

2 **Parallel WSAR for Solving Permutation Flow Shop Scheduling**
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10 † Presented at the title, place, and date.

Abstract: This study presents a coalition-based parallel metaheuristic algorithm for solving Permutation Flow Shop Scheduling Problem (PFSP). The novel approach incorporates five different single-solution based metaheuristic algorithm (SSBMA) (Simulated Annealing Algorithm, Random Search Algorithm, Great Deluge Algorithm, Threshold Accepting Algorithm and Greedy Search Algorithm) and a population based algorithm (Weighted Superposition Attraction-Repulsion Algorithm) (WSAR). While SSBMAs are responsible for exploring the search space, WSAR serves as a controller that handles the coalition process. SSBMAs perform their search simultaneously through MATLAB parallel programming tool. The proposed approach tested on PFSP against the state of the art algorithms in the literature. Moreover, the algorithm is also tested against its constituents (SSBMAs and WSAR) and its serial version. Non-parametric statistical tests are organized to compare the performance of the proposed approach statistically with the state of the art algorithms, its constituents and its serial version. The statistical results prove the effectiveness of the proposed approach.

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1. Introduction

Optimization is finding the solution that gives the best result in the solution space of a problem. In other words, it is to achieve the best solutions under the given conditions. Today, different optimization algorithms are used to solve many optimization problems [1–4]. These algorithms can be classified in two groups as exact algorithms and approximate algorithms. Exact algorithms search the entire search space and try every possible alternative solution. Even if they provide the optimal solution, they need long runtime, especially as the size of the problem grows. On the other hand, approximate algorithms perform their solution space search through some logical operators. Although they do not guarantee optimal solution, they provide near optimal solutions in reasonable time. Through this superiority, most of the researchers prefer approximate algorithms in optimization problem solving.

Approximate algorithms are classified into two groups as heuristic and metaheuristic algorithms. While a heuristic algorithm's structure is problem-specific, a metaheuristic algorithm's structure is generic, allowing it to be applied to any optimization problem. Metaheuristic algorithms are more flexible than heuristic algorithms in that they can handle any problem. They can also provide better solutions to optimization problems than heuristic algorithms. Metaheuristic algorithms, on the other hand, may have drawbacks such as early convergence and poor speed, and a metaheuristic algorithm may be superior to other metaheuristic algorithms.

1 The No Free Lunch Theorem [5] must also be mentioned at this point in order to
2 underline the logic for integrating diverse search techniques within the framework of cre-
3 ating successful optimization methods. According to this theorem, no optimization
4 method beats all remaining solution processes for all optimization problems, and there is
5 no statistical difference between the performances of different metaheuristics when all
6 optimization problems are solved [6]. It is a result that implies that the computing cost of
7 finding a solution for optimization problems is the same for any solution technique. This
8 theorem can be a base point to combine various metaheuristic algorithms to tackle opti-
9 mization problems more effectively. It will take substantial time to combine the various
10 metaheuristic algorithms and run them sequentially [7]. Most of the metaheuristic algo-
11 rithms are designed to run sequentially, parallel execution of metaheuristic algorithms
12 can increase solution quality while shortening run time [8][9].

13 This research is the outcome of an attempt to combine several metaheuristics in order
14 to reveal a high level of synergy and, as a result, deliver sufficient performance while
15 solving optimization problems.

16 This paper provides a new framework for addressing Permutation Flow Shop Sched-
17 uling Problem (PFSP) based on a coalition of diverse metaheuristics in a parallel compu-
18 ting environment. To implement the multiple metaheuristic algorithms in parallel, a new
19 optimization system combining different single solution based metaheuristic algo-
20 rithms(SSBMA) (Simulated Annealing Algorithm (SA), Random Search Algorithm(RS),
21 Great Deluge Algorithm(GD), Threshold Accepting Algorithm (TA) and Greedy Search
22 Algorithm (GS)) and a controller (Weighted Superposition Attraction algorithm) is de-
23 signed.

24 The remaining of the paper is organized as follows. In Section 2, parallel computing
25 is explained and in Section 3, the proposed optimization approach (p-WSAR) is intro-
26 duced. In Section 4, PFSP is presented and experimental results are reported. Finally, con-
27 cluding remarks are presented in Section 5.

28 **2. Parallel Computing**

29 Parallel computing is a type of computing architecture in which many processors execute
30 or process an application or computation simultaneously. Parallel computing helps us do
31 large computations by dividing the workload among multiple processors, all working on
32 at the same time. Most supercomputers use parallel computing principles to work. Paral-
33 lel computing is also known as parallel processing. For this to happen, we need to
34 properly empower resources to execute concurrently. Parallel computing can reduce so-
35 lution time, increase energy efficiency in our application, and allow us to tackle bigger
36 problems. It is a computational technique developed to solve complex problems faster
37 and more efficiently [10] [11].

38 **3. p-WSAR Algorithm**

39 The p-WSAR algorithm is introduced in this section. p-WSAR is comprised of five
40 SSBMAs namely, Random Search (RS) [12], Threshold Accepting (TA) [13], Great Deluge
41 [14], Simulated Annealing (SA) [15], Greedy Search (GS) [16] and a controller WSAR [17].
42 p-WSAR mainly has three stages namely search stage, information sharing stage and re-
43 production stage. In the search stage, all of the SSBMAs explores the solution space in
44 parallel. After exploring the solution space, they share their findings with other SSBMAs
45 through WSAR algorithm superposition principle. One can see the details of the superpo-
46 sition principle in the following study [17]. Then, all SSBMAs moves through their next
47 positions. In the last stage, SSBMAs' parameters reproduced. This iterative process lasts
48 until the termination criteria is met. Notations of p-WSAR algorithm is given below.
49 The main stages of the WSAR algorithm and flow chart of the algorithm are depicted in
50 Figure 1 and Figure 2 respectively.

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PROCEDURE p-WSAR
Randomly generate initial solutions
Match randomly generated initial solutions with randomly selected SSBMAs
WHILE iteration<Maxiter
  WHILE termination conditions for SSBMAs are not met
    Run SSBMAs in parallel
  END WHILE
Sort the solutions returned by SSBMAs according to their fitness values
Determine attractive and repulsive superpositions
Calculate attractive and repulsive superpositions' fitness
  PARFOR each solution returned by SSBMAs //parallel FOR
    IF solution_fitness < Superposition fitness
      Randomly move solutions
    ELSE
      Move towards superposition
    END IF
    Randomly generate a new set of parameters for SSBMAs
    Randomly match solutions with SSBMAs
  END FOR
END WHILE
END PROCEDURE
    
```

Figure 1. Main steps of p-WSAR

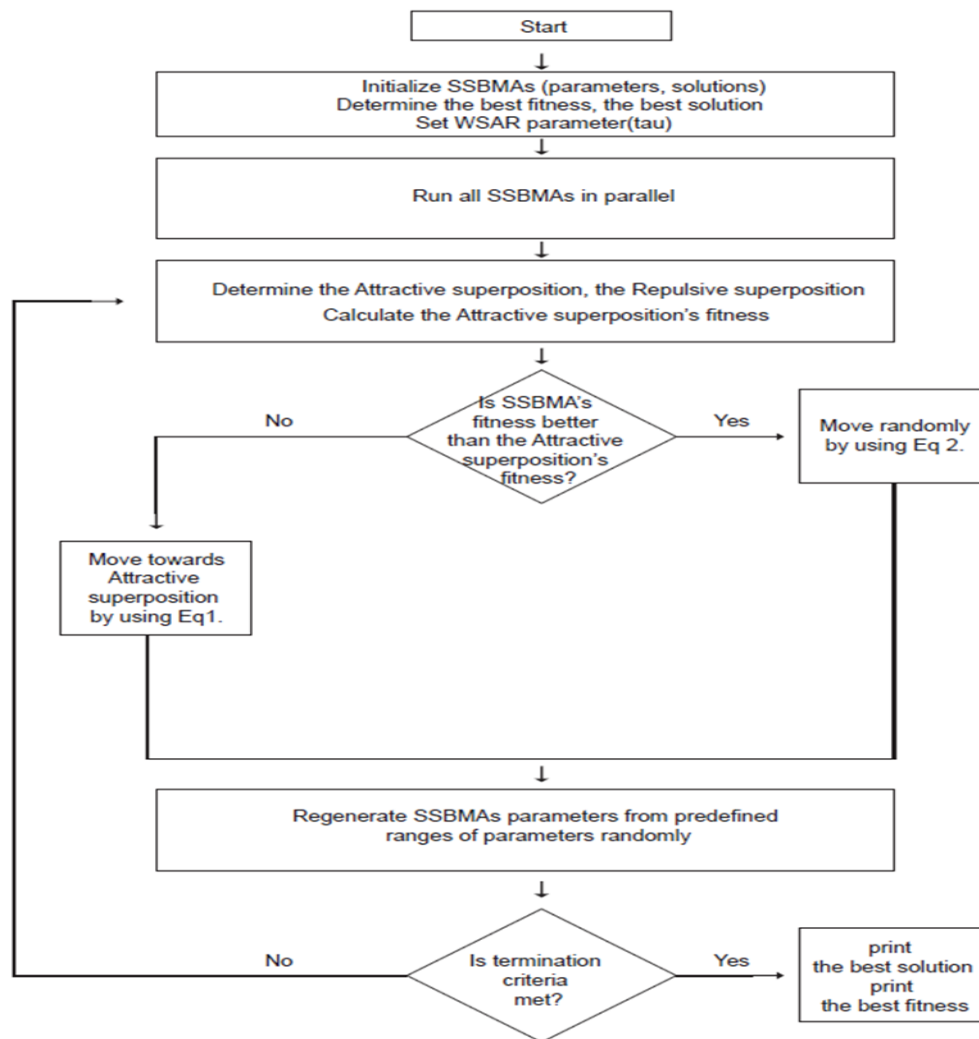


Figure 2. The flow chart of the p-WSAR algorithm

4. Permutation Flow Shop Scheduling Problem and Experimental Results

In this section, firstly PFSP is introduced and then, experimental results are given

4.1 Permutation Flow Shop Scheduling Problem (PFSP)

The PFSP has a set of m machines and a group of n jobs. Every job is made up of m operations that must be accomplished on several machines. For each of the n jobs, the machine ordering for the process sequence is the same. Each machine may only conduct one operation at a time, and all jobs are completed sequentially according to a permutation schedule. It is assumed that no machine problems would occur during the manufacturing stage, thus all of the machines will be ready to process activities. Operation preemption is also disallowed. The goal is to design a schedule that reduces the total job completion time (makespan) while adhering to the preceding assumptions.

A permutation type n -dimensional real-number vector can be utilized in the PFSP to determine the job process sequence. After identifying the job order, the makespan can be calculated using the "completion time matrix approach," which Onwubolu and Davendra proposed [18].

4.2 Experimental Results

The p-WSAR's performance in PFSP is evaluated using the Taillard [19] benchmark instances, which are divided into 12 groups of problems. 5 of these problems are selected to test p-WSAR's performance against some state of the art algorithms and WSAR. These problems' size (PS: (J*M)) and well known solutions (WKS) is given in Table 1. The best, the worst and the average performance of 30 runs of each algorithm is recorded. In all of the instances, p-WSAR is able to find better solutions than other algorithms.

Table 1. Comparison of p-WSAR with some state of the art algorithms and WSAR

Problems	Algorithm	TLBO[20]	HPSO[21]	NPSO[22]	WSAR	p-WSAR
ta001 PS:(20*5) WKS:1278	Best	1278	1278	1278	1278	1278
	Worst	1297	1278	1297	1297	1278
	Average	1287.2	1278	1279.9	1278.6	1278
ta011 PS:(20*10) WKS:1582	Best	1586	1582	1582	1586	1582
	Worst	1618	1596	1639	1618	1582
	Average	1606	1587.3	1605.8	1592.2	1582
ta031 PS:(50*5) WKS:2724	Best	2724	2724	2724	2724	2724
	Worst	2741	2724	2729	2729	2724
	Average	2729.4	2724	2725	2724.6	2724
ta051 PS:(50*20) WKS:3771	Best	3986	3923	3938	3969	3902
	Worst	4095	3963	3989	4063	3923
	Average	4029.7	3944.6	3964.3	4015.9	3916
ta061 PS:(100*5) WKS:5493	Best	5493	5493	5493	5493	5493
	Worst	5527	5493	5495	5495	5493
	Average	5499.4	5493	5493.2	5493.2	5493

In addition, the performance of p-WSAR is statistically compared with the other algorithms through nonparametric statistical tests by using average values. Table 2 indicates that (based on the Friedman test results) p-WSAR surpasses the other algorithms. Furthermore, according to the Wilcoxon signed-rank test, the difference between p-WSAR and HPSO is found negligible as the $p > 0.1$. Besides p-WSAR is performed slightly better than TLBO, NPSO, WSAR as $p < 0.1$.

Table 2. Non-parametric test results on Taillard Instances

Friedman test average rankings			Wilcoxon signed-rank test between p-WSAR and state of the art algorithms	
Algorithms	Sum of Ranks		p-WSAR vs.	p-value
TLBO	5.0	(5)	TLBO	0.0625
HPSO	1.7	(2)	HPSO	0.5
NPSO	3.7	(4)	NPSO	0.0625
WSAR	3.3	(3)	WSAR	0.0625
p-WSAR	1.3	(1)		

Another computational study is organized to test the performance of p-WSAR with its constituents (SSBMAs) in terms of solution quality. The results are presented in Table 3, and Table 4. According to the computational results, p-WSAR' performance is far beyond its constituents (SSBMAs). Besides, in respect of the non-parametric statistical tests, p-WSAR is able to produce more effective results than its constituents. Also, there is statistically significant difference between the performance of the p-WSAR and its constituents since p-value is < 0.1.

Table 3. Comparison of p-WSAR with SSBMAs

Problems	Algorithm	SA	RS	GD	TA	GS	p-WSAR
ta001 PS:(20*5) WKS:1278	Best	1286	1294	1278	1278	1284	1278
	Worst	1297	1302	1297	1284	1292	1278
	Average	1292.2	1296.5	1279.9	1280.6	1287.8	1278
ta011 PS:(20*10) WKS:1582	Best	1606	1616	1596	1592	1608	1582
	Worst	1620	1650	1616	1618	1642	1582
	Average	1610	1632.4	1610.7	1608	1624	1582
ta031 PS:(50*5) WKS:2724	Best	2804	2942	2806	2864	2916	2724
	Worst	2908	3026	2846	2938	3002	2724
	Average	2856	2978	2824.6	2886	2984	2724
ta051 PS:(50*20) WKS:3771	Best	4206	4807	4402	4622	4424	3902
	Worst	4240	6240	4803	5162	6024	3923
	Average	4222.4	5465.8	4627	4838.6	5146	3916
ta061 PS:(100*5) WKS:5493	Best	6122	8640	6248	6125	7426	5493
	Worst	6378	9026	6414	6642	8424	5493
	Average	6564.3	8924.7	6344.9	6348.4	8012.6	5493

Table 4. Non-parametric test results on Taillard Instances p-WSAR vs. SSBMAs

Friedman test average rankings			Wilcoxon signed-rank test between p-WSAR and state of the art algorithms	
Algorithms	Sum of Ranks		p-WSAR vs.	p-value
SA	3.4	(4)	SA	0.0625
RS	5.8	(6)	RS	0.0625
GD	2.6	(2)	GD	0.0625
TA	3.2	(3)	TA	0.0625
GS	5.0	(5)	GS	0.0625
p-WSAR	1.0	(1)		

5. Conclusion

In this research, multiple metaheuristic algorithms are combined to build a coalition for tackling PFSP. The suggested methodology uses WSAR as the controller to run multiple single solution based metaheuristic algorithms (SSBMA) in parallel. The suggested method is put to the test on some of the Taillard instances. According to the results, the proposed approach is capable of finding the best solutions. Furthermore, the proposed approach surpasses its constituents. The proposed approach's motivation is supported by the computational results. Applying the proposed approach to the other type of problems is planned as a future research.

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