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Concise Review of Classical Guitar Modelling Technologies

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Abstract: Analytical modeling and numerical simulation of multiphysics coupled systems is an exciting research area, even when it comes to intrinsically linear or linearized formulations, as is usually the case with coupled vibroacoustic problems. The combined effect of many localized geometrical miss-modeling with significant uncertainty in mechanical characterization of some organic materials yields large discrepancies in the natural frequencies and mode shapes obtained. The main goal of this work is to compare two basic approaches for the modeling of stringed musical instruments in the frequency domain: simplified lumped-parameters analytic modeling, considering only the most influencing degrees-of-freedom, and discretized finite element modal analysis. Thus, the focus is on a review of some key references in this field, including previous work by the authors, which should shed some light on some of the most relevant questions surrounding this problem.

Keywords: Classical guitar; lumped-parameters modeling; finite element method; vibroacoustics; frequency domain; musical instruments; luthier.

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1. Introduction

Due to the diversity of phenomena involved in the modeling of musical instruments, and the difficulties inherent in formulating the vibroacoustics and various underlying possible dissipating effects, and despite the availability of some classical and computational approaches, this task remains a significant challenge after many years. It's amazing that, despite the huge global popularity of plucked stringed instruments, such as the guitar, scientific research into the piano and violin began much earlier, paving the way for the application of similar principles to the modeling and study of classic guitars.

One may wonder, however, to what extent are analytical models completely *exact*? Though convergence towards the *analytic* solution is mathematically granted for methods such as, e.g., the finite or boundary elements [3], to what extent can we call this result *correct*? This article is intended to discuss the above questions contrasting two major approaches when it comes to acoustic guitar modelling. On one hand, a quantitative analysis can follow a familiar approach among engineers and more practically-oriented researchers: the quest for an *equivalent model* capable of yielding a sufficiently accurate approximate result, given that model parameters lie inside a broad-enough range of permitted variations – this paradigm is eventually called *gray box modelling*, because it does not rely entirely neither on experimental identification nor on analytical modelling. Usefulness has proven to be the key feature here, with the loss of accuracy perceived as a minor drawback in comparison with some gains expected for a computationally inexpensive model. On the other hand, lies the demand for high fidelity descriptions of physical reality, where a number of numerical methods could be employed to discretize the system into a set of algebraic linear equations – finite element (FE) models standing by far as a preferred option. Thus, bigger descriptions can be obtained, even though they should unavoidably require greater sets of input data certainly contaminated by some amount of uncertainty. Thus, model updating has become a standard practice both in academic research and industry (commercial software) applications [4]. It is important to note from its definition, however, that the whole model uncertainty remains distributed over the parameters in an unpredictable, possibly overlapped manner, consequently rising serious concerns about uniqueness of obtained solutions [5]. And these issues become evidently more severe for huge complex models with possibly hundreds of free parameters. Thus, its clear that a delicate trade-off emerges at this point for those situations where a smaller, less detailed model, could still bring useful results while providing valuable gains from narrower bounds on uncertainty propagation.

