Testing Noncommutative Spacetimes and Violations of the Pauli Exclusion Principle with underground experiments

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We propose to deploy limits that arise from different tests of the Pauli Exclusion Principle in order: i) to provide theories of quantum gravity with an experimental guidance; ii) to distinguish among the plethora of possible models the ones that are already ruled out by current data; iii) to direct future attempts to be in accordance with experimental constraints. We firstly review experimental bounds on nuclear processes forbidden by the Pauli Exclusion Principle, which have been derived by several experimental collaborations making use of different detector materials. Distinct features of the experimental devices entail sensitivities on the constraints hitherto achieved that may differ one another by several orders of magnitude. We show that with choices of these limits, renown examples of flat noncommutative space-time instantiations of quantum gravity can be heavily constrained, and eventually ruled out. We devote particular attention to the analysis of the κ-Minkowski and θ-Minkowski noncommutative spacetimes. These are deeply connected to some scenarios in string theory, loop quantum gravity and noncommutative geometry. We emphasize that the severe constraints on these quantum spacetimes, although cannot rule out theories of top-down quantum gravity to whom are connected in various way, provide a powerful limitations of those models that it will make sense to focus on in the future.

PACS numbers:

I. INTRODUCTION

The Pauli Exclusion Principle (PEP) is a direct implication of the Spin Statistics theorem (SST) stated by Pauli in Ref. [1]. PEP automatically arises from anti-commutation properties of fermionic creation and annihilation operators in the construction of the Fock space of the theory. In turn, the SST was proven by assuming Lorentz invariance. This certainly implies that the PEP is closely connected to the structure of the space-time itself. The PEP is indeed a successful fundamental principle not only when addressed from theoretical quantum field theory considerations, but also in high precision agreement with every atomics, nuclear and particle physics experimental data. In other words, if the PEP is violated, the violating channels must be parametrized by very tiny coupling constants in front of the PEP-violating operators. This possibility was suggested within an effective field theory approach in Refs. [2–10].

The possibility of renormalizable PEP-violating operators might be seen as "un-aesthetic" and un-natural. However, the possibility of non-renormalizable effective operators induced by a PEP-violating new physics scale is still an open and natural possibility, which is predicted by many possible models of quantum gravity realizing an ultraviolet completion. A possible way to violate the PEP is, of course, to relax the main hypothesis on the basis of the Spin Statistics theorem. For example, as mentioned above, the theorem in its standard enunciation — namely in terms of commutation relation for bosonic ladder operators, and anticommutation relation for fermionic ladder operators — is no more valid if Lorentz invariance is relaxed. Lorentz symmetry is one of the basis of the Standard Model of particle physics: its explicit violation must allow any possible Lorentz Violating and CPT violating renormalizable operators. Even finetuned to very small couplings, the latter operators will introduce new UV divergent diagrams in the Standard Model sector, affecting the basic requirement of unitarity of the theory. This is why the Spin Statistics theorem, as a companion of Lorentz symmetry, is considered a *milestone* of the Standard Model. Notice furthermore, as pointed out in Ref. [11], that Lorentz violating effects — for instance induced by the Planck scale in quantum gravity — might manifest themselves in the propagation of low-energy particles with a sizable magnitude that in some cases is already ruled out by experimental data[1].

Nonetheless, the eventuality that the Lorentz Symmetry is dynamically or spontaneously broken at a very high energy scale Λ_{UV} is still open, and this must turn into the generation of non-renormalizable operators suppressed as inverse powers of Λ_{UV}. For example, many quantum gravity theories predict a *noncommutative* space-

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[1] It was shown in Ref. [11] that only a strong and unnatural fine-tuning of the bare parameters of the theory may prevent from Lorentz violations at the percent level. Nonetheless, this analysis anyway does not take into account the possibility of a deformation of the Lorentz symmetries.

time geometry close to the Planck length scale. The idea that the space-time can be noncommutative was accredited to *W. Heisenberg* in Ref. [12] and elaborated later on in Refs. [13, 14]. After few decades it was realized that noncommutativity of space-time can be rediscovered within the context of both[2] string theory [16–20] and loop quantum gravity [21–27]. Besides these two frameworks, many other studies have shed light on the emergence of deformed symmetries as a feature of effective theories that can be derived from (more fundamental?) non-perturbative models of quantum geometry — see *e.g.* Ref. [28].

