New explicit asymmetric hopscotch methods for the heat conduction equation

Mahmoud Saleh[1,*], Endre Kovács [1]

[1] Institute of Physics and Electrical Engineering, University of Miskolc;
* Corresponding author: mhmodsalh84@gmail.com

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Abstract: This study aims at constructing new and effective fully explicit numerical schemes for solving the heat conduction equation. We use fractional time steps for the odd cells in the well-known odd-even hopscotch structure and fill it with several different formulas to obtain a large number of algorithm-combinations. We generate random parameters in a highly inhomogeneous spatial distribution to set up discretized systems with various stiffness ratios and systematically test these new methods by solving these systems. The best six algorithms were tested against some conventional methods in case of large systems. The results showed that they are accurate, stable and of the second order algorithms.

Keywords: odd-even hopscotch methods; heat equation; explicit time-integration; stiff equations; unconditional stability

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OUTLINE

Description of the method

Numerical experiments

Supplementary Materials

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Description of the method

- Unlike original Hopscotch, we used ten different formulas
- The first formula is derived from CNe method [1]:

$$u_i^{n+1} = u_i^n e^{-2r} + \left(\frac{u_{i+1}^n + u_{i-1}^n}{2}\right)(1 - e^{-2r})$$

- The other nine formulas are derived from Theta method [2]:

$$u_i^{n+1} = u_i^n + r\theta(u_{i-1}^n - 2u_i^n + u_{i+1}^n) + (1-\theta)\left(u_{i-1}^{n+1} - 2u_i^{n+1} + u_{i+1}^{n+1}\right)$$

$$\theta \in \left[0, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, 1\right]$$

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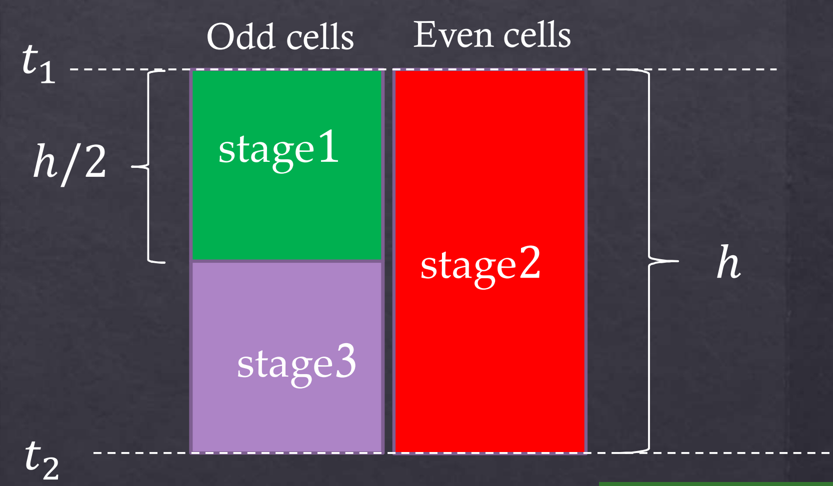
Description of the method

Stages

- In each stage , one can use any of the previous 10 formulas
- For the odd cells, we take half time step
- For even cells we take full time step
- We have 1000 possible methods
- We selected the best 6 methods

asymmetric hopscotch structure



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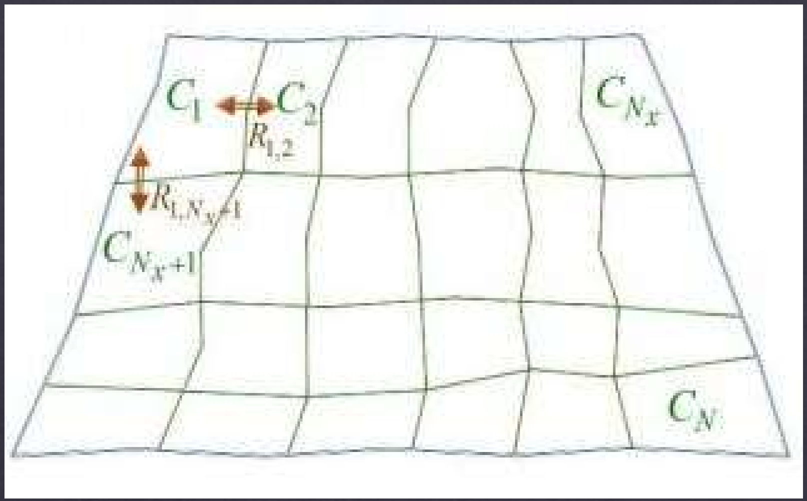
Numerical Experiment



$$c\rho\frac{\partial u}{\partial t} = \nabla(k\nabla u)$$

- 2D inhomogeneous system was tested
- Large system with N= 1000 cells
- We considered the following norms when calculating the errors:

$$Error(Average) = \frac{1}{N}\sum_{0\le j\le N} |u_j^{ref}(t_{fin}) - u_j^{num}(t_{fin})|$$

$$Error(Energy) = \frac{1}{N}\sum_{0\le j\le N} C_j\ |u_j^{ref}(t_{fin}) - u_j^{num}(t_{fin})|$$

Top 6 method structures

Method/Stage	Stage 1	Satge 2	Stage 3
A1	$\theta = 0$	$\theta = 1/_2$	$\theta = 1$
A2	$\theta = 1/_5$	$\theta = 1/_2$	$\theta = 4/_5$
A3	$\theta = 1/_4$	$\theta = 1/_2$	$\theta = 3/_4$
A4	CNe	$\theta = 1/_2$	CNe
A5	CNe	$\theta = 1/_2$	$\theta = 1/_2$
A6	$\theta = 1/_3$	$\theta = 1/_2$	$\theta = 2/_3$

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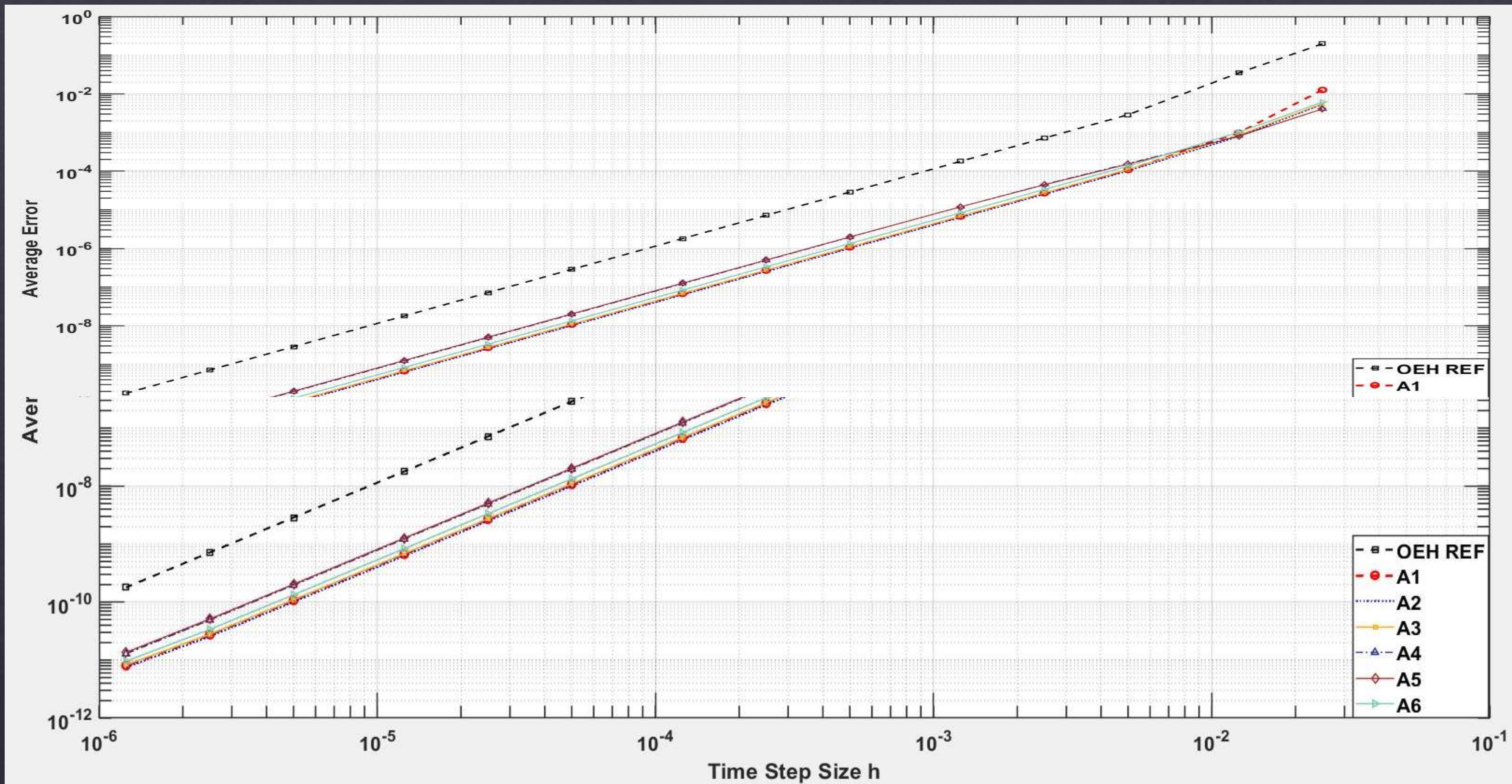
Numerical Experiment I



