



Reduction of Rocket Dispersion using Model Predictive Lateral Pulse Jet Control †

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Abstract: Uncontrolled ground to ground fire rockets produce high impact point dispersion due to various sources of error and atmospheric uncertainties. To increase a high single-shot hit probability, the rocket systems must be equipped with an on-board control mechanism. This paper investigates the use of lateral pulse jet control to reduce the dispersion of fire rockets. The numerical simulation of rocket trajectory was used to predict the impact point from an arbitrary state. The magnitude and direction of pulse jet thrust were based on difference between the predicted impact point and target location. The pulse jet thrust was correlated with a given set of trajectory states using the multiple linear regression. A comparison of dispersion for the uncontrolled and controlled impact points was proceeded through a Monte-Carlo simulation. It was shown that the dispersion radius which was defined as the radius of circle centered at the mean impact point and containing 50% of the impact points for the controlled case was noticeably smaller than the uncontrolled case.

Keywords: Rocket Dispersion; Lateral Pulse Jet; Multiple Regression; Circular Error Probable

1. Introduction

The unguided artillery rockets typically produce a widely dispersed impact point due to various sources of error. The error sources include rocket production inaccuracy, launcher deflection, rocket tip-off, fin and thrust misalignment, and wind and atmospheric variations [1]. A number of studies on active control for artillery rocket to reduce the dispersion have been proposed. Jitraphai [2] showed a drastic reduction in impact point dispersion using a lateral pulse jet mechanism coupled to a trajectory tracking flight control system. Parametric trade studies of pulse jet mechanism that effected the impact point dispersion were also conducted. Burchett [3] developed a specialized control technique that blended model predictive control and projectile linear theory with a lateral pulse jet mechanism. The closed-form solution of projectile linear theory was used to map the current state of rocket to the target plane and base control action on projected miss distance and direction. Ollerenshaw [4] employed the control scheme similar to Burchett employed, but to predict the future state along trajectory projectile and used the canards as continuous control input to minimize an error between predicted projectile states and a desired trajectory leading to the target.

This paper presents one of the alternative ways to reduce the rocket dispersion using the combination of six-degree of freedom (6DOF) trajectory simulation and lateral pulse jet thrust model. The 6DOF trajectory simulation was used in prior to predict the impact point and the lateral pulse jet thrust model was built based on difference between the predicted impact point and target location. To fire a pulse jet from an arbitrary state, the desired pulse jet thrust magnitude was correlated with a given set of trajectory states using the multi linear regression. The proposed method was verified by the comparison of dispersion radius between the controlled and uncontrolled case.

2. Six-Degree of Freedom Model

The 6DOF rocket model has been treated in large numbers of literature. A fire control program [5] developed in author's laboratory also employed the 6DOF model in the simulation of rocket trajectory. The 6DOF model in this program can be adequately represented under realistic conditions by inputting different data such as propellant thrust profile, mass profile, moment of inertia profile, center of gravity, aerodynamic data and meteorological data. In this study, the program was mainly used to compute the projected impact point in horizontal target plane. Then, the program is later modified to determine the pulse jet force magnitude required for correcting the miss distance. The schematic of rocket with pulse jet force components and an axis identified is illustrated in Figure 1 (a). The range and drift error in horizontal target plane is denoted in Figure 1 (b). The rocket configuration considered here was 2.9 m. long, had a 0.122 m. diameter and weighted 70.3 kg. In the study case, the pulse jet force was applied at the mass center, thus the rocket dynamic could be considered as the motion of point mass. And, each pulse jet could only fire one single time.