Several studies were hitherto devoted also to the analysis of the physical meaning of deformed symmetries in spacetime, as *e.g.* in Refs. [29–37]. Some possible phenomenological consequences were also investigated in Refs [38–41], at least for those cases of noncommutativity that are the most manageable, namely those specific classes of models deforming the Lorentz symmetry to a noncocommutative space-time deformed symmetry group called κ-Poincaré and θ-Poincaré. Among these latter, there exists a specific class of models enjoying θ-Poincaré symmetries, which can preserve the unitarity of S-matrix in the Standard Model sector. This comes with a restriction [42] on the components of the space-time noncommutative matrix $\theta^{\mu\nu}$. Under these assumptions, noncommutative quantum field theories with the Groenewold-Moyal product will not lead to catastrophic violation of the probability conservation principle [43–47].

For these θ-deformed theories, and for the class of deformations that enjoy κ-Poincaré symmetries [48], it is possible to show that deformed versions of the CPT theorem exists, or anyway that a deformed notion of discrete C, P and T symmetries can be recovered. This entails the introduction of a deformed SST, which encodes deformed commutation and anticommutation quantization rules, and thus deviation from the standard CPT theorem, which is nevertheless predicted to be small [49]. These deviations consequently lead to a violation of the standard PEP. Furthermore, it was enlightened in a series of work that CPT violation does not necessarily lead to violations of Lorentz invariance [50], and vice versa [50, 51], dismantling the bases of a well celebrated no-go theorem that was instead predicting this link, based on standard relativistic quantum field theory. These results call for an investigations of PEP directly at the level of the Fock space of the theory, where the breakdown or deformation of the theory induces deviations from ordinary statistics. In such a large panorama of possibilities, an effective parametrization approach is highly motivated, in order to experimentally distinguish among different scenarios that are theoretically allowed. We will account for deviations from commutation/anti-commutation relations of the creation and annihilation operators, which act on the vacuum in the Fock space of the theories, and then show how the cases of the theories enjoying κ-Poincaré symmetries θ-Poincaré lie in a specific class of parametrization that allows a phenomenological falsification of (standard) PEP violations.

II. PARAMETRIZATION

Operatively, deviations from the PEP in the commutation/anti-commutation relations can be parametrized — see *e.g.* Refs. [8–10] — by introducing a deviation function $q(E)$, *i.e.*

$$a_i a_j^\dagger - q(E) a_j^\dagger a_i = \delta_{ij}\,, \tag{1}$$

$$q(E) = -1 + \beta^2(E), \quad \text{and finally} \quad \delta^2(E) = \beta^2(E)/2\,.$$

Among the possible parameterizations of the function $\delta^2(E)$, we propose a class corresponding to models that, depending on the order in the inverse powers of the energy scale of Lorentz violation, are classified at the k-th order as

$$M_k: \quad \delta^2(E) = c_k \frac{E^k}{\Lambda^k} + O(E^{k+1})\,.$$

The phenomenological method we deploy here naturally takes into account, through an analytic expansion driven by dimensional analysis, the corrections to the standard statistics that may arise, in the infrared limit, from UV-complete quantum field theories. This parametrization can capture every possible first term of the power series expansions in E/Λ, for every possible deformation function $q(E)$ in Eq. (1). In other words, constraints on $\delta(E)$ can be translated into constraints on the new physics scale within the framework of the M_n parametrization.

III. LIMITS ON PEP VIOLATING PROCESSES BY UNDERGROUND EXPERIMENTS

In order to investigate the aforementioned models we start referring to results obtained by the underground experiments. Fig. 1 shows the most stringent limits on the relative strength (δ^2) for the searched non-paulian transitions.

