Volumetric Entropy Associated with Short Light Pulses Propagation in Gaseous Media

Abu Mohamed Alhasan

Physics Department, Assiut University
Assiut 71516, Egypt; am.alhasan.sq@gmail.com
Current address: Bağlar Mahallesi, 31500 Ryhanlı, Hatay, Turkey.

Contents

- Introduction
- The atomic System
- Surface and Volumetric Entropy
- Short pulse propagation of sigma minus light polarization in the sharp line limit (SSL)
- Numerical Results
- Conclusions

- *One of the fundamental interests in laser pulse propagation through atomic or molecular media relies on the control of propagation in terms of the area of the electric field envelope over time.*
- *That is meaningful as the initial phases of the atomic dipoles are ignored, and the pulses are kept at resonance with their respective optical transitions. Otherwise, the integrated pulse envelope over time becomes complex. Therefore, the real-valued area is not defined precisely.*
- *We seek local stabilization of the propagated pulses in terms of volumetric entropy of the Bloch-ball.*
- *This study proposes supremum and infimum identifiers to quantify the divergence of the upper and lower limits of the integral over the complex envelope of the pulse.*
- *The adopted volumetric entropy overcame the implementation of the complex-valued information measure on entropy.*

Overview

- *We investigate intense-short pulses propagation through two-level atomic vapors*
- *Considered: D_1 line in the fine structure of sodium or rubidium*
- *The time evolution of the dressed atom is based upon the density matrix approach*
- *The fields are described by the reduced-Maxwell Field equations*
- *The pulses are off-resonant to their respective optical transitions*

Background

- *McCall and Hahn demonstrated theoretically and experimentally the propagation of ultra-short pulses pulse within an attenuating medium of two level atoms.*
- *McCall and Hahn derived the Area Theorem which governs ultra short pulses propagation.*
- *The theorem showed the transparency of pulses with initial area N(2π), with N an integer. The phenomenon is called Self Induced Transparency (SIT)*
- *The area theorem predicts the pulse break up such as $2N\pi$ area propagation leads a train of (2π) pulse propagation.*
- *The area theorem requires inhomogeneous broadening and the lack of the atomic relaxations.*

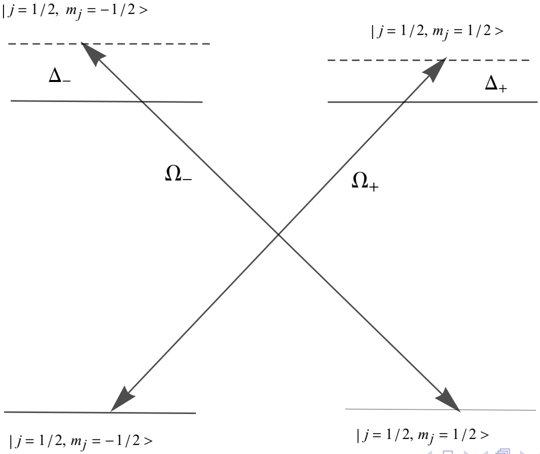
Motivations: Experimental and Theoretical

- *S. L. McCall and E. L. Hahn, Self Induced Transparency by pulsed coherent light, Phys. Rev. Lett.* **18**, *908 (1967).*
- *S. L. McCall and E. L. Hahn,Self-Induced transparency, Phys. Rev.,* **183**, *457 (1969).*
- *S. L. McCall and E. L. Hahn, Pulse-Area-Pulse-Energy Description of a Traveling-Wave Laser Amplifier, Phys. Rev. A* **2**, *861 (1970),*
- *G.L. Lamb, Jr, Analytical Descriptions of Ultrashort Optical Pulse Propagation in a Resonant Medium, Rev. Mod. Phys.* **43**, *99 (1971)*
- *H. M. Gibbs and R. E. Slusher, Self-Induced Transparency in Atomic Rubidium, Phys. Rev. A,* **6**, *2326 (1972).*

- *The state of the atom is described by the Liouville-von Neumann equation for the density matrix as* $-i\partial_t\rho = \mathcal{L}\rho = (\hat{H} + i\hat{\Phi})\rho$
- $\mathcal{L}$, $\hat{H}$ *and* $\hat{\Phi}$ *stand for the system Liouvillian, Hamiltonian and the relaxation superoperators, respectively.*
- *Ref. Fiutak J, Van Kranendonk J. The effect of collisions on resonance fluorescence and Rayleigh scattering at high intensities.* J. Phys. B: Atom. Mol. Phys. **1980**, 13, *2869–2884; https://doi.org/10.1088/0022-3700/13/15/006*
- *Ref. Alhasan, A.M.; Fiutak, J.; Miklaszewski, W. The influence of the atomic relaxation on the resonant propagation of short light pulses.* Z. Phys. B - Condensed Matter **1992**, 88, *349–358; https://doi.org/10.1007/BF01470924*