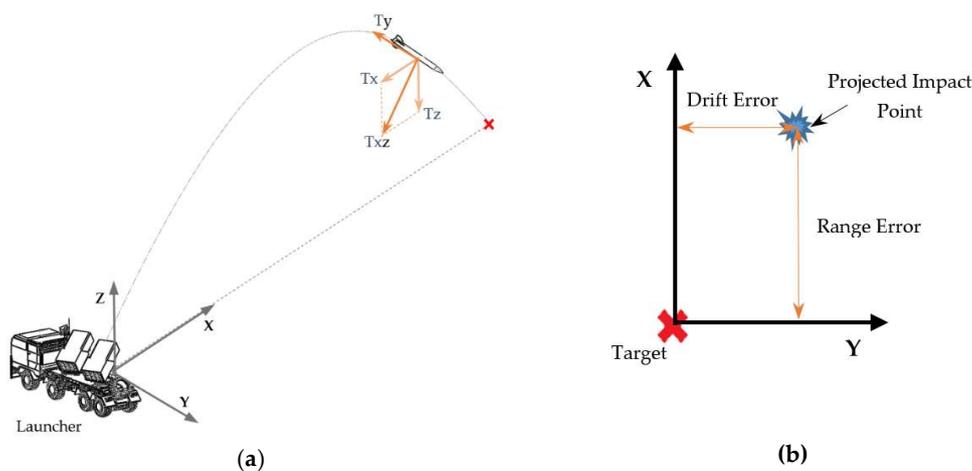


Figure 1. (a) Schematic of coordinate of launcher and rocket with pulse jet thrust; (b) Range error and drift error in the horizontal target plane.

The schematic of 6DOF trajectory simulation is shown in a dash box in Figure 2. The 6DOF model is numerically integrated using a fourth-order Runge-Kutta algorithm. By having prior knowledge of the projectile of the representative rocket that the total flight time duration is 114 sec and the time at the highest altitude is 50 sec, hence in this study the pulse jet firing was chosen during descending flight at the time of 80.0 – 80.6 sec and the trajectory states were measured at the time of 75 sec.

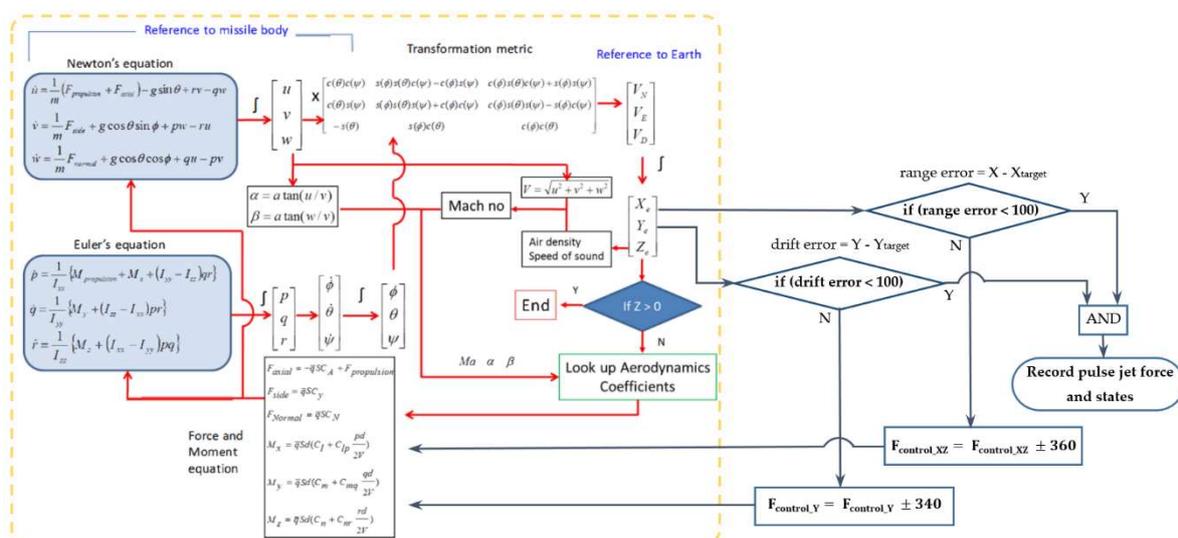


Figure 2. Schematic of 6DOF trajectory simulation [5] modified to compute the desired pulse jet magnitude.

