coils. Striggow and Dankert [1] first introduced the theory of this sensor type in 1985. Since then, various patents (e.g. US4740755A, US5793214A, US6414493B1, US7965167B2) evolved based on this theory and current research is still being conducted [2].

With transformer type sensors not being the only electromagnetic sensors to measure conductivity, the research conducted here focuses on alternative inductive sensors based on magnetic flux through the fluid. This paper discusses the theory of solenoidal coils used for conductivity measurement, which in the following are referred to as eddy current sensors. Eddy current sensors so far have been mostly empirically investigated on but showed with their simple construction promising test results [3,4,5,6]. This paper describes the basic functionality of eddy current sensors and introduces a new way of modeling for these sensor types. Eddy Current Sensors are used through phase differences [5,6] or amplitude values [3,4]. The model description is derived from Maxwell's equations and the effects are supported by FEM-Simulations.

## 2. Eddy Current Sensor Model

The toroidal coils of the transformer type sensors (Fig 1a) are coupled by the current through the water as a third winding. The magnetic flux is bound in the ferrite cores. In contrast, eddy current sensors (Fig. 1b) are based on alternating magnetic flux through the water, which induce the name giving eddy currents in the fluid. The alternating magnetic flux is generated by a simple solenoid [3,4] or a planar coil [5,8] fully submerged in the fluid. The current density of the eddy currents depends on the water conductivity. This effect enables the related measurement. The eddy current effect will also have an influence on transformer type sensors, but for them the coupling through conductive water is more dominant.
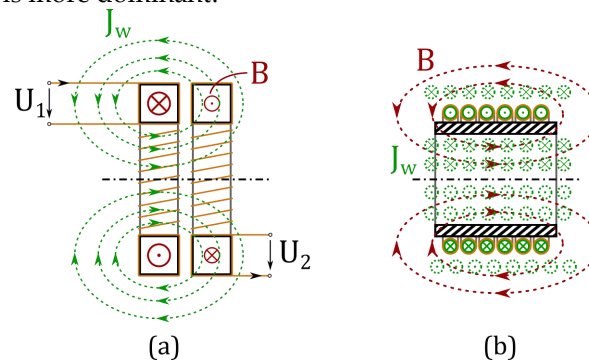


Figure 1: Schematic diagram of (a) a transformer type sensor and (b) an eddy current sensor. Green vectors display the current density, red the magnetic field strength. The sensors are surrounded by a conductive fluid.

### 2.1 Working principle

Due to Lenz law, the induced eddy currents generate a magnetic counter field, which weaken the main magnetic field of the primary coil. This damping influences the coils impedance and therefore the coupling of two coils. The result is an inversely proportional relationship between the strength of the B-field and the conductivity. Besides the conductivity, the strength of the magnetic field depends on the permeability $\mu_r$ of the liquid. This effect is negligible when measuring NaCl concentrations e.g. ocean salinity [7]. For other salt solutions like $CuSO_4$ however, the solutions concentration has a major influence on the permeability and has to be considered [8]. In the following the model description will be derived for the impedance of a single solenoid. Later this model can be used to describe the magnetic coupling between two coils.

### 2.2 Equivalent Model

In the following, the damping effects of eddy currents are established using Maxwell's equations. An equivalent model representation is then derived. The model of the eddy current sensor is based on a standard RLC equivalent circuit of a coil. A single solenoid is represented by the inductance L, copper resistance $R_{Cu}$ and parasitic effects $C_p$. Due to the influence of the eddy currents on the magnetic field, the inductance L has to depend on the water conductivity $\sigma$. It is considered that the coil is perfectly isolated from the fluid and no direct current can flow from the copper coil through the water. The E-Field, which leads to the eddy currents, is induced by the total B-Field in the fluid and can be described by Faraday's law of induction.

$$\text{rot } \vec{E} = -\frac{\delta B}{\delta t} \tag{1}$$

Due to the fluids conductivity $\sigma$ the eddy currents again induce a counteracting magnetic field $B_E$ which can be represented by inserting eq. (1) in Ampere's circuital law.

$$\text{rot } \vec{B_E} = \mu_0 \cdot \left( -\sigma \cdot \frac{\delta B}{\delta t} - \epsilon_0 \frac{\delta^2 B}{\delta t^2} \right) \tag{2}$$

