

# Exactly Solvable Bose and Fermi many-body Hamiltonian with higher order terms based on the S2 symmetry

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It is shown that the two component Fermi or Bose many-body Hamiltonian, such as the two-orbit fermion pairing and two-site Bose-Hubbard model with arbitrary finite higher order terms can always be solved exactly by using Bethe ansatz vector construction based on the permutation of two components of bosons or fermions involved. As examples of the solution, the extended one-dimensional dimer Bose-Hubbard model with multi-body interactions and the mean-field plus orbit-dependent non-separable pairing model with two non-degenerate j-orbits are demonstrated with the eigenstates and the eigen-energy and the related Bethe ansatz equations. It is shown that the main feature of the solutions lies in the fact that the Bethe ansatz vectors can be expressed in terms of binomials of the boson or fermion operators times the related symmetric functions. As the consequence, two-component quantum many-body systems, such as the extended Lipkin-Meshkov-Glick model with higher-order interactions, can be solved in a similar way.

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