A new generalized log-logistic distribution with increasing, decreasing, unimodal, bathtub hazard rates: properties and applications

Abstract: A new generalization of the log-logistic distribution with increasing, decreasing, unimodal and bathtub hazard rates was proposed and studied, extending the log-logistic distribution by adding an extra shape (or skewness) parameter to the existing distribution, leading to greater flexibility in analysing and modeling various data types. Some of its mathematical and statistical properties were derived and model parameters estimated using the classical method especially the maximum likelihood approach and the Bayesian approach. The new hazard rate can be "increasing", "decreasing", "unimodal", and "bathtub" shapes. The flexibility and usefulness of the proposed distribution was applied to three different real-life data sets with symmetric and asymmetric shapes and as well as different failure rate shapes and compared to other competitive parametric survival models. Finally, the proposed distribution is applied to regression survival analysis and verified that it is closed under both proportional hazard and accelerated failure time models that is a great contribution to the field of survival and reliability analysis and other disciplines in the areas of economics and demographic studies.
Keywords: hazard rate function, log-logistic distribution; maximum likelihood estimation; generalized log-logistic distribution; covid-19 data; Monte Carlo Simulation.

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