Proceeding Paper

Effects of Exceptional Points on the Optical Properties of a Quantum Dot in a Microcavity [†]

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Abstract: We theoretically study the effect of quantum exceptional points on the optical properties of a quantum dot placed inside an optical microcavity and interacting with a weak probe field. Using a quantum approach, we study the steady state behavior of the system and calculate the optical susceptibility. By separating the total susceptibility to two equivalent susceptibilities, corresponding to fictitious free quantum emitters, we show exceptional points drastic effect to the optical properties of the system close to the region where the exceptional point is formed. We further examine the optical properties of the system in the regions of the parameter space that arise from the exceptional condition, namely the strong coupling regime and the weak coupling regime.

Keywords: exceptional point; quantum dot; optical microcavity; cavity induced transparency

1. Introduction

Exceptional Points (EPs) have been studied in a plethora of classical optical systems [1–4] and can dramatically modify the optical response of the studied system. Recently, there is increasing interest in applying the EP formalism in fully quantum systems to achieve efficient control at the quantum level and show the drastic effect EPs have on the system's properties [5–8]. In this work, we theoretically study the effect of quantum EPs on the optical properties of a quantum dot placed inside an optical microcavity and interacting with a weak probe field. For the analysis of the system, we use a quantum approach, where we model the quantum dot as a two-level system and describe the light-matter interaction with the proper master equation, including the spontaneous decay and the pure dephasing of the quantum dot, as well as the decay of the optical cavity. We also define the effective non-Hermitian Hamiltonian of the system and derive the necessary conditions for the formation of an EP, the point where the eigenvalues of the Hamiltonian coalesce.

We then study the steady state behavior of the system and calculate the optical susceptibility from the coherences between the vacuum and the system, which can lead to cavity induced transparency [9,10]. By separating the total susceptibility to two equivalent susceptibilities, corresponding to fictitious free quantum emitters, we show EPs drastic effect to the optical properties of the system close to the region where the exceptional point is formed. We further examine the optical properties of the system in the regions of the parameter space that arise from the exceptional condition, namely the strong (coherent) coupling regime and the weak (incoherent) coupling regime. In conclusion, we give an overview of the effect of EPs in a purely quantum system and show how that effect translates in the observable, which are the classical properties of the system.

2. Materials and Methods

We consider a system consisting of a quantum dot, modeled as a two-level quantum emitter (QE), placed inside an optical microcavity. The QE is coupled to the electromagnetic field inside the cavity with coupling rate g. The QE is also subject to decay γ due to spontaneous emission and a pure dephasing rate γ^* (in our analysis we take $\gamma^* = 10^4 \gamma$, to be in agreement with solid state QEs like quantum dots). Additionally, the cavity decays in free space with a decay rate κ. Lastly, we consider an external probe electromagnetic field, $E = \mathcal{E}e^{i\omega_p t} + c.c.$, that interacts with the QE. Thus, the Hamiltonian is $H_0 = \omega_a \sigma^+ \sigma^- + \omega_c a^\dagger a + g\sigma^+ a + \Omega e^{i\Delta t}\sigma^+ + h.c.$, where $\Omega = \mu\mathcal{E}$ corresponds to the Rabi frequency of the $e \to g$ transition, μ is the dipole moment of the QE and $\Delta = \omega_p - \omega_a$ is the detuning. In this work we take $\hbar = 1$. We can define an effective non-Hermitian Hamiltonian that encompasses both the coherent and the dissipative terms of our system, given by $H_{eff} = H_0 - (i\kappa/2)a^\dagger a - (i\gamma/2)\sigma^+ \sigma^- - (i\gamma^*/2)\sigma_z^\dagger \sigma_z$.
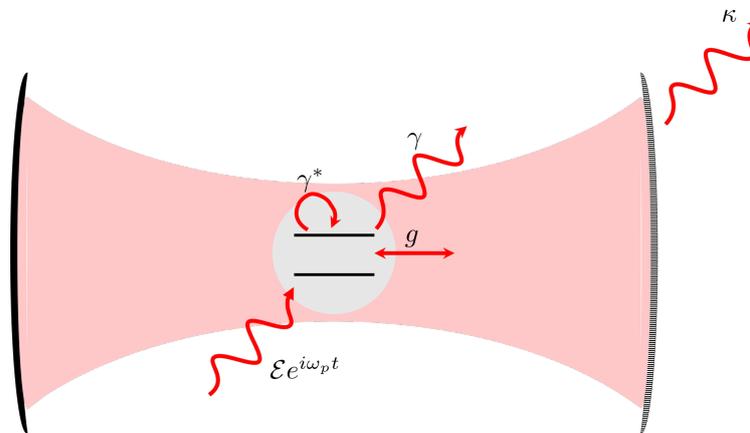