To determine the pulse jet magnitude, a steady increasing or decreasing number of pulse jet force was added to the force equation of motion until the impact point was within a specific miss distance, as shown schematically in Figure 2. Once a computation was completed, the last pulse jet magnitude and projectile states were recorded as the data for use later. In the next section, the method to model the relationship between the pulse jet thrust and projectile states is discussed in detail.

3. Lateral Pulse Jet Thrust Model

The pulse jet thrust model is the correlation between the desired pulse jet thrust and a given set of projectile states. To build an accurate model, the amount of data in a whole range of values should be enough. In this study, the number of data generated from 6DOF simulation is in total of 1090. The data is split into two datasets. The first of 1040 data is to train the model and the second of 50 data is to test the model. Table 1 shows the example data of the first five rows. Each row represents the data from a single completed computation as the scheme in Figure 2.

Table 1. Example data of pulse jet thrust and projectile states.

| | Ty | Txz | X | Y | Z | u | v | w | ax | ay | az | theta | psi | phi | p | q | r | pdot | qdot | rdot |
|---|-------|-------|----------|---------|-----------|---------|--------|---------|--------|--------|-------|--------|--------|--------|--------|--------|--------|-------|--------|--------|
| 0 | 2040 | 2160 | 26467.87 | -532.16 | -12495.01 | 265.039 | -5.328 | 227.564 | -2.292 | 0.048 | 7.838 | -0.709 | -0.020 | 2.379 | 20.098 | 0.015 | 0.014 | 0.386 | 0.295 | -0.310 |
| 1 | 2720 | 2880 | 26093.47 | -795.69 | -13370.50 | 267.721 | -8.166 | 219.270 | -1.992 | 0.062 | 8.170 | -0.686 | -0.031 | -1.363 | 18.794 | -0.003 | -0.020 | 0.396 | -0.403 | 0.086 |
| 2 | -340 | -1080 | 25173.19 | 158.42 | -13078.01 | 257.066 | 1.672 | 222.833 | -1.947 | -0.008 | 8.115 | -0.714 | 0.005 | 2.496 | 18.828 | 0.016 | 0.011 | 0.433 | 0.246 | -0.328 |
| 3 | 1700 | 3960 | 26730.85 | -469.80 | -12737.76 | 269.380 | -4.733 | 224.887 | -2.264 | 0.042 | 7.915 | -0.695 | -0.018 | -0.338 | 20.640 | -0.019 | -0.006 | 0.389 | -0.146 | 0.418 |
| 4 | -4420 | 360 | 25939.88 | 1237.47 | -12592.78 | 261.168 | 12.468 | 226.898 | -2.187 | -0.100 | 7.905 | -0.715 | 0.048 | 2.591 | 20.454 | 0.018 | 0.011 | 0.420 | 0.228 | -0.373 |

One of machine learning algorithms called multiple linear regression was employed to model the relationship between the pulse jet thrust (Ty and Txz) and a given set of projectile states (X, Y, Z, u, v, w, ax, ay, az, theta, psi, phi, p, q, r, pdot, qdot and rdot). The algorithm was written in Python language using Anaconda software package. Figure 3 shows the correlation heatmap representing the correlation coefficient between two different variables. It is seen that the pulse jet thrust Ty is highly correlated with the states Y, v, ay and psi. And, the pulse jet thrust Txz is highly correlated with the state X, Z, u, w, ax, az and theta.

| | Ty | Txz | X | Y | Z | u | v | w | ax | ay | az | theta | psi | phi | p | q | r | pdot | qdot | rdot |
|-----|--------|-------|-------|--------|--------|-------|--------|---------|---------|-------|--------|--------|--------|-------|-------|--------|-------|---------|-------|--------|
| Ty | 1 | 0.022 | 0.024 | -1 | 0.0012 | 0.019 | -1 | 0.00082 | -0.0057 | 0.99 | -2e-05 | 0.0096 | -1 | 0.072 | 0.018 | 0.018 | 0.038 | -0.0015 | 0.036 | -0.011 |
| Txz | -0.022 | 1 | 0.54 | -0.022 | -0.56 | 0.97 | -0.022 | -0.59 | 0.39 | 0.023 | 0.52 | 0.8 | -0.023 | 0.039 | 0.14 | -0.023 | 0.033 | 0.18 | 0.042 | 0.015 |

Figure 3. Correlation coefficient between lateral pulse jet thrust and projectile states.