- Moderately stiff system was tested
- Best 6 methods vs original Hopscotch (OEH REF)
- Second order methods

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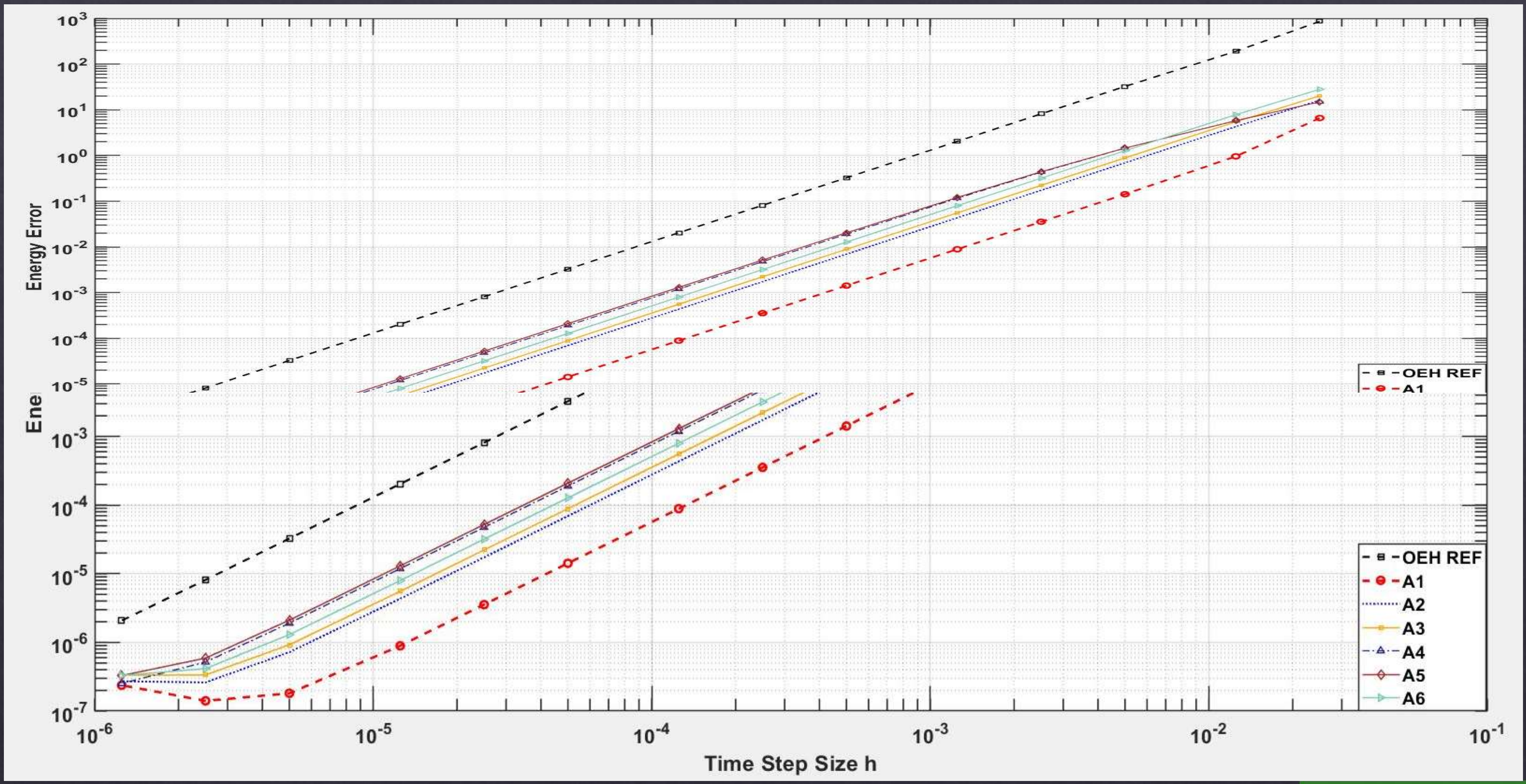
Numerical Experiment I



