Label Symmetry Subgroups and the Conversation of Generalized Cross Helicities of Non-Barotropic MHD

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Noether's theorem connects symmetries to conservation laws in various physical systems. Among the unique symmetries of continuous matter are labelling symmetries which are manifested by Arnold's diffeomorphism group. A special symmetry subgroup of the diffeomorphism is the translation of labelling. This subgroup is connected to conservation laws which suffer a topological interpretation. For example, in ideal barotropic fluids the metage translation symmetry subgroup is connected through Noether's theorem to the conservation of helicity. Helicity is a measure of the knottiness of vortex lines and thus a topological constant of motion. The same is true for barotropic or incompressible magnetohydrodynamics (MHD) in which the same subgroup leads to the conservation of cross helicity. Although standard cross helicity is not conserved in non-barotropic MHD it was shown that a new kind of cross helicity which is conserved in the non-barotropic case can be introduced. This conservation law was deduced from the variational principle using the Noether's theorem. The symmetry subgroup associated with the new cross helicity was magnetic metage translations. Also, we show that additional labelling translations symmetries exist which are connected to new and different topological conservation laws.