Fig. 1: Excitation of duplicated two-level atom

J angular momentum quantum number; m_j magnetic quantum number
$\Omega_\pm$ Rabi frequency of the circularly polarized fields
$\Delta_\pm$ corresponds to detuning



- There are two atomic populations
  $n_1 = \sqrt{2}\rho_1$,
  $n_2 = \sqrt{2}\rho_2$
- Two components for the atomic orientation
  Ground state orientation: $\sqrt{2}\rho_3$
  Excited state orientation: $\sqrt{2}\rho_4$
- There are two complex coherences: ρ_5 and ρ_6

Space and time variables

- Variables: space z, time t
  c is the light speed
- For space $\eta = z + ct$
- The retarded time $t^{'} = (t - z/c)$.
- $\partial_\eta = \partial_z + 1/c\,\partial_t$
- Dimensionless variables
  For space $\zeta = \alpha^{'}\eta$,
  For time $\tau = \gamma t^{'}$.
- $\alpha^{'}$ denotes the absorption coefficient of one of the pulses at $z = 0$.

The Atom-Field coupling dimensionless variable

- the atom-field coupling is denoted by $v = \frac{d_r E}{\sqrt{3}}$,
- d_r is the reduced dipole moment and
- E is the electric field envelope.
- We have used $\hbar = 1$.
- The Rabi frequency associated the atomic transition
- $\Omega = \kappa v$, where κ is the proportionally constant.
- The dimensionless atom-field coupling: $\mathrm{v} = v/\gamma$.
- γ is the spontaneous decay of the atomic excited state.

The Reduced-Maxwell field equations for a pulse with left hand circular polarization

$$
\begin{aligned}
&\frac{\partial v(z,t)}{\partial z} = \alpha' \rho_5(z,t),\\
&\frac{\partial \rho_1(z,t)}{\partial t} = \gamma\rho_2(z,t) + \frac{1}{\sqrt{2}}[v^*(z,t)\ \rho_5(z,t) + v(z,t)\ \rho_6(z,t)],\\
&\frac{\partial \rho_2(z,t)}{\partial t} = -\gamma\rho_2(z,t) - \frac{1}{\sqrt{2}}[v^*(z,t)\ \rho_5(z,t) + v(z,t)\ \rho_6(z,t)],\\
&\frac{\partial \rho_3(z,t)}{\partial t} = -\frac{\gamma}{3}\rho_4(z,t) + \frac{1}{\sqrt{2}}[-v^*(z,t)\ \rho_5(z,t) - v(z,t)\ \rho_6(z,t)],\\
&\frac{\partial \rho_4(z,t)}{\partial t} = -\gamma\rho_4(z,t) + \frac{1}{\sqrt{2}}[-v^*(z,t)\ \rho_5(z,t) - v(z,t)\ \rho_6(z,t)],\\
&\frac{\partial \rho_5(z,t)}{\partial t} = -(\Gamma - i\Delta)\rho_5(z,t) + \frac{1}{\sqrt{2}}[-v^*(z,t)\ \rho_1(z,t) + v(z,t)\rho_2(z,t) +\\
&v^*(z,t)\ \rho_3(z,t) + v(z,t)\ \rho_4(z,t)],\\
&\frac{\partial \rho_6(z,t)}{\partial t} = -(\Gamma + i\Delta)\rho_6(z,t) + \frac{1}{\sqrt{2}}[-v(z,t)\ \rho_1(z,t) + v^*(z,t)\ \rho_2(z,t) +\\
&v(z,t)\ \rho_3(z,t) + v^*(z,t)\ \rho_4(z,t)].
\end{aligned}
\tag{1}
$$

The temporal pulse profile at $z = 0$ and its area

$$v(t) = \tfrac{64\sqrt{2}\pi}{27} v_0 (t/T_p)^2 e^{-8/9\pi (t/T_p)^2}, \quad \theta(z) = \kappa \int_0^\infty v(t)\,dt \tag{2}$$

- The pulse is characterized by its mean amplitude v_0 and mean duration T_p as

$$v_0 = \int_0^\infty v^2(t)\,dt / \int_0^\infty v(t)\,dt, \quad T_p = (\int_0^\infty v(t)\,dt)^2 / \int_0^\infty v^2(t)\,dt. \tag{3}$$

- We consider off-resonance propagation, thus we have a complex field and its time integral is a complex valued.