Figure 1. Illustration of the system under consideration. A quantum dot (grey) is placed inside an optical microcavity of one electromagnetic mode (pink). The QE is coupled to the cavity mode with coupling constant g and to an external probe field. Also, the QE experiences decay γ due to spontaneous emission and pure dephasing with rate γ^*. Lastly, the cavity is open and thus has decay rate κ.

We investigate the eigenvalues of the effective Hamiltonian for the case of a weak probe field, meaning $\Omega \ll g, \gamma, \kappa$. The point where the eigenvalues coalesce corresponds to the condition $2g/|\gamma + \gamma^* - \kappa| = 1/2$, which is independent of the probe field. In order to study the optical properties of the system, we need to study its dynamics and steady state, by solving the Lindblad equation and evaluating the coherences with the vacuum. We find that the susceptibility is given by

$$\chi = \xi \frac{\Delta + i\frac{\kappa}{2}}{g^2 - \left(\Delta + i\frac{\gamma+\gamma^*}{2}\right)\left(\Delta + i\frac{\kappa}{2}\right)}. \tag{1}$$

Here, $\xi = N\mu/\varepsilon_0$, where N is the atomic density of the quantum dot and ε_0 is the vacuum permittivity. The imaginary part of the susceptibility is a measure of the probe absorption of the system and the real part shows the dispersion relation of the system.

We separate the total susceptibility of the system in two parts, corresponding to two "free QE" susceptibilities, such that $\chi = \chi_1 + \chi_2$, which are given by

$$\chi_1 = \xi \left(\frac{i(\gamma + \gamma^* - \kappa)}{2\sqrt{Z}} - 1 \right) \left(\frac{1}{2\Delta - \sqrt{Z} + i(\gamma + \gamma^* - \kappa)/2} \right), \tag{2}$$

$$\chi_2 = \xi \left(\frac{i(\kappa - \gamma - \gamma^*)}{2\sqrt{Z}} - 1 \right) \left(\frac{1}{2\Delta + \sqrt{Z} + i(\gamma + \gamma^* - \kappa)/2} \right), \tag{3}$$

where $Z = 4g^2 - (\gamma + \gamma^* - \kappa)^2/4$. These "free QE" susceptibilities are of the form $\chi = \xi \frac{A}{(\Delta - \Delta_0) + iw/2}$, whose real and imaginary parts are the well known dispersion $\chi^r = \xi \frac{A(\Delta - \Delta_0)}{(\Delta - \Delta_0)^2 + (w/2)^2}$ and Lorentzian absorption $\chi^i = \xi \frac{A(w/2)}{(\Delta - \Delta_0)^2 + (w/2)^2}$ curves respectively, with amplitude A which is a complex number.