Several methods of experimental investigations for testing PEP have been used so far. The VIP experiment [52] uses a method of searching for PEP forbidden atomic transitions in copper; the limits on the probability that PEP is violated by electrons are reported in Fig. 1. The experimental method consists in the injection of "fresh" electrons into a copper strip, by means of a circulating current, and in the search for the X-rays following the possible PEP forbidden radiative transitions that occur if one of these electrons is captured by a copper atom and

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[2] The fact that it appears in both theories may not be just a coincidence, in light of a new **H**-duality conjecture that has been recently formulated in Ref. [15].
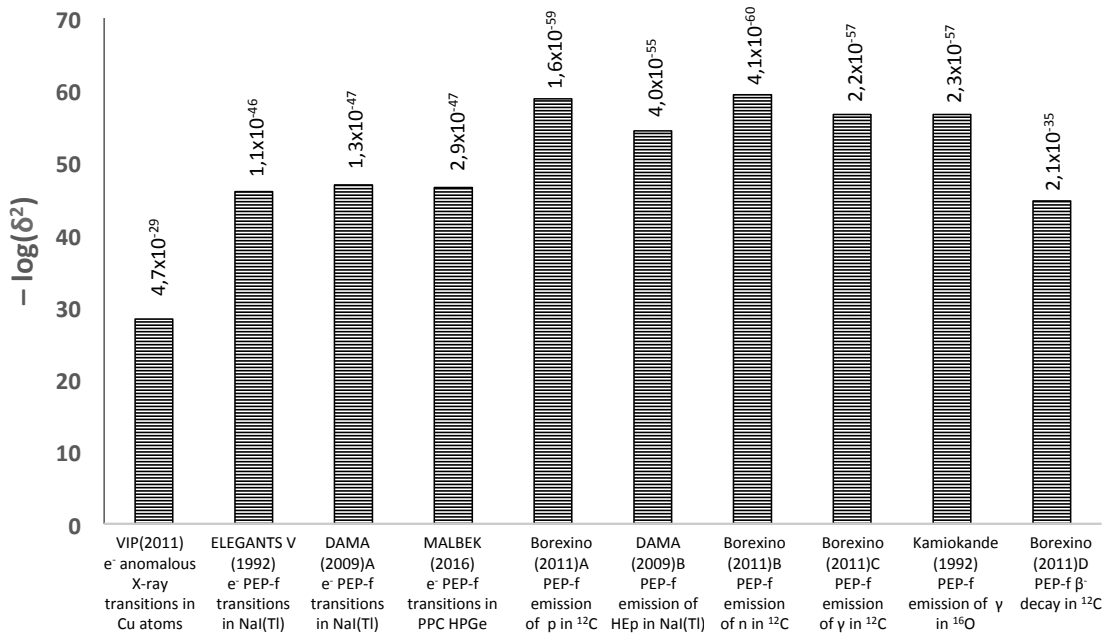
FIG. 1: Limits at 90% C.L. on various PEP violation channels in logarithmic scale, displaying the $-\log \delta^2$ for different experimental collaborations: VIP(2011) [52]; ELEGANTS V (1992) [53]; DAMA(2009)A [54]; MALBEK(2016) [55]; Borexino(2011)A [56]; DAMA(2009)B [54]; Borexino(2011)B [56]; Borexino(2011)C [56]; Kamiokande(1992) [57]; Borexino(2011)D [56].

cascades down to the already-filled 1S state. In particular, the experiment is searching for the K_α (2P $\rightarrow$ 1S) transition. The energy of this PEP forbidden transition (7.729 keV) would differ from the normal K_α transition energy (8.040 keV) by a Δ term (about 300 eV) due to the presence of the other electrons in the already-filled shell. This energy shift can be detected by the high resolution CCD devices.