The Complex Pulse Area and its integral inequalities

- We have the integral inequalities as

$$|\int_{T'} \mathrm{v}_{\pm}(\zeta,\tau)d\tau|^2 \leq \int_{T'} |\mathrm{v}_{\pm}(\zeta,\tau)|^2 d\tau, \tag{4}$$

$$|\int_{T'} \mathrm{v}_{\pm}(\zeta,\tau)d\tau| \leq \int_{T'} |\mathrm{v}_{\pm}(\zeta,\tau|d\tau. \tag{5}$$

- Here we assume that $|\mathrm{v}_{\pm}(\zeta,\tau)|$ is absolutely integrable, i.e

$$\int_{-\infty}^{\infty} |\mathrm{v}_{\pm}(\zeta,\tau)|d\tau < \infty \tag{6}$$

The upper-limit and lower-limit of the integrated pulse envelope

The inequalities hold for each integrable complex-valued function. The first limit defines the lower limit as

$$A_L(\zeta) = \vartheta_{min}(\zeta) = \liminf \vartheta(\zeta) = \left| \int_{T'} \kappa \, \mathsf{v}(\zeta, \tau) d\tau \right|. \tag{7}$$

The second limit defines the upper limit as

$$A^U(\zeta) = \vartheta_{max}(\zeta) = \limsup \vartheta(\zeta) = \int_{T'} \kappa \, |\mathsf{v}(\zeta, \tau)| d\tau. \tag{8}$$

We use both two limits to investigate the propagation of optical pulses.

Volumetric Entropy and Surface-metric Entropy

Let us define the spatiotemporal probability for obtaining a surface on the Bloch-ball and its associated surface-entropy as:

$$\begin{gathered} P_A(z,t) = A(z,t)/\sum_{t'} A(z,t'), \\ \mathbb{S}(z) = -\int P_A(z,t)\ \log_2(P_A(z,t))dt. \end{gathered} \tag{9}$$

Similarly, we can define the spatiotemporal volume probability distribution and its associated volumetric entropy as

$$\begin{gathered} P_V(z,t) = V(z,t)/\sum_{t'} V(z,t') \\ \mathbb{E}(z) = -\int (P_V(z,t)\log_2(P_V(z,t))dt, \end{gathered} \tag{10}$$

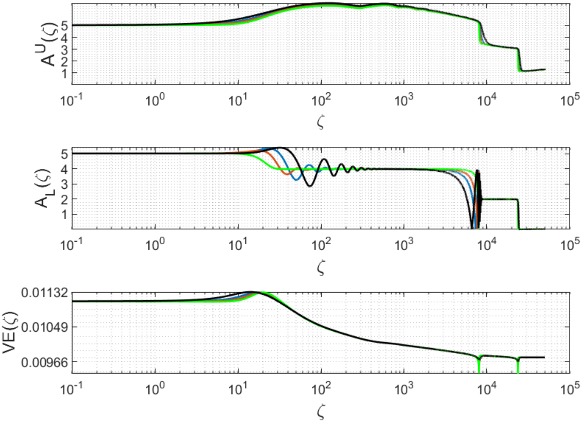
$A(z,t)$ is the area of the Bloch-ball
$V(z,t)$ is the volume of the Bloch-ball.

Propagation with an initial area close to 5π values

- LBCR(UBCR) Lower (Upper) Bounds of the Complex Area integral
- Ref. Figure 2
  The LBCR displays four different sections in space
  1st. Propagation that is close to the initial value
  It is different from SIT predictions for an attenuating media
  2nd. LBCR shows stabilization on the 4π value
  3rd. It shows collapse to the 2π value
  4th. It shows collapse to the 0π value
- A new feature is given by the behavior of UBCR
  1st. The UBCR indicates enhancement and transition towards increasing the initial area
  2nd. It shows collapse to nearby 3π value
  3rd. It shows collapse to the π value
  Both bounds shows an abrupt transition to stabilized area values of $\pi(0\pi)$ at the same space point

Fig. 2: LBCR and UBCR space dependence

The initial area $\vartheta = 5.025$ in π units. $\Delta = 5$; $\Delta = 1$; $\Delta = 2$; $\Delta = 3$.