3. Results and Discussion

In Figure 2 we plot the imaginary part of the susceptibilities corresponding to the absorption of each "free QE" and of the total system for weak cavity decay $\kappa = 0.001\gamma$ and strong dephasing rate $\gamma^* = 10^4\gamma$, as a function of the coupling g and the detuning Δ. We clearly see that for $2g/|\gamma + \gamma^* - \kappa| = 1/2$ the "free Q.E." susceptibilities tend to infinity while the system's susceptibility is finite and has two well separated, symmetric peaks. Thus, we can split the parameter space in two regions, the region of incoherent coupling $2g/|\gamma + \gamma^* - \kappa| < 1/2$, and the region of strong coupling $2g/|\gamma + \gamma^* - \kappa| > 1/2$. Special treatment will be given to the region where $2g/|\gamma + \gamma^* - \kappa| \to 1/2$. In the incoherent coupling region, χ_1^i is a narrow Lorentzian curve centered around $\Delta = 0$ with negative amplitude, while χ_2^i is a wide Lorentzian curve centered around $\Delta = 0$ with positive amplitude. As g decreases, χ_1^i becomes narrower and with smaller amplitude while χ_2^i practically stays the same, thus the "dip" of the total susceptibility χ^i at zero detuning becomes less visible. In the coherent coupling regime, χ_1^i and χ_2^i are well separated and symmetric around zero detuning, making the total susceptibility χ^i to have two distinct symmetric brunches. More specifically, for large enough coupling, only the Lorentzian part contributes thus each curve has as center the detuning $\Delta = \pm\sqrt{Z}/2$ and width $w = (\gamma + \gamma^* + \kappa)/2$. While the width stays the same as the coupling increases, their amplitude decreases as approximately $1/g$.
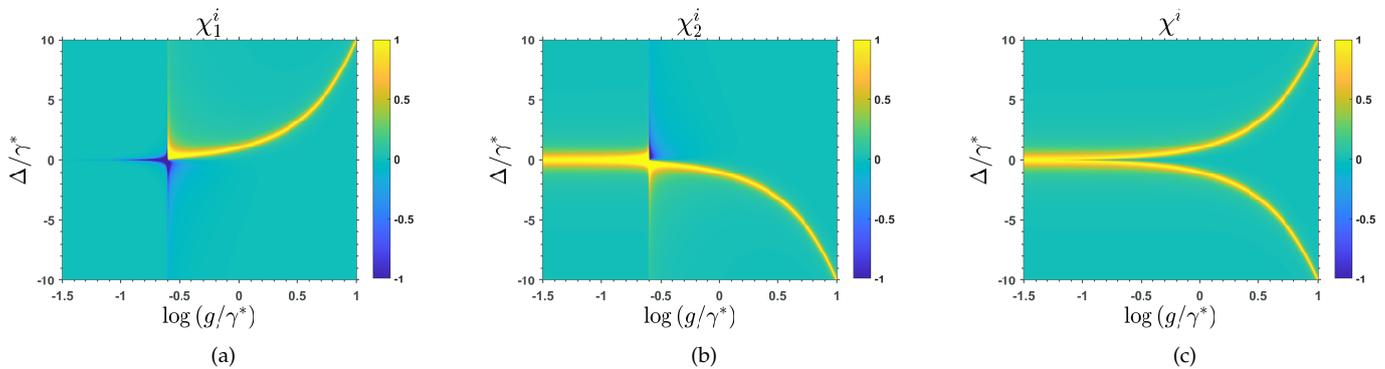

Figure 2. Imaginary part of the susceptibility of the equivalent "free QE" 1 and 2 and for the total system for small cavity decay $\kappa = 0.001\gamma$ and strong dephasing $\gamma^* = 10^4\gamma$ in the parameter space of the atom-cavity coupling g and the detuning of the probe field Δ.

Lastly, we investigate the behavior of the susceptibilities close to the EP as a function of the detuning (see Figure 3). We observe that the closer we get to the EP, the larger the "free-QE" susceptibilities become, and thus exactly at the EP they diverge to infinity. Additionally, the way the "free-QE" susceptibilities diverge depends on the sign of the detuning and on the region from which we approach the EP (depends on the kind of the limit $2g/|\gamma + \gamma^* - \kappa| \to 1/2^{\pm}$). When the system approaches the EP from the incoherent coupling region, the two susceptibilities correspond to a Lorentzian function (Figure 3), which are centered at zero detuning and have comparable and opposite amplitudes. At the limit, χ_1^i diverging to $-\infty$ and χ_2^i diverging to $+\infty$ for all Δ, but the total susceptibility remains a positive finite number. Thus, the total susceptibility has a wide "dip" at zero detuning and two symmetric peaks. When the system approaches the EP from the coherent coupling region, both susceptibilities correspond to Lorentzian dispersion functions (Figure 3). Similarly to the previous case, at the limit, χ_1^i is diverging to $+\infty$ for negative detuning and to $-\infty$ for

