

A Stochastic Model for Forest Growth: Framework of Multivariate Diffusion Process

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Abstract. Stochastic differential equations have been intensively used to analyze data from physics, finance, engineering, medicine, biology, and forestry. This study proposes a general stochastic dynamical model of a forest stand development which includes random forces governing the dynamic of multivariate distribution of tree size variables. The dynamic of the multivariate probability density function of tree size components (diameter, height, crown base height, crown width and so on) in a stand is described by a mixed effect parameters Gompertz-type multivariate stochastic differential equation (SDE). The advantages of SDE method are that it do not need to choose many different equations to be tried, it relates the tree size components dynamic against the age dimension (time), and consider the underlying covariance structure driving changes in the tree (stand) size variables. SDE model allows us a better understanding of biological processes driving the dynamics of natural phenomena. The new derived multivariate probability density function and its marginal univariate, bivariate and trivariate distributions, and conditional univariate, bivariate and trivariate distributions can be applied for the modeling of stand attributes such as the mean diameter, height, crown base height, crown width, volume, basal area, slenderness ratio, their increments and much more. This study introduces general multivariate mutual information measures based on the differential entropy to capture multivariate interactions between state variables. The purpose of the present study is therefore to experimentally confirm the effectiveness of using multivariate mutual information measures to reconstruct multivariate interactions in state variables.

Key words: multivariate Gompertz-type stochastic differential equation; marginal distributions; conditional distributions; Shannon entropy; multivariate interaction