The field strength $B_E$ acts against the field $B_C$ emitted by the coil. For a coil with $w$ windings, a flux area $A$ and a magnetic resistance $R_{mag}$, the current through the coil $i$ leads to:

$$B_c = \frac{i \cdot w}{A \cdot R_{mag}} \tag{3}$$

The total B-Field through the water is then given by a differential equation as the sum of $B_C$ and $B_E$:

$$B = \frac{i \cdot w}{A \cdot R_{mag}} - \mu_0 \cdot (\sigma \cdot \frac{\delta B}{\delta t} + \epsilon_0 \frac{\delta^2 B}{\delta t^2}) \tag{4}$$

After Laplace transformation and multiplication with the magnetic flux area $A$, the magnetic flux $\phi$ can be derived:

$$\phi = \frac{i \cdot w}{R_{mag} \cdot (1 + \mu_0 \cdot \sigma \cdot s + \mu_0 \cdot \epsilon_0 \cdot s^2)} \tag{5}$$

Since the induced voltage is given by $U = w \cdot \frac{\delta \phi}{\delta t}$ and the impedance is $Z = \frac{U}{i}$, the impedance of the coil is given by

$$Z = \frac{L \cdot s}{1 + \mu_0 \cdot \sigma \cdot s + \mu_0 \cdot \epsilon_0 \cdot s^2} \tag{6}$$

where $L = \frac{w^2}{R_{mag}}$ is the constant inductance of the coil. The derived impedance transfer function in eq. (6) models the eddy current effects and represents the conductivity dependent inductance of the coil. The TF can be decomposed into another equivalent RLC parallel circuit where the resistance and the capacitance are represented by

$$R_E = \frac{L}{\mu_0 \cdot \sigma} \text{ and } C_E = \frac{\mu_0 \cdot \epsilon_0}{L} \tag{7}$$

Comparing eq. (2) to eq. (6 and 7) shows that the capacitance $C_E$ in the model represents the displacement current and the resistance, the inverse of the conductivity. Applying the derived transfer function to the equivalent circuit of a coil leads to the eddy current model of a single coil shown in fig. 2b. An eddy current sensor can also be implemented as a transformer by using two solenoids coupled through the water. Fig. 2b shows the equivalent circuit of such a system.
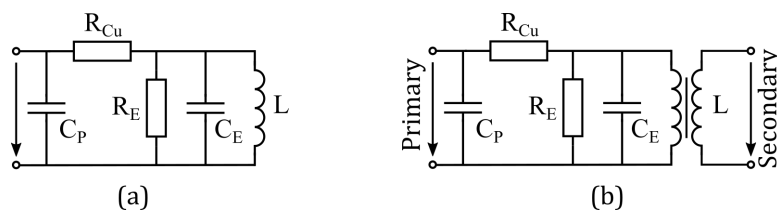


Figure 2: Equivalent circuit of (a) a single coil and (b) a coil pairing in conductive fluid, modeling the eddy current effect

For a constant permeability $\mu_r$, $R_{mag}$ and the inductivity $L$ stays constant. Therefore, the resistance $R_E$ is proportional to $\frac{1}{\sigma}$. An increase of the conductivity leads to a decrease of the equivalent impedance and therefore to damping of the output signal. A changing permeability would have an impact on the inductivity $L$, but is not considered since the changes for seawater are negligible.

## 3. Test and Simulation Results

For model verification, the frequency response of the circuit impedance in fig. 2b was calculated using Matlab. The results were compared to a FEM simulation and a prototype test. All tests calculated the impedance based on output voltage divided by input current.