2. Simplified Guitar Models

Even as a complex and truly instigating system, the construction of musical instruments still relies almost solely upon traditional knowledge in the great majority of famous luthieries, with an increasing number of academic researchers trying to establish this bridge out of a personal passion [12]. Things were slightly different to a guitar's close relative: the violin, been for long time a learned instrument, it has naturally attracted researchers' attention earlier than its plucked-string cousin. Thus, Schelleng [10] proposed, already in 60's, an analogue 2nd order electro-magnetic resistance–inductance–capacitance – RLC – circuit that could emulate the elasto-acoustic interactions of top plate mechanical properties – mass distribution, stiffness and damping – with the air confined inside and surrounding the resonant chamber. Firth [11] adapted the analogue circuit to the dynamics of acoustic guitar, relating electrical quantities to the most clearly important structural and acoustical parameters. Although valuable for those familiar with electrical–mechanical analogy (a standard theoretical apparatus in classical control theory) this approach eventually gave place to a rather direct description of the coupled phenomena. Thus, Caldersmith [12] devised, constructed and measured a simple resonator composed by parts with similar functions as those pointed in [11], and Christensen [14], relying upon these concepts and related equations [10–12], presented a simplified 2 degrees-of-freedom (DOF) model derived from purely mechanical considerations, focusing on the relations between 1st and 2nd coupled modes and their intermediate Helmholtz anti-resonance, where almost all model parameters (with exception of those related to damping) can be obtained from simple measurement of these 3 frequencies. This work greatly simplified the resulting model, thereby bringing timely applicability and physical insight to subsequent developments. On this way, Caldersmith [13] introduced a straightforward yet important modelling refinement by taking into account the contribution of back plate flexibility in the coupled dynamics, resulting in a 3 DOF system for the resonant chamber, and French [6,15] tested a procedure for identification of model parameters and prediction of structural modification based on the sensitivities of eigenvalues for both 2 and 3 DOF models. Finally, the last extension to this simplified modelling approach was presented in [16], where additional mass and motion DOF were considered for the ribs, neglecting, however, any associated stiffness to this region. Following [14], here as well model parameters can be obtained, with exception to dissipative coefficients, from simple measurements. Top and back plate bending are represented by that of a thin axisymmetric disk corresponding approximately to the region circumscribed by the lower bout, in an average condition between hinged and clamped. This way theoretical approximate expressions could be obtained for the enclosed air volume variation as the product of equivalent plate area and displacement.

2.1. Formulation of Simplified Models and Definition of Parameters

In what follows, the equations of these simplified models discussed above will be presented, trying to keep clarity and conciseness such as possible. The meaning of each index adopted in the development of equations below is shown in Table 1 (for explanation on guitar components refer to Figure 1 below). Table 2 bellow presents the parameters appearing in the equations of 4 DOF model [16]; it should be noted that those appearing in the equations for 2 and 3 DOF models [6,13,14] are also covered in this table, but some table entries do not apply for the simpler models. Damping coefficients are presented separately because these parameters are to be defined through experimental modal analysis.

Table 1. Meaning of the indexes used in the various vibroacoustic models of the classic guitar.

Index	Meaning
t	Top plate
a	Air column through the sound hole
b	Back plate
r	Ribs (i.e., the remaining parts moving as a whole)

Table 2. Equivalent parameters of all 3 models.

Symbol	Description	Unit
V	Air volume inside the resonant chamber	m^3
m_a	Equivalent mass of the air column	kg
m_t	Equivalent mass of the top plate	kg
m_b	Equivalent mass of the back plate	kg
m_r	Equivalent mass of the ribs	kg
A_a	Equivalent area of the air column	m^2
A_t	Equivalent area of the top plate	m^2
A_b	Equivalent area of the back plate	m^2
k_t	Equivalent stiffness of the top plate	N/m
k_b	Equivalent stiffness of the back plate	N/m
ζ_r, η_r	r-th mode damping coeff. ($r = 1, .., 4$)	–

About the damping parameters in Table 2, it is important to emphasize at this point that these coefficients alone would imply, for instance, a purely viscous, or viscoelastic/hysteretic dissipation behaviour, which is clearly a non-physical assumption considering the predicted forms of energy loss in the modelling of vibro-acoustic coupled systems; i.e., the energy balance in this cases should include, at least, some sort of viscous, viscoelastic and acoustic radiation forms of damping [7,17]. Indeed, for those parameters related to energy dissipation, that equivalent modelling approach mentioned earlier is almost always adopted – with damping ratios, loss factors or another dissipation quantifier accounting alone for all forms os energy loss.

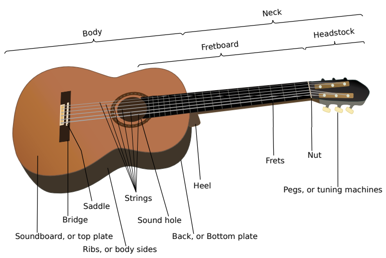

Figure 1. Acoustic guitar parts. [Source: adapted from original image due to William Crochot, distributed under CC BY-SA 3.0 license; Wikimedia Commons].