PEP forbidden radiative atomic transitions are also searched for in Iodine atoms deploying NaI(Tl) detectors, as done in DAMA/LIBRA (DAMA(2009)A in Fig. 1) [54] and ELEGANTS V [53] experiments, and in Germanium atoms in PPC HPGe detectors of the MALBEK experiment [55] (see Fig. 1). In such cases, when a PEP-violating electronic transition occurs, X-rays and Auger electrons are emitted by the transition itself and by the following rearrangements of the atomic shell. The detection efficiency of such radiation in the NaI(Tl) detectors of DAMA/LIBRA is $\simeq 1$ at the low energy of the process. Thus, all the ionization energy for the considered shell is detected, but it is actually shifted by a Δ term due to the presence of the other electrons in the already filled shells. Generally, in this class of experiments the K-shell is considered, as it provides the largest available energy in the subsequent X-rays /Auger-electrons radiation emission. However, stringent limits (not reported in Fig. 1) were also obtained by DAMA/NaI looking for

transitions to L-shell in Iodine atoms [58], providing 4-5 keV radiation emission, thanks to the low energy thresholds of such NaI(Tl) detectors.

The most stringent constraint on this class of PEP violations in atomic transitions comes from the DAMA/LIBRA experiment, a 250 kg array of highly radiopure NaI(Tl) detectors hosted in the Gran Sasso National Laboratory. DAMA/LIBRA searched for PEP violating K-shell transitions in Iodine using the data corresponding to 0.53 ton$\times$yr; a lower limit on the transition lifetime of 4.7×10^{30} s has been set, giving $\delta^2 < 1.28 \times 10^{-47}$ at 90% C.L. [54]. This value is reported in Fig. 1.

A similar experiment, MALBEK, has been using a high-purity germanium (HPGe) detector with an energy threshold suitable for observing the transition from L- to K- shells in germanium. In this case, the energy of the transition has been calculated to be 9.5 keV [55], once shifted down by the Δ term. The obtained limit on δ^2 is also reported in Fig. 1.

A different approach for studying PEP violating processes has been exploited by DAMA/LIBRA collaboration (DAMA(2009)B in Fig. 1) [54]. Specifically, PEP violating transitions in nuclear shells of ^{23}Na and ^{127}I are investigated by studying possible protons emitted with $E_p \geq 10$ MeV. In such a case, events with only one detector fires, that is each detector has all the others as

veto, are considered to search for high energy protons. The rate of emission of high energy protons ($E_p \geq 10$ MeV) due to PEP violating transitions in ^{23}Na and ^{127}I was constrained to be $\lesssim 1.63 \times 10^{-33}$ s^{-1} (90% C.L.) [54]. This corresponds to a limit on the relative strength of the searched PEP violating transitions: $\delta^2 \lesssim 4 \times 10^{-55}$ at 90% C.L. (see Fig. 1).

Moreover, the Pauli exclusion principle has also been tested with the Borexino detector [56], considering the nucleons in the ^{12}C nuclei. The research profits of the extremely low background and of the large mass (278 tons) of the Borexino detector. The exploited method is to look for γ, $\beta^\pm$, neutrons, and protons, emitted in a PEP violating transition of nucleons from the $1P_{3/2}$ shell to the filled $1S_{1/2}$ shell and the following limits on the lifetimes for the different PEP violating transitions were set [56] (all the limits are at 90% C.L.): $\tau(^{12}\text{C} \to ^{12}\widetilde{C}+\gamma)$ $\geq 5.0 \times 10^{31}$ yr, $\tau(^{12}\text{C} \to ^{12}\widetilde{N}+\text{e}^- + \bar{\nu}_e) \geq 3.1 \times 10^{30}$ yr, $\tau(^{12}\text{C} \to ^{12}\widetilde{B}+\text{e}^+ + \nu_e) \geq 2.1 \times 10^{30}$ yr $\tau(^{12}\text{C} \to ^{11}\widetilde{B}+\text{p})$ $\geq 8.9 \times 10^{29}$ yr, and $\tau(^{12}\text{C} \to ^{11}\widetilde{C}+\text{n}) \geq 3.4 \times 10^{30}$ yr.

These limits correspond to constraints on the relative strengths for the searched PEP violating electromagnetic, strong and weak transitions: $\delta^2_\gamma \leq 2.2 \times 10^{-57}$, $\delta^2_N \leq 4.1 \times 10^{-60}$, and $\delta^2_\beta \leq 2.1 \times 10^{-35}$ (see Fig. 1) [56].

Finally, we report here the results obtained by the large underground water Cherenkov detector, Kamiokande [57], where anomalous emission of γ rays in the energy range $19 - 50$ MeV has been searched for. No statistically significant excess was found above the background; this allows to set a limit on the lifetime of PEP violating transitions to $9.0 \times 10^{30} \times$ Br(γ) yr per oxygen nucleus, where Br(γ) is the branching ratio of the ^{16}O decay in the γ channel. In the case the PEP violating transitions is due to the p-shell nucleons, then the limit is $1.0 \times 10^{32} \times$ Br(γ) yr. Thus, the limit at 90% C.L. of the relative strength for forbidden transitions to normal ones is $\delta^2 < 2.3 \times 10^{-57}$ [57], it is also shown in Fig. 1.

IV. IMPLICATIONS TO PLANCK-SCALE DEFORMED SYMMETRIES

We can start considering a generic model, with the assignments M_k, for $k \in \mathbb{N}$. On these latter, using the DAMA/LIBRA results as example, the following constraints can be derived:

$$\delta^2 \leq 4 \times 10^{-55} \longleftrightarrow \delta^2(E) = c_k \frac{E^k}{\Lambda^k} \leq 4 \times 10^{-55}. \quad (2)$$

We are interested in those cases that are mostly motivated by quantum gravity scenarios. This corresponds to select $\Lambda = M_{Pl} \simeq 1.22 \times 10^{19}$ GeV. A straightforward estimate of k can be then achieved, which has already dramatic consequences for several models of quantum gravity. Since nuclear decays processes taking place in the detector have an energy whose order of magnitude is few times 10^{-3} GeV, we may consider $E = 10 \pm 1$ MeV. For

a set of heuristic choices of $c_k = \{1, 4, 10\}$, this implies immediately that at 90% C.L. only $k > k^*$ power suppressions are still experimentally allowed, with respectively[3] $k^* = \{2.58, 2.61, 2.63\} \pm 0.01$. The exclusion limits on the k–Λ plane are displayed in Fig. 2, in which we use the accurate values for E that pertain the different experiments analyzed, and set the coefficients[4] $c_k = 1$. The most stringent constraints on the k–Λ parameters' plane, obtained by the above-mentioned experimental limits on the relative strength for non-paulian transitions, are provided by the Borexino [56], Kamiokande [57] and DAMA [54] collaborations.

A different scenario arises while working at a scale of energy $E \simeq 10$ keV, which is induced by transitions from k-electronic shells. This provides the upper bound $\delta^2 \simeq 10^{-47} - 10^{-48}$, which is less stringent than the former one. Nonetheless, it still entails rejection of PEP violating terms that are suppressed at the second order in Λ, and at the same time are regulated by coefficients c_n of order one.

Below we focus on PEP violations that arise in the aforementioned models of noncommutative spacetime, with particular focus on models endowed with κ-Poincaré deformed symmetries, and θ-Poincaré deformed symmetries. Notice that the latter can be recast in the language of the noncommutative geometries à la Connes [59], while for the former the equivalence has been proven hitherto under certain restrictions [60, 61]. Besides the links to noncommutative geometry, noncommutative spacetimes have also been derived in several frameworks of quantum gravity, most notably in string theory and loop quantum gravity. In the former scenario the Groenewold-Moyal noncommutativity is induced by the expectation value of background B fields [20], while in the latter several instantiation of κ-deformation have been so far derived, with mesoscale deviations from Lorentz invariance — see e.g. [22–24, 26]. Although these derivations are not yet decisively answering the question about the low energy limit of string theory and loop quantum gravity, the constraints that we are providing here will have the undoubtable advantage to furnish a guidance for the development of theoretical models of quantum gravity.

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[3] The propagation of the error only affects the last digit of k^*, and is effectively independent on these heuristic choices of c_k, which capture the range of values in the literature — see e.g. Refs. [29, 38, 47, 49, 63–65]. On the other hand, the theoretical ambiguity on c_k may affect the estimate of k^* up to 2% of its value.

