On Basis Invariants of the Symmetry Groups Generalized N-cube

Marina Bershadsky

August 2021

Abstract: Some properties of the basic invariants of the symmetry groups $G(m,p,n)$, B_n^m were described by O. I. Rudnitskii. Here we continue to study and expand these properties. We study the properties of the basis invariants of the symmetry groups of the complex polytope $\frac{1}{p}\gamma_n^m$ and the generalized n-cube γ_n^m, as well as its subgroups D_m^n. We give an explicit construction of all the basis invariants of odd degree of these groups. This invariants of the symmetry groups $G(m,p,n)$, B_n^m are under construction on the basis of Pogorelov's polynomials and it is possible to construct in explicit form *all* generators of the algebra $I^{B_n^m}$.

Conclusion:

1. $\Delta R_{mr}^1 = J_{m(r-1)}^* = -2\frac{(m(p+1))!}{(mp)!}\sum_{i=1}^n x_i^{m(r-1)}$

2. Since $J_{m(r-1)}^* = A\sum_{i=1}^n x_i^{m(r-1)}$, the form $J_{m(r-1)}^*$ is a basic invariant of an odd degree mr of group B_n^m.

3. Thus, it is proved that, on the basis of Pogorelov's polynomials, it is possible to construct in explicit form all generators of the algebra $I^{B_n^m}$.

Reference:

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