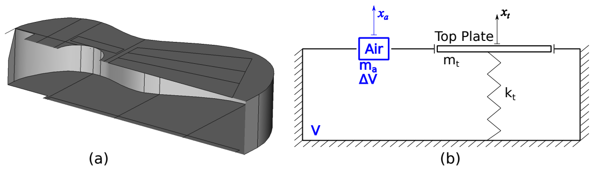

Figure 2. Schematic of the simple 2 DOF model: (**a**) Cut plane view of a guitar resonant chamber 3D CAD model, constructed with shells, planes and lines for subsequent FE analysis. (**b**) Representation as a simple 2 DOF model, considering only the movement of top plate and air column.

2.1.1. Simplified 2 Degrees-of-Freedom Model

Now, suppose the tensions in guitar strings transmit a force of magnitude f through the bridge (see Figure 1), that can be represented as acting pointwise, up and down, in the centroid of top plate. Thus, the equations of movement representing the dynamic coupling with the air column DOF displayed in Figure 2 can be written as:

$$m_t \ddot{x}_t + k_t x_t - \Delta p A_t = f, \; m_a \ddot{x}_a - \Delta p A_a = 0. \tag{1}$$

Considering the adiabatic, linearised form for the equations of acoustics, the pressure variation Δp may be expressed in terms of the total volume V inside the resonant chamber, the sound velocity c and mass density ρ of the surrounding air, and of the (idealised) chamber volume increment $\Delta V = A_t x_t + A_a x_a$ [8]:

$$\Delta p = -\rho c^2 \frac{A_t x_t + A_a x_a}{V}. \tag{2}$$

Then, substituting in Equation 1 and putting in matrix form, yields:

$$\begin{bmatrix} m_t & 0 \\ 0 & m_a \end{bmatrix}\begin{bmatrix} \ddot{x}_t \\ \ddot{x}_a \end{bmatrix} + \begin{bmatrix} k_t + \frac{\rho c^2 A_t^2}{V} & \frac{\rho c^2 A_t A_a}{V} \\ \frac{\rho c^2 A_t A_a}{V} & \frac{\rho c^2 A_a^2}{V} \end{bmatrix}\begin{bmatrix} x_t \\ x_a \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}. \quad (3)$$

Quantities appearing in the left-hand side of Equation 3 could be viewed separately into two classes: those described in Table 2 depend on geometric and/or material properties of guitar components, and thus shall be treated as adjustable parameters of the equivalent system, while the remaining ones (speed of sound and mass density) are physical properties of surrounding air, and so it is more reasonable to consider them as system-independent constants. On the other hand, driving force f would be truly impossible to obtain, both experimentally or analytically, because this excitation motion is rather imposed by an effectively distributed conjugate on the interfacing area between the top plate and the bridge; i.e., its much higher stiffness impels it to rotate when forced to and fro by the tension of guitar strings, acting some distance above the plate. Theoretically, once the plate is adequately approximated (to some level of accuracy) as a clamped or hinged plate [16] an equivalent pivot distance could be considered, via static equilibrium of moments, in order to compare the driving force over the top plate with the tension in guitar strings. This approach was not explored up to this moment, however, to the author's knowledge. Nevertheless, analytical values for modal parameters can be readily extracted from this matrix equation, and in this way it has been extensively used for model comparison [12,14,15], structural modifications prediction based on eigenvalues sensitivity [6,15] or frequency domain model updating [1].