positive detuning, while χ_2^i diverging to $-\infty$ for negative detuning and to $+\infty$ for positive detuning. Again, the total susceptibility is finite and we see that it is a continuous function $(\lim_{g \to g_{EP}^{\pm}} \chi = \chi)$.
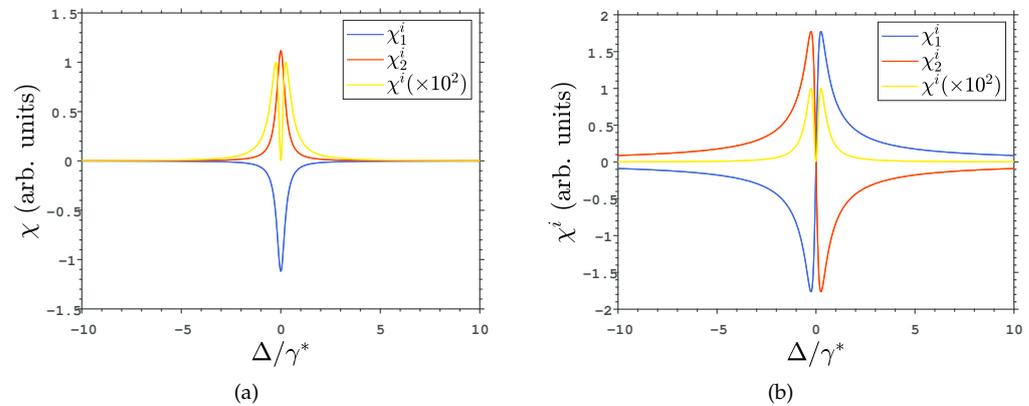

Figure 3. Imaginary part of the susceptibility of the equivalent "free QE" 1 and 2 and for the total system for small cavity decay $\kappa = 0.001\gamma$ and strong dephasing $\gamma^* = 10^4\gamma$ as a function of Δ for parameters that approach the EP from (**a**) the left $(g/|\gamma + \gamma^* - \kappa| \to 1/4^-)$ and (**b**) from the right $(g/|\gamma + \gamma^* - \kappa| \to 1/4^+)$ respectively.

Abbreviations

The following abbreviations are used in this manuscript:

EP   Exceptional Point
QE   Quantum Emitter

References

1. Bender, C.M.; Boettcher, S. Real Spectra in Non-Hermitian Hamiltonians Having PT Symmetry. *Phys. Rev. Lett.* **1998**, *80*, 5243–5246. https://doi.org/10.1103/PhysRevLett.80.5243.
2. El-Ganainy, R.; Makris, K.G.; Khajavikhan, M.; Musslimani, Z.H.; Rotter, S.; Christodoulides, D.N. Non-Hermitian physics and PT symmetry. *Nat. Phys.* **2018**, *14*, 11–19.
3. Özdemir, Ş.K.; Rotter, S.; Nori, F.; Yang, L. Parity–time symmetry and exceptional points in photonics. *Nat. Mater.* **2019**, *18*, 783–798.
4. Parto, M.; Liu, Y.G.N.; Bahari, B.; Khajavikhan, M.; Christodoulides, D.N. Non-Hermitian and topological photonics: optics at an exceptional point. *Nanophotonics* **2021**, *10*, 403–423. https://doi.org/doi:10.1515/nanoph-2020-0434.
5. Minganti, F.; Miranowicz, A.; Chhajlany, R.W.; Nori, F. Quantum exceptional points of non-Hermitian Hamiltonians and Liouvillians: The effects of quantum jumps. *Phys. Rev. A* **2019**, *100*, 062131.
6. Chen, W.; Abbasi, M.; Joglekar, Y.N.; Murch, K.W. Quantum Jumps in the Non-Hermitian Dynamics of a Superconducting Qubit. *Phys. Rev. Lett.* **2021**, *127*, 140504. https://doi.org/10.1103/PhysRevLett.127.140504.
7. Wu, Y.; Liu, W.; Geng, J.; Song, X.; Ye, X.; Duan, C.K.; Rong, X.; Du, J. Observation of parity-time symmetry breaking in a single-spin system. *Science* **2019**, *364*, 878–880.
8. Naghiloