

# Symmetries in Yetter-Drinfel'd-Long categories

Dongdong Yan

School of Mathematics, Southeast University, Nanjing 210096, Jiangsu, China, ydd150365@163.com

**Abstract:** Symmetric categories have been of great interest in quantum algebra and mathematical physics. Cohen and Westreich in 1998 studied symmetries in the Yetter-Drinfel'd category over a Hopf algebra under some conditions. Pareigis in 2001 found the necessary and sufficient condition for  ${}^H\mathcal{YD}$  to be symmetric. Later, Panaite et al. in 2010 proposed the definition of pseudosymmetric braided categories which can be viewed as a kind of weakened symmetric braided categories, and showed that the category  ${}^H\mathcal{YD}$  is pseudosymmetric if and only if it is commutative and cocommutative. Let  $H$  be a Hopf algebra and  $\mathcal{LR}(H)$  the category of Yetter-Drinfel'd-Long bimodules over  $H$ . We first show that the Yetter-Drinfel'd-Long category  $\mathcal{LR}(H)$  is symmetric if and only if  $H$  is trivial in four different methods, and that  $\mathcal{LR}(H)$  is pseudosymmetric if and only if  $H$  is commutative and cocommutative. We then introduce the definition of the  $u$ -condition in  $\mathcal{LR}(H)$  and give a necessary and sufficient condition for  $H_{i}$  ( $i=1,2,3,4$ ) to satisfy the  $u$ -condition. Then we study the relation between the  $u$ -condition and the symmetry of  $\mathcal{LR}(H)$ .

**Keywords:** symmetric category, Yetter-Drinfel'd-Long category, the  $u$ -condition, pseudosymmetry

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