- More accurate than the original method
- Unconditionally stable

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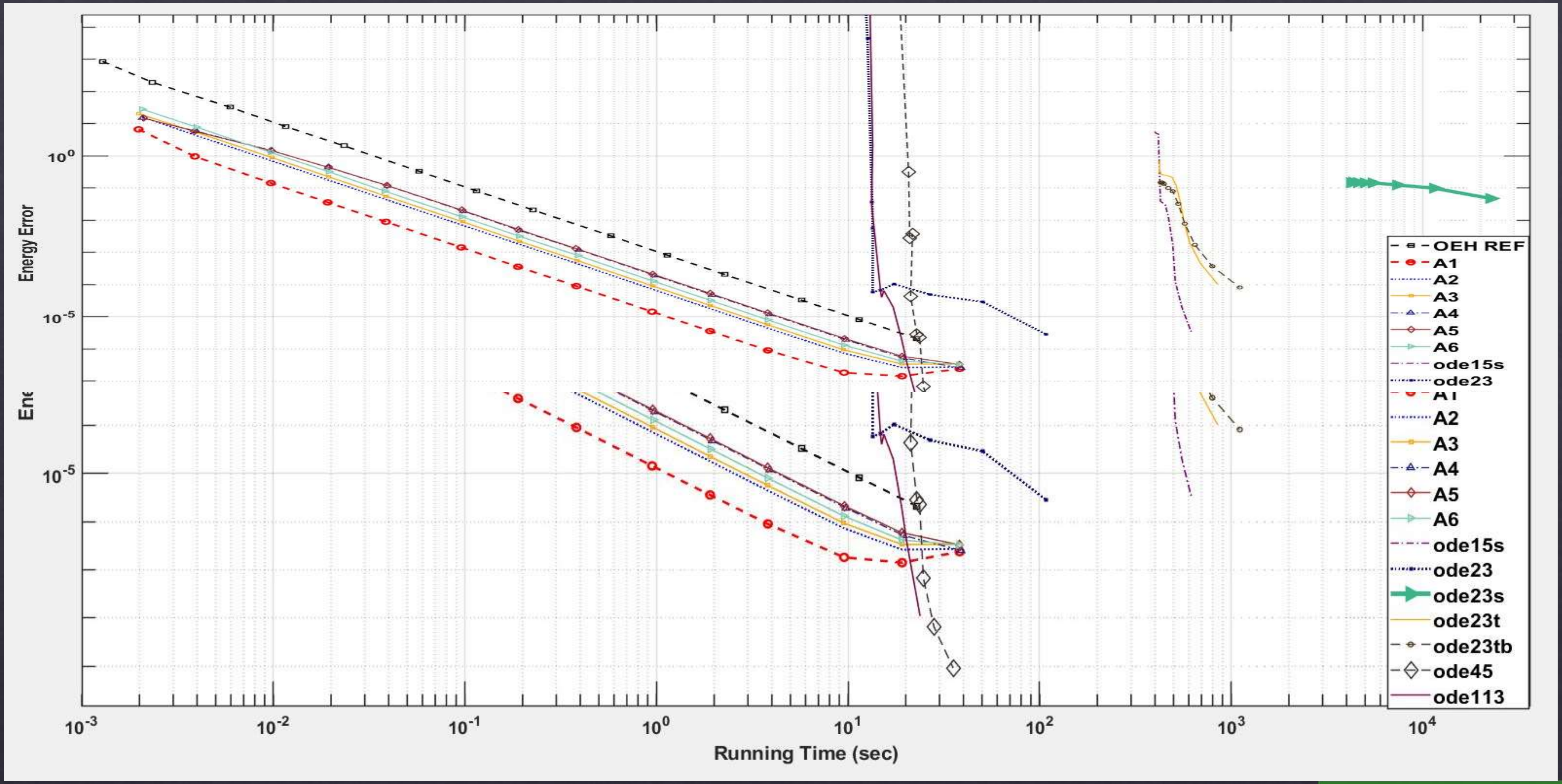
Numerical Experiment I