Note that the correlation coefficient between two state variables is not shown here due to space limitation. If the correlation between two state variables was greater than 0.7 or less than -0.7, it would cause collinearity which resulted in distorted model prediction [6]. For this reason, the state variables correlating with the pulse jet thrust Ty were reduced to only state Y, since the correlation with each other among the group of states Y, v, ay and psi were almost 1. In the same manner, the state variables correlating with the pulse jet thrust Txz were reduced to only state X, Z and u, since the correlation with each other among the group of states u, w, ax, az, and theta were more than 0.75. To perform the algorithm, the regression model can be expressed as follows:

$$Ty = -3.667Y - 20.414, \quad (1)$$

$$Txz = 3.147X - 1.68Z + 78.577u - 122754.45, \quad (2)$$

The models were tested with 50 unseen data to measure the predictability of models. The explained variance score of both models appeared approximately 0.99 which indicated the models had a good predictability. The pulse jet thrust in equation (1) and (2) were considered to be the control force and put into the 6DOF equation of motion. It was assumed that the trajectory states were sensed by an onboard inertial measurement unit (IMU).

4. Results and Discussion

The comparison of impact point dispersion between the uncontrolled case and controlled case was proceeded through a Monte Carlo simulation. Ten uncertainties that caused the dispersion, for example, launcher orientation, initial spin rate, drag coefficient and wind speed were selected in the simulation. All uncertainties were described by normal distribution with six standard deviation. The sample size of trajectory was 50. After running the simulation, the random spot of impact point for uncontrolled and controlled case was shown in Figure 4. Circular error probable (CEP) which is defined as the radius of circle containing 50% of the impact points and centered at the mean impact point was used to measure the dispersion for determining rocket accuracy. The result showed the great improvement in the CEP from 448 m for the uncontrolled case to 27 m for the controlled case.

It should be noted that the amount and quality of data influenced the accuracy of this control scheme. Therefore, an attention on data generation from 6DOF model should be paid. The input to the 6DOF model should correspond with an actual condition such as thrust profile, mass profile, and meteorological data. Moreover, from the scheme in Figure 2, the data could have been generated to be more accurately by decreasing a miss distance criterion or a specific amount of pulse jet force.

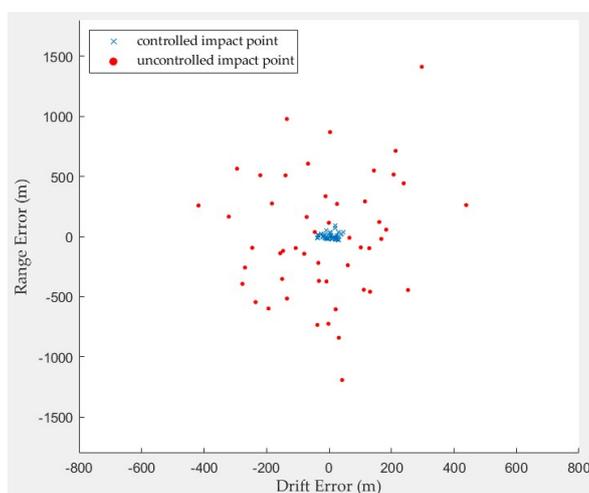


Figure 4. Uncontrolled dispersion (CEP = 448 m) and controlled dispersion (CEP = 27 m)

5. Conclusions

The proposed control scheme to reduce the rocket dispersion by the 6DOF trajectory simulation combined with lateral pulse jet model was elaborately described. The scheme was verified by comparison of dispersion radius between the uncontrolled and controlled case. The result showed that the rocket dispersion could be reduced significantly. Since the accuracy of the control scheme depends on the amount and quality of data, better prepared data could further reduce the dispersion.

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