[4] As remarked in the previous footnote, this choice is motivated by the fact that in the literature about noncommutative spacetimes the c_k coefficients are order 1.
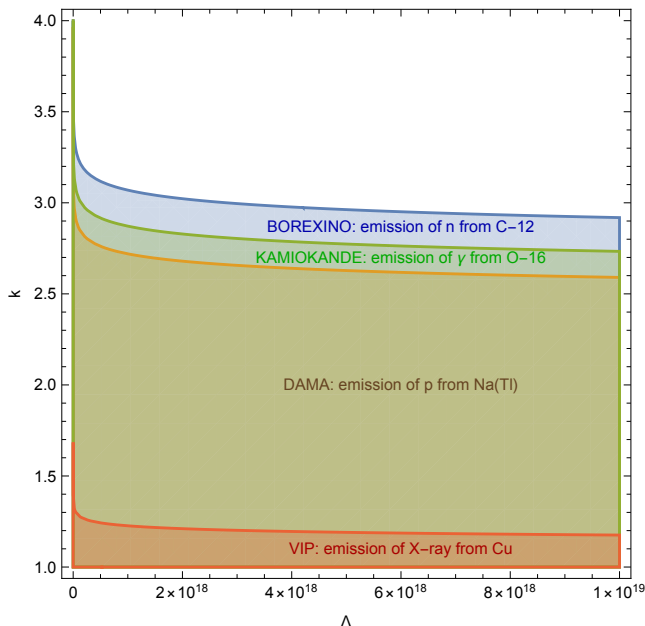
FIG. 2: Exclusion limits at 90% C.L. on the k–Λ parameters' plane. Most stringent constraints are provided by the Borexino [56], Kamiokande [57] and DAMA [54] collaborations.

A. The case of the κ-Poincaré group

Small departures from locality — an essential requirement for micro-causality in standard quantum field theory — may be kinematically or dynamically generated in some quantum gravity scenarios, and have been shown in [62] to be connected to the emergence of deformed κ-Poincaré symmetries. This is a non-trivial Hopf-algebra of symmetries dual to the κ-Minkowski non-commutative spacetime. The latter is characterized by commutation relations among the spacetime point coordinates of the type

$$[x_i, x_j] = 0 \qquad \text{and} \qquad [x_i, x_0] = \frac{\iota}{\kappa} x_i \,,$$

where κ denotes a scale of energy assumed to be order of the Planck scale in effective quantum gravity frameworks. There exists at least a basis of the Hopf algebra in which the Lorentz sector is standard, but the action of the Lorentz generators on the translation subgroup is κ-deformed. For instance, in the *bicrossproduct basis* there is one $\mathcal{K}$-deformed commutator, namely

$$[P_l, N_j] = -\iota\delta_{lj}\left(\frac{\kappa}{2}\left(1 - e^{-\frac{2P_0}{\kappa}}\right)\frac{1}{2\kappa}\mathbf{P}^2\right) + \frac{\iota}{\kappa}P_l\,P_j \,,$$

where P_μ denote spacetime translation generators and N_l boost generators. Even in this basis, in which translation generators remain commutative, the co-product map Δ acquires a κ-deformation. This is a remarkable deviation from standard properties. When P_μ are represented as derivatives acting on "coordinates-ordered" exponentials

[48], it is trivial to recognize that Δ generalizes the Leibnitz rule of derivatives' action. The κ-deformed Leibnitz rule can then be inferred by the only κ-deformed co-product, *i.e.*

$$\Delta(P_j) = P_j \otimes \mathbb{1} + e^{-\frac{P_0}{\kappa}} \otimes P_j \,.$$

The fate of discrete symmetries in the κ-Poincaré setting was addressed in Ref. [48], while a detailed analysis of the fate of the CPT theorem for κ-Poincaré symmetries and of its consequences is still missing. Nonetheless, a phenomenological analysis of deviations from the standard case is still possible. Moving from the parametrization in Eq. (1), by straightforward dimensional arguments we can express

$$\delta^2(E) = c_1\kappa E \,, \tag{3}$$

where it is assumed that $\kappa \simeq M_{Pl}^{-1}$. This implies automatically the rejection of every model available in the literature that predicts a c_1 non-vanishing and of order one.

Following a constructive procedure, we can show that most part of the models hitherto addressed in the literature — see *e.g.* Refs. [29, 38, 63–65] — either reproduce the case c_1 order one, or they fall in the class of a vanishing c_1, for which they cannot be falsified at the level of PEP violations. For instance, in Refs. [29, 38, 63] $c_1 = 1$, and consequently the models are ruled out. While in Refs. [64, 65], where $c_k = 0$, for $k \in \mathbb{N}$, the commutation relations are unmodified. This scenario can be then falsified up to the second order in the ratio E/M_{Pl}, but is not distinguishable from the standard unmodified case.

B. The case of the Groenewold-Moyal plane

The algebra $\mathcal{A}(R^d)$ of commutative functions on a smooth d-dimensional space-time manifold can be mapped into that one of noncommutative functions on the Groenewold-Moyal plane $\mathcal{A}_\theta(R^d)$, if the star-product is considered $(\alpha * \beta)(x) = \left(\alpha\,e^{\frac{\iota}{2}\overleftarrow{\partial_\mu}\theta^{\mu\nu}\overrightarrow{\partial_\nu}}\beta\right)(x)$, where $\theta^{\mu\nu} = -\theta^{\nu\mu}$ and $x = (x^0, ..., x^{d-1})$. Accordingly, the Groenewold-Moyal (GM) multiplication map m_θ reads

$$m_\theta(\alpha \otimes \beta) = m_0\left(e^{-\frac{\iota}{2}\theta^{\mu\nu}(-\iota\partial_\mu)\otimes(-\iota\partial_\nu)} \cdot (\alpha \otimes \beta)\right) \,, \tag{4}$$

where $m_0(\gamma \otimes \delta)(x) \equiv \gamma(x)\delta(x)$ stands for the standard point-wise multiplication rule.

Introducing the invertible element of the $\mathcal{R}$-matrix

$$\mathcal{F}_\theta = e^{-\frac{\iota}{2}(-\iota\partial_\mu)\theta^{\mu\nu}\otimes(-\iota\partial_\nu)} \,,$$

the GM multiplication rule can be recast as $m_\theta(\alpha \otimes \beta) = m_0(\mathcal{F}_\theta\alpha \otimes \beta)$. The invertible element of the $\mathcal{R}$-matrix enters in a natural way the twisted deformation of the Fock space of scalar field theory, with spin zero, and thus the commutation relations of the ladder operators, *i.e.*

$$a(p)a^\dagger(q) = \tilde{\eta}'(p,q)\tilde{F}_\theta^{-2}(-q,p)a^\dagger(q)a(p)$$

$$+2p_0\delta^d(p-q)\,, \qquad (5)$$

where $\tilde{\eta}'$ approaches the constant $+1$ in the low energy limit — this is formally equivalent to the commutative limit $\theta^{\mu\nu} \to 0$. Anti-commutation relations for free spinor fields are equal to the ones given in Eq. (5), provided that $\tilde{\eta}'$ approaches -1 in the low energy limit.

We may expand Eq. (5) at first order in $\theta^{\mu\nu}$, neglecting orders $O(\theta_{\mu\nu}\theta^{\mu\nu})$. This corresponds to a second order expansion in Λ, since $\theta^{\mu\nu}$ has dimension of length square. This immediately entails $c_1 \equiv 0$, and allows to set $c_2 = 1$ provided that $\theta^{12} = \theta^{13} = \theta^{23} = 1/(3\Lambda^2)$. Focusing on the data provided by DAMA (2009) B [54], and accounting for an isotropic distribution of the protons' momenta, we obtain that

$$\delta_\theta^2 = \left(\frac{E}{\Lambda}\right)^2\,,$$

the exponent of which can be confronted with the values of k excluded in Fig. 2. Thus this model seems to be already excluded by present data. This is as transparent as surprising, since it was never pointed out in the wide literature devoted to non-commutative space-times.

