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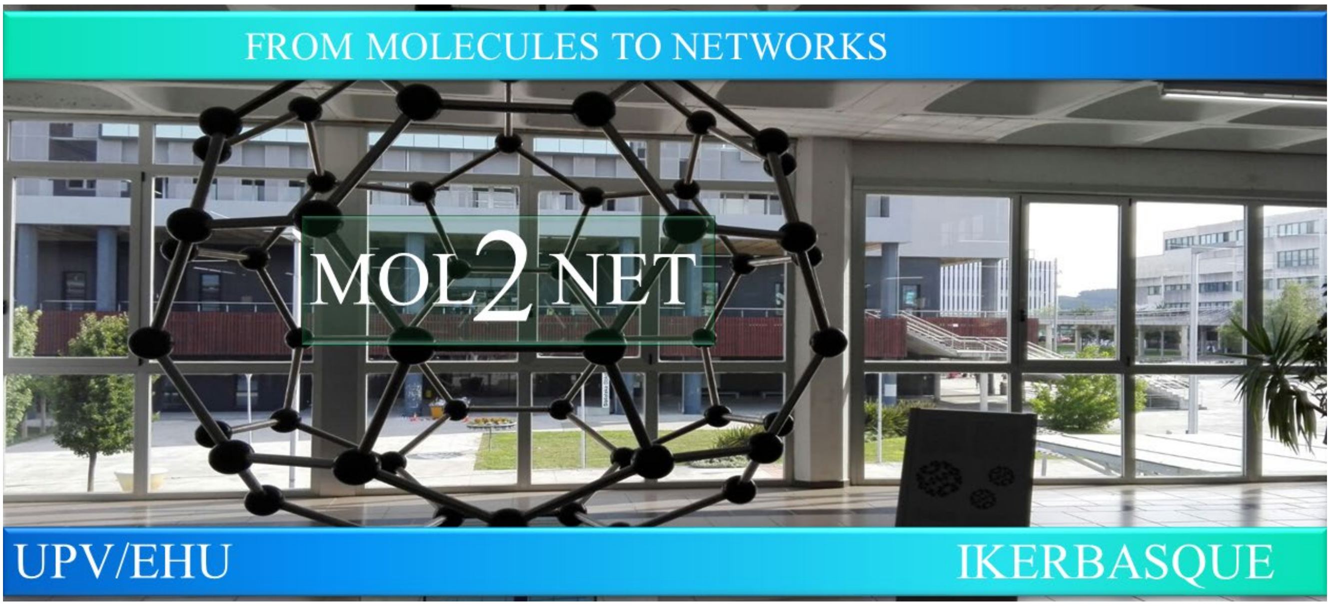

A probability problem suitable for Problem-Based Learning

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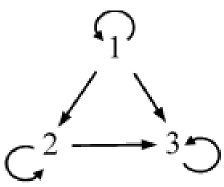
Graphical Abstract



Abstract.
We propose a simple probability problem for undergraduate level. This problem involves different branches of Mathematics, such as Graph Theory, Linear Algebra or hypergeometric sums, hence it is quite suitable to be used as Problem-Based Learning. In addition, the problem allows several variations so that it may be proposed to different groups of students at the same time. Finally, we propose that the students interpret the result obtained as combinations with repetition.

Introduction

Problem *If we choose a sequence of m natural numbers,* $a_1, a_2, \ldots, a_m$*, at random, such that* $1 \leq a_i \leq n$*, what is the probability to obtain* $a_1 \leq a_2 \leq \cdots \leq a_m$*?*

Results and Discussion

In https://doi.org/10.46583/nereis_2021.13.782, the solution to the problem is derived with the aid of Graph Theory, Linear Algebra and hypergeometric sums, to finally obtain,

$$p = \frac{(n+m-1)!}{m!(n-1)!\, n^m}.$$

The above result suggests the following shortcut to the solution. Indeed, according to the solution, we see that the number of favorable events *f* coincide with the combinations with repetition formula CR_n^m , i.e. the number of combinations of *n* elements taken *m* at a time, allowing repetition of elements [3, Sect. 6.2],

$$f = \binom{m+n-1}{m} = CR_n^m.$$

To show this coincidence, consider that we have *n* different natural numbers: $\alpha_1, \alpha_2, \ldots, \alpha_n$, where $\forall i, \alpha_i \leq n$, and then chose *m* of these numbers, allowing repetition. The number of the different selections we can make is given by the combination of repetition formula, as aforementioned. Define as the number of times you have chosen number $n(\alpha_k)$. For instance, taking $n = 3$ and $m = 6$, a possible combination is,

α_1	α_2	α_3
x	**xx**	**xxx**

where $n(\alpha_1) = 1, n(\alpha_2) = 2, n(\alpha_3) = 3$. Now, order the natural numbers α_k in such a way that if $n(\alpha_i) \geq n(\alpha_j)$ then $\alpha_i < \alpha_j$. Therefore, in the above example, we have

3	2	1
x	**xx**	**xxx**

That is to say, we have the combination: 1-1-1-2-2-3. Obviously, this combination is one of the possible favored events in the initial problem, thus the number of all these favored events is given by the combinations with repetition formula.

It is worth noting that if the restriction of the problem is $a_1 < a_2 < \cdots < a_m$, then the favored events are given by the number of combinations formula, i.e.

$$f = C_n^m = \binom{m}{n}.$$

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