Do the symmetries of product spaces hold the key to unification?

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June 27, 2021

We explore the symmetry groups of changes of basis on tangent spaces and show that these seem to point towards a new kind of Kaluza-Klein theory.

These transformations, for any pseudo-Riemannian spacetime, form a general linear group. This has a pseudo-orthogonal subgroup which preserves pseudo-orthonormality that is, it maps frame bases to other frame bases.

On product spaces, there is a class of coordinate system which respects the factor spaces. In such coordinate systems, the full group of general linear symmetries is non-linearly realised, with only the transformations in the tangent spaces to the factor spaces realised linearly. Higher-dimensional tensors decompose into multiplets for the factor spaces. The metric takes a block diagonal form.

For a Cartesian product space, each block of the metric varies only with the coordinates for the corresponding factor space, but his is not true for more general product spaces. We focus on spaces for which the blocks are independent of the coordinates on the compact factor spaces (a higher-dimensional version of Kaluzas cylinder condition), but all of them vary with the familiar four dimensions of spacetime. We show that in such a case, the covariant derivatives of vectors take exactly the right form for coupling to gravity and internal symmetries. We illustrate this for U(1) and SU(2). The gauge fields are components of the Levi-Civita spin connection (components which are absent in a Cartesian product space) and their field strengths are components of the Riemann curvature tensor in particular coordinates.

This model appears to exploit a loophole in ORaifeartaighs no-go theorem. This is because transformations which appear as translations in the decompactification limit are realised as additional rotational transformations on the compact space.