C. Quantum gravity with lower energy scales

We can resort to the experimental bound in (2) in order to constrain departures from the standard spin-statistics theorem within those theoretical frameworks that predict a lower energy scale of quantum gravity. Several models fit this scenario, notably the proposals that took into account an eventual role of large scale extra dimensions in the resolution of the hierarchy problem — see *e.g.* Refs [66–68]. It is then straightforward to check that any violation of PEP could arise up to the ninth order in the ratio E/Λ, within those proposals where the scale of quantum gravity is lowered down to the threshold hitherto achievable on terrestrial experiments, $\Lambda \simeq 10$ TeV. This rules out any reliable model of extra dimension that would break Lorentz invariance and would predict violations of PEP.

V. CONCLUSIONS AND OUTLOOKS

Although a direct connection between deformation of space-time symmetries and quantum gravity has not yet been decisively proved, nonetheless there are many results in the literature that provide a clear instantiations of space-time symmetry deformation or space-time symmetry breakdown regulated by the Planck scale. Making contact in particular with those models that predict a deformation of the energy-momentum dispersion relations for one-particle states, and thus entail a deformation of the Fock space states and of the SST, we developed a framework to falsify these scenarios accounting for possible PEP violations.

We emphasize that the phenomenological analysis we developed here differentiates from previous phenomenological investigations accounting for the one-particle Hilbert space structure of quantum field theories on non-commutative space-times. Constraints on energy-momentum dispersion relations do apply only to certain classes on non-commutative space-times. For instance, quantum field theories endowed with κ-Poincaré symmetries, in which the algebra and the mass Casimir are deformed, provide an arena to test deformations of the energy-momentum dispersion relations. On the other hand, quantum field theories endowed with θ-Poincaré symmetries can be falsified only by looking at deformations of the Fock space structure, including eventual violations of the Pauli exclusion principle.

The tightest constraints on in-vacuo dispersion relations that are sensitive to the Planck-scale, as discussed within the phenomenological models in Refs [69–74], are provided for photons by the observation of TeV flares originated from active galactic nuclei at redshift smaller than 1 — see e.g. Refs. [74–79]. Taking into account deformation's effects that are linear in the Planck length scale, the bounds can reach $1/10$ of the Planck scale. On the other hand, the best constraints on anomalous in-vacuo dispersion that are quadratic in the Planck length-scale may be obtained from the detection of neutrinos[5] emitted by gamma ray bursts, with energies between 10^{14} and 10^{19} TeV — see e.g. Refs. [74, 80–86]. This clearly shows the relevance of our analysis with respect to the constraints previously discussed in the literature. Our analysis indeed provides either a restriction of the dimensionful parameters entering the UV-complete theories to be tested or a rejection/acceptance of their theoretical predictions. For instance, for string theory we can only restrict the values of the parameters involved in the theoretical construction, while in the case of loop quantum gravity, the only dimensionful scale is the Planck-scale, and all the order-one dimensionless parameters are fixed by the theory. Thus, with our analysis we were able to provide for all these attempts a restriction of the universality classes that are allowed on the theoretical side, and rule-out values of the parameters that are either the most natural ones — from a theoretical perspective — to be considered, or the only ones that can be considered.

Dedicated measurements can be planned in forthcoming updates of DAMA/LIBRA and other experiments. This may provide the chance of constraining M_n with $n \geq 3$ contributions, which are suppressed by the n-th power of the energy scale Λ. In particular, an increase of sensitivity in δ^2 would trigger the possibility of constraining third order suppressed PEP violating terms. For completeness we also mention the potentiality of a very interesting result on this topic from data collected

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[5] Indeed for these neutrinos there is also the possibility to compare the arrival times with that one of low-energy photons.

by Super-Kamiokande.

Acknowledgments

We wish to acknowledge G. Amelino-Camelia and M. Arzano for insightful comments on the draft.

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