### 3.1 Eddy Current Sensor Test

An eddy current sensor was build according to fig. 3. Two solenoids coiled on a pipe and coated by an insulation. A test was executed, where sensor was submerged in an aquarium with 35 liters of NaCl solution. In three steps the conductivity was increased, and measured by a reference sensor to 0.65 S/m, 3.7S/m and 9.2 S/m. The test showed a damping of the coupling with increase of water conductivity. This result differs to the observations by Parra in [3]. However, a failed test with missing insulation showed an increased coupling between primary and secondary coil with increasing conductivity, which was also observed in some of Para's experiments.
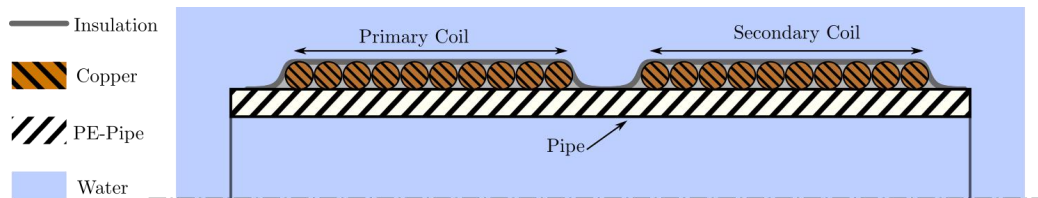


Figure 3: Schematic of the implemented eddy current sensor

### 3.2 FEM Model

The FEM-simulation used a 3D-model based on the prototype concept of fig. 3 and was conducted using Ansys Maxwell. In the simulation, the sensor was surrounded by water with the same three conductivity as in section 3.1. The FEM-Simulation confirmed the damping behavior for eddy current models. Parasitic effects were not simulated and only the linear effect was represented. The amplification and other effects around the resonance point could not be validated.

### 3.3 Results

The results from the sensor test and the FEM simulation were compared to the RLC equivalent model representation. Since the parameters of all three tests were different, the results were normed to their resonance peak to be comparable. The normed solutions are plotted together in Fig. 4. All three frequency responses for the impedance show a similar behavior. An increase in conductivity leads to a decreased impedance. In the resonance point of the model and the prototype test, the relative difference has a high similarity. Since the parasitic effects were not simulated in the FEM simulation, the resonance peak is missing. Still the FEM results show the same damping behavior, only in smaller dimensions.
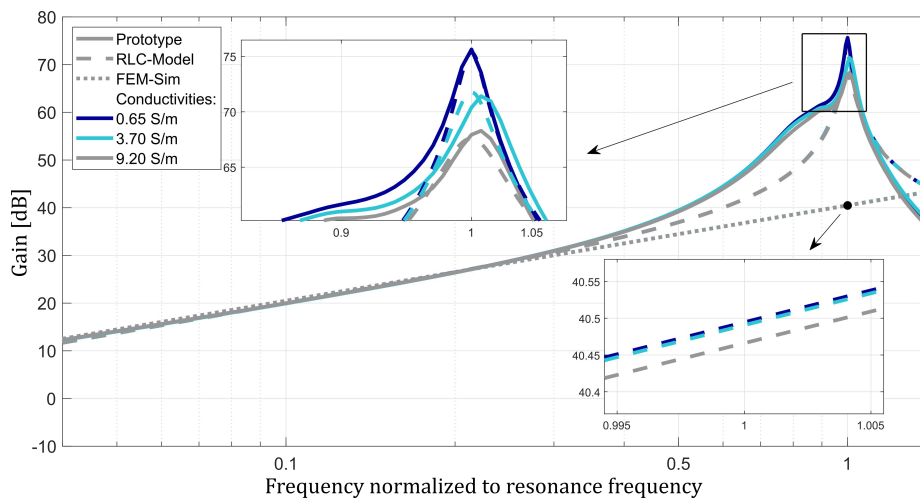


Figure 4: Comparison of simulation results to prototype tests with different conductivity values

## 4. Discussion

The results show that the here derived equivalent model can represent the eddy current effect for this sensor type. The theory of the ongoing effects was confirmed by practical tests and FEM-Simulations. An improved parameterization of the model is necessary to ensure a representative simulation. This can be realized by carrying out further experimental tests and comparing them with the model representation. Furthermore, our results show that the eddy current effect leads to a damping of the measurement signal when increasing the conductivity of the water. In some frequency range this is deviating from observations made in existing publications [3] and could not be explained finally in the course of this research. For final confirmation, a deeper analysis of the settings, and the frequency dependent dominant effects is necessary.

## 5. Conclusion

The outcome of this paper gives a better model understanding of the eddy current sensor type which can lead to the design of optimized sensors. Based on the here derived results further development and investigation needs to be done to fully evaluate the potential and usage of eddy current sensors as an alternative to transformer type sensors. Besides accuracy, the power consumption and cost effectiveness will be evaluated in future work. Furthermore, the influence of the eddy current effect on transformer-type sensors could investigated.

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