- Faster than the original method
- Faster than Matlab built-in routines

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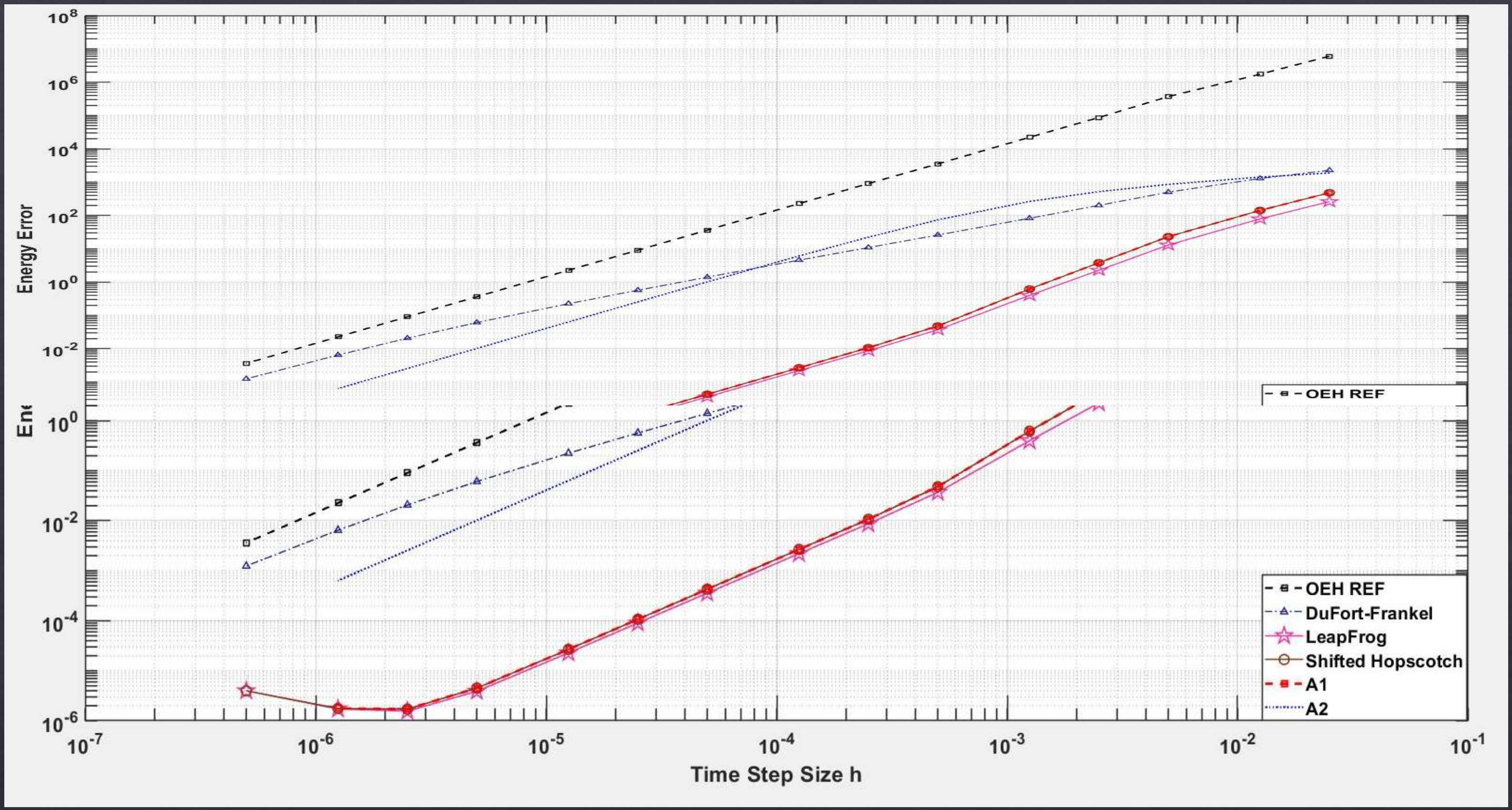
Numerical Experiment II



- Very stiff system was tested
- Best 2 methods vs some conventional methods
- Our previous works, Shifted-Hopscotch and Leapfrog-Hopscotch, were comparec

