Abstract

In addition to its infinite hierarchy of local, commuting symmetries, the Korteweg-de Vries equation (KdV) has scaling and Galilean symmetries, and these can be generalized, by application of the recursion operator, to a second infinite hierarchy, known as the "additional symmetries" of KdV. Unlike the standard symmetries, the additional symmetries are nonlocal, and do not commute, either amongst themselves or with the standard local symmetries. In previous work, we explained how the standard symmetries can be obtained by power series expansion of a single symmetry depending on a parameter, which can be identified as an infinitesimal double Backlund transformation. We called this the Gardner method for symmetries, as it is similar to the original construction of an infinite set of conserved quantities for KdV given in [Miura, Gardner, Kruskal 1968]. We show that a similar generating function can be found for the additional symmetries, and it is also naturally expressed in terms of the functions appearing in Backlund transformations. We determine the Lie algebra of the generating functions of the standard local symmetries and the additional symmetries.

In the early 1990s various groups showed the existence of 4 further nonlocal hierarchies of symmetries generated by application of the inverse recursion operator to the trivial or the scaling symmetry. We discuss the generating function approach to these. Only a single extra generating function needs to be introduced, also corresponding to an infinitesimal double Backlund transformation. The 6 hierarchies are obtained by expansion of 3 generating functions for small and large values of the parameter. There is substantial subtlety in identifying the commutator algebra of the generating functions, related to the general difficulty of correctly defining nonlocal symmetries.