The Upper and Lower Limits of The Pulse Energy

The lower limit can read as

$$E_{r_min}(\zeta) = \frac{\liminf E(\zeta)}{E_0} = \frac{|\int_{T'} \kappa \, \mathsf{v}(\zeta,\tau) d\tau|^2}{E_0}, \tag{11}$$

and the upper limit can read as

$$E_{r_max}(\zeta) = \frac{\limsup E(\zeta)}{E_0} = \frac{\int_{T'} |\kappa \, \mathsf{v}(\zeta,\tau)|^2 d\tau}{E_0}. \tag{12}$$

Volume and Surface Entropy Gained Across Propagation

- We notice that the isentropic period for entropy corresponds to energy depletion without oscillation in both the two limits
- By isentropic entropy, we mean that the change in entropy is small
- The upper energy limit is related to the average power of the complex-field intensity
- The Fourier transform of the absolute square of the field amplitude gives the autocorrelation function
- The lower area limit of the complex area integral gives a resemblance to the IST and SIT for an attenuating medium
- However, the local energy of these results does not show the legacy of the power spectrum and the Wiener–Khinchin theorem.

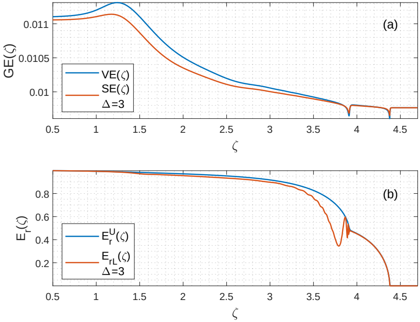


Figure: In $(3.a)$: The space-dependent geometrical entropy, $GE(\zeta)$ as volume-entropy, $\mathbb{E}(\zeta) = VE(\zeta)$, and surface entropy, $\mathbb{S}(\zeta) = SE(\zeta)$, in the course of propagation. The initial area $\vartheta = 5.025$ in π units.
In $(3.b)$: The space-dependent energy: upper limit of relative energy, $E_r^U(\zeta)$, and lower limits of relative energy, $E_{r_L}(\zeta)$, in the course of propagation. The initial area $\vartheta = 5.025$ in π units.

- It is now more suitable to deal with compound pulse propagation in gaseous media considering the hyperfine structure
- The compound pulse comprises a detuned fundamental pulse and another weak pulse rotating around the central frequency
- We used such performance to reveal the light-storage effect and the speed reduction in sodium atoms. The number of density matrices was 28 components (Alhasan et.al 2003).
- In That work, the fundamental pulse has four detunings away from the central atomic line.
- Yet another complicated system is one including the hyperfine structure and for arbitrary polarization where the number of the density matrix components is 136 components (Dangel 1997).

- Alhasan, A.M.; Fiutak, J. Density matrix description of light storage and speed reduction in the sodium atom. 246 Proc. SPIE 2003 5258, IV Workshop on Atomic and Molecular Physics; https://doi.org/10.1117/12.544418
- Dangel, S. et.al Semiclassical theory for the interaction dynamics of laser light and sodium atoms including the hyperfine structure. Phys. Rev. A 1997 56, 3937–3949; https://link.aps.org/doi/10.1103/PhysRevA.56.3937

Concerning: Upper and Lower Limits of the Complex Area Integral

- We have facilitated a mathematical analysis procedure for complex-area propagation of short optical pulses
- The governing equations are the reduced form of Maxwell Bloch equations in a two-level atom media incorporating the atomic relaxations.
- We have shown that both the upper and lower area bounds signify transitions to lower values of the area of each type
- The lower bounds give resemblance to the results of SIT
- Our results can be considered as an extension to SIT to limit beyond its justifications
- The upper limits show transitions that become un-relevant to SIT, especially for an attenuating media

Concerning: Surface and Volume Entropy

- Volume entropy and Surface Entropy
- It shows the combined information carried out by the upper and lower area transitions.
- The volume entropy exposes dips at the area transitions.
- The space length of the area window displays a measure for the stable transparency of the optical pulse.
- Our mathematical procedure for the complex analysis avoids using complex information measures.
- The proposed technique can be used in studying light storage and its retrieval by even a single pulse through multilevel atoms.

Thank You!

Funding : This research received no external funding
Conflicts of Interest : The author declare no conflict of interest

am.alhasan.sq@gmail.com