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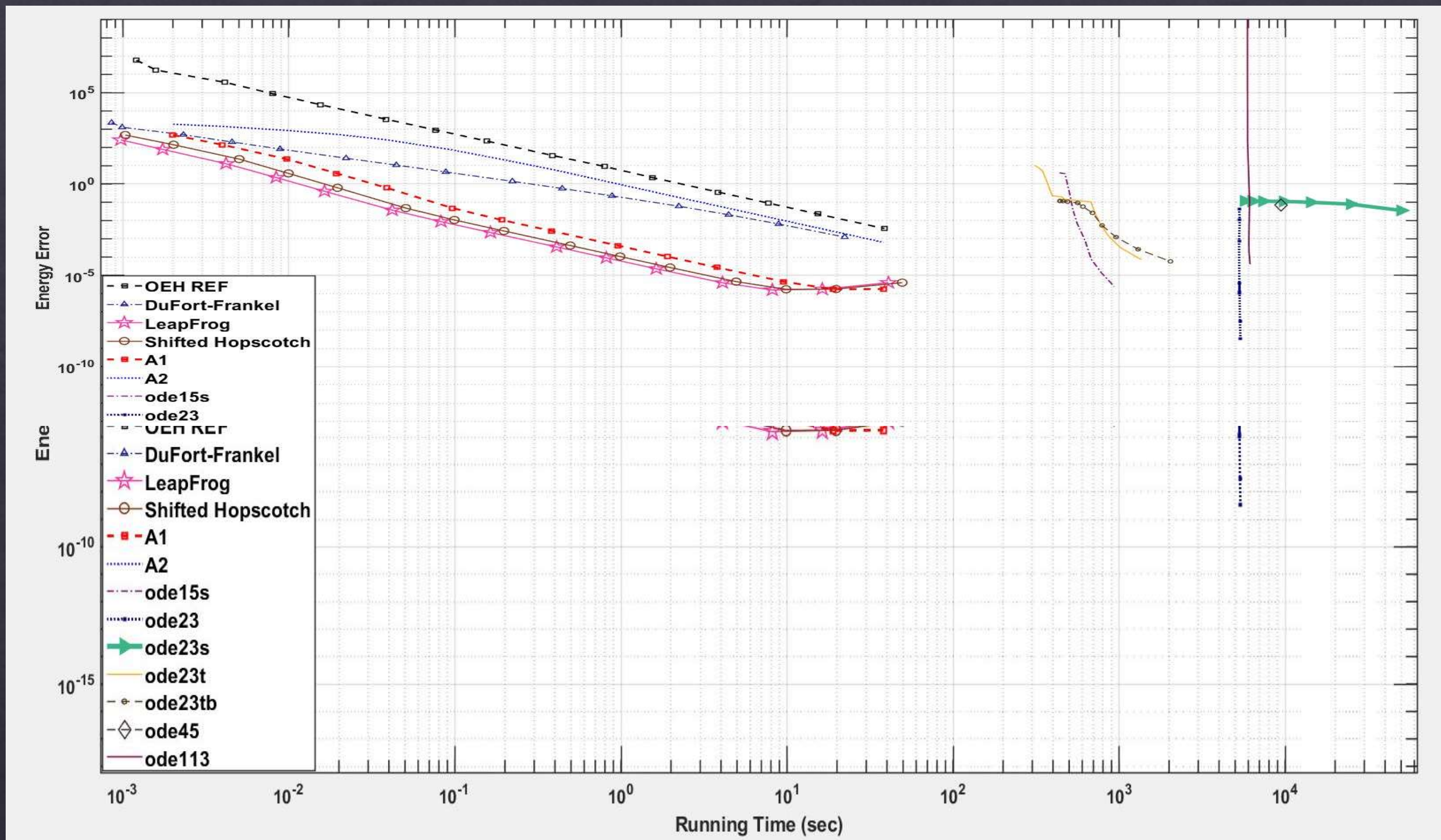
Numerical Experiment II



- Faster than the original Hopscotch
- Faster than Matlab routines

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Thank you for your attention

Supplementary Materials

1. Endre Kovács, New Stable, Explicit, First Order Method to Solve the Heat Conduction Equation, JOURNAL OF COMPUTATIONAL AND APPLIED MECHANICS 15 : 1 pp. 3-13. , 11 p. (2020)
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