Optimal Ellipsoid Approximations in Control Theory

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Abstract

Minimum volume ellipsoids containing a given set arise often in control theory observed as a problem that solves differential equation with inputs and outputs. Generally, the problem is described by a differential inclusion $\dot{x} \in F(x(t), t)$, where F is a set valued function on $\mathbb{R}^n \times \mathbb{R}_+$. An ellipsoid is given itself with a symmetric, positive definite matrix Q such that $\mathcal{E} = \{\xi \in \mathbb{R}^n, (\xi - \xi_0)^T Q^{-1}(\xi - \xi_0) \leq 1\}$.

If a linear differential inclusion is given by $\dot{x} \in \Omega x$, $x(0) = x_0$, and with $\Omega \subseteq \mathbb{R}^{n \times n}$, then sufficient condition for the system stability is to find a positive definite symmetric matrix P such that the quadratic function $V(\xi) = \xi^T P \xi$ decreases along every nonzero state trajectory.

Specific linear differential inclusions are described, such as the linear time-invariant, Polytopic, norm-bound with additional output that affects the additional input in bounded measure or diagonal norm-bound bounds of input and output functions are given componentwise.

In our work we interpreted stability conditions of above systems in terms of ellipsoid that is invariant to a solution of a differential inclusions: if $x(t_0) \in \mathcal{E}$ then $x(t) \in \mathcal{E}$ for every $t \geq t_0$.

Key words: control theory; minmal volume ellipsoid; symmetric matrix; linear matrix inequalities.