Unlike mentioned references, the previous equations were written in undamped form. Although this choice was made primarily for the sake of cleanness in the presentation, it has a more profound justification stemming from authors' previous works [2] that is: given the levels of uncertainty in these simplified models, the use of more complex, non-proportional damping formulation would not cause any perceivable alteration in measurable outputs. Thus, since a proportionally damped analytical system produces the same eigenproperties as its undamped version, and tacking into consideration the prominence of modal parameters over the forced response for the envisaged applications, damping factors can be confidently considered here as fine tuning adjustments. Among the myriad of available methods to extract damping coefficients (along with mode shapes and natural frequencies) from measured responses, Subspace-Stochastic Identification (SSI) has proven to be a very convenient, output-only, alternative [2].

2.1.2. Simplified 3 and 4 Degrees-of-Freedom Models

From the definitions presented above for 2 DOF, modelling extensions can be readily devised. Figure 3 shows schematic representations for two of such possible expansions, resulting in 3 and 4 DOF models, respectively. The most obvious approach then is to consider the flexibility of back plate in a symmetrical manner as was made for the top plate. This in turn adds up three more free parameters: for mass, stiffness and one more modal damping also, as the system could now vibrate on three linearly independent modes. Thus, resultant mass and stiffness matrices for 3 DOF model are written below, following [6], where displacements vector is

$$\mathbf{x} = \begin{bmatrix} x_t \\ x_a \\ x_b \end{bmatrix}, \mathbf{M} = \begin{bmatrix} m_t & 0 & 0 \\ 0 & m_a & 0 \\ 0 & 0 & m_b \end{bmatrix}, \mathbf{K} = \begin{bmatrix} k_t + \frac{\rho c^2 A_t^2}{V} & \frac{\rho c^2 A_t A_a}{V} & \frac{\rho c^2 A_t A_b}{V} \\ \frac{\rho c^2 A_t A_a}{V} & \frac{\rho c^2 A_a^2}{V} & \frac{\rho c^2 A_a A_b}{V} \\ \frac{\rho c^2 A_t A_b}{V} & \frac{\rho c^2 A_a A_b}{V} & k_b + \frac{\rho c^2 A_b^2}{V} \end{bmatrix}. \quad (4)$$

The next expansion presented, the model considering mass and motion for the ribs, is a little bit trickier and thus needs a deeper attention. First of all the motion allowed for the 'ribs' in fact means that the whole guitar is free to move. Thus, this is not exactly about freeing a motion in the same sense as it was done for the back plate, but rather to permit a fully-free boundary condition. Although governing equations of the 4 DOF model in [16] are not explicitly presented in matrix form, here mass and stiffness matrices were extracted and are thus presented, where displacements vector is

$$\mathbf{x} = \begin{bmatrix} x_t \\ x_a \\ x_b \\ x_r \end{bmatrix}, \mathbf{M} = \begin{bmatrix} m_t & 0 & 0 & 0 \\ 0 & m_a & 0 & 0 \\ 0 & 0 & m_b & 0 \\ 0 & 0 & 0 & m_r \end{bmatrix}, \mathbf{K} = \begin{bmatrix} k_t + \frac{\rho c^2 A_t^2}{V} & \frac{\rho c^2 A_t A_a}{V} & \frac{\rho c^2 A_t A_b}{V} & -k_t - \frac{\rho c^2 A_t^2}{V} + \frac{\rho c^2 A_t A_b}{V} - \frac{\rho c^2 A_t A_a}{V} \\ \frac{\rho c^2 A_t A_b}{V} & \frac{\rho c^2 A_a A_b}{V} & k_b + \frac{\rho c^2 A_b^2}{V} & k_b + \frac{\rho c^2 A_b^2}{V} - \frac{\rho c^2 A_a A_b}{V} - \frac{\rho c^2 A_t A_b}{V} \\ -k_t & 0 & k_b & k_t + k_b \\ \frac{\rho c^2 A_t A_b}{V} & \frac{\rho c^2 A_a A_b}{V} & \frac{\rho c^2 A_b^2}{V} & \frac{\rho c^2 A_b^2}{V} - \frac{\rho c^2 A_a A_b}{V} - \frac{\rho c^2 A_t A_b}{V} \end{bmatrix}. \tag{5}$$

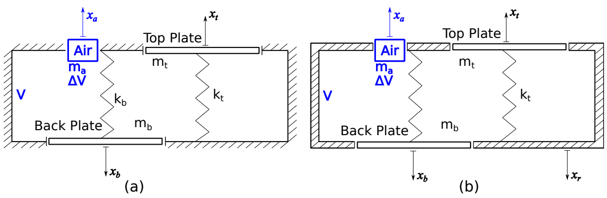

Figure 3. Schematic of 3 and 4 DOF simplified models; (**a**) 3 DOF: allowing for motions on top and back plates, and air column through the hole. (**b**) 4 DOF: basically allowing for the same motions of the 3 DOF model, plus one for the entire structure (free condition).

3. Finite Element Modelling of Acoustic Guitar Vibroacoustics

Many factors contributed to the popularity of FE modelling in the last two or three decades, but most of all, possibly, is the great availability of sufficient computational resources. Thus, it is not surprising that nowadays the bulk of academic research in vibro-acoustic of musical instruments makes use of it in greater or lesser extent. Brooke [22] was a pioneering work on the application of FE to guitar coupled vibro-acoustics. The simplified models were employed alongside with FE modelling of the top plate and boundary element (BE) model of surrounding air, thus achieving an organic coupling between lumped parameters and domain discretization approaches. Elejabarrieta, Ezcurra and Santamaría [18,19] performed fluid-structure interaction FE analyses, both for fluid and structural domains. Fluid domain was considered only in a small air column, in a similar fashion as was done for the lumped models discussed earlier. Neck and headstock participations were disregarded. 8474 brick elements were used to model the structure, while the mesh on fluid domain was adjusted to give sufficiently accurate results up to 1 kHz. Comparison of coupled and uncoupled mode shapes confirmed the physical insights brought by lumped models and the importance of Helmholtz frequency to the lower range coupled dynamics. Chaigne and collaborators [9,20,21] successfully employed FE and finite differences in a full non-linear transient simulation, where considerable effort is put in precise definition of purely mathematical aspects related to the problem well-posedness and stability of the proposed solution procedure. Fictitious domain method was used to avoid what they called an 'ectoplasm', referring to the rather artificial definition of boundaries on the fluid domain. Structural flexibility of the resonant chamber is considered only within the top plate, disregarding fan bracing. Both these simplifications may left aside important aspects of top plate dynamic behaviour. It is interesting to note that, despite the clear relevance of such a complete description, these important features are especially focused at by lumped parameters models presented above. Other instruments, closely related to the acoustic guitar, were successfully modelled via FE analysis as well, and we pinpoint here references [23] on Brazilian *viola caipira* and [24] on Colombian *bandola*. Both present detailed equations and solution procedure for the full coupled FE analysis and solve equations using ANSYS commercial software, but in [24] a convenient symmetrized form for the coupling matrices is introduced, and an effective length is considered for the air column [18,19].

4. Results and Discussion

This section is intended to show some results obtained by the authors with both kinds of modelling techniques discussed up to this point. Löw [1], updating/identification of the 3 DOF model parameters was performed, following [6], but using some heuristic optimization algorithms instead to minimize the sum of squared differences between analytical and experimental FRFs. Masses were measured with 0.1 grams accuracy, and stiffness parameters were previously estimated by static force/displacement testing, as suggested by [11,14,15]. Figure 4 below compares the updated *theoretical* model with experimental data; a modified particle swarm algorithm was used in error minimization. Experimental data was acquired on a low quality, large-scale manufactured instrument.

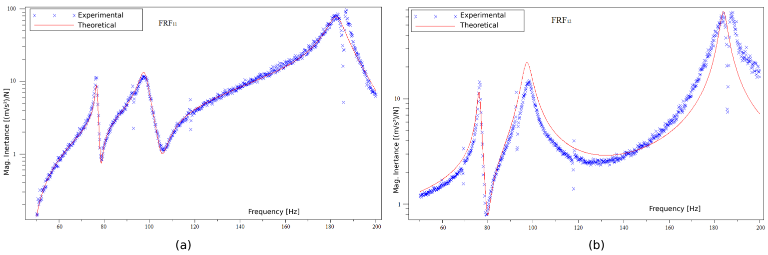

Figure 4. 3 DOF model analytical vs. experimental FRFs: (**a**) top plate; (**b**) forced on top plate and measured on the back plate.

Table 3. Natural frequencies of the classic guitar with the updated 3DOF and FE models: analytical / experimental results (relative deviation).

3 DOF [Hz]	FE [Hz]
76.7 / 76.6 (0.1%)	75.9 / 71.88 (5.5%)
97.7 / 98.0 (-0.3%)	110.2/ 108.4 (1.6%)
182,6 / 187.2 (-2.4%)	169.3 / 209.4 (-19.1%)

For the sake of comparison, some more recent results [2] obtained for a Giannini GWNE15, a Brazilian mid quality, partially hand-made guitar, are presented below, this time parametrically modelled in ANSYS commercial software. The mesh used 17001 shell elements, considering wood plates orthotropic properties. Fan bracing, struts and neck were modelled with shell elements as well. Figure 5 below shows the FE mesh, and Figure 6 shows structural (*in vacuo*) modes of top and back plates. Table 3 compares experimental and analytical natural frequencies for both models showed earlier. Bigger discrepancies in FE model are primarily due to not considering the air-structure coupling, so it is difficult to establish mode matching. Even so, it is clear that the greater number of available vibration modes can be used to better represent effects associated with higher frequency range dynamics, such as the timber of instruments, and radiation efficiency.

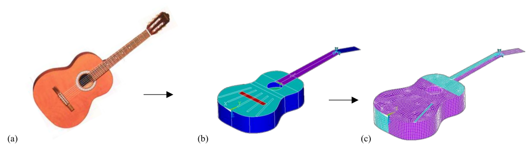

Figure 5. (**a**) Giannini catalog photo for model GWNE15 ; (**b**) 3D CAD model; and (**c**) resultant mesh for numerical analysis of Giannini guitar.

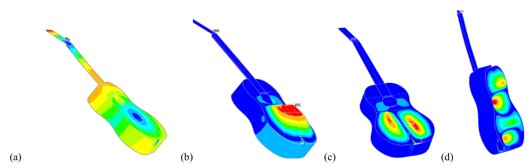

Figure 6. Undamped structural modes: (**a**) in phase top plate mode (75.9 Hz); (**b**) anti phase top plate mode (110.2 Hz); (**c**) first dipole top plate mode (169.3 Hz) and (**d**) quadrupole back plate mode (174.1Hz).

The simplicity and low computational burden of the lumped parameter 3D model permitted an easy application of an optimization routine to curve-fit structural parameters to reproduce low frequency range dynamic behaviour with good agreement. Phase angles were not taken into account in the error function and some acceptable agreement was verified as well, thus bringing reliability to the obtained parameters values. On the other hand, a numerical approximate solution of partial differential equations makes it easy to visualize mode shapes, obtain dozens of natural frequencies, maybe more than one hundred if one needs and has sufficient computational resources, accounting for global and local contributions as well. It should be mentioned, however, that FE results are largely affected by a significant number of purely numerical issues, such as element distortion (close to singular, or even negative jacobian determinant), and spurious modes due to artificial stiffness in finite element formulation.

5. Conclusion

The use of finite element (FE) element modeling is unquestionably advantageous for systems with a high degree of complexity, coupling diverse physical phenomena, where interesting applications for numerical analysis of musical instruments have been presented. Although many of the FE codes are very user-friendly and non-commercial versions may be found, these not accessible to the majority of the luthiers and the process is far too complex for them. This article describes the classical lumped mass approach and the widely used FE analysis for modelling and simulation of the acoustic guitar, highlighting the advantages and disadvantages of each. There is no undisputed standard for coupled vibroacoustic analysis, especially among users and manufacturers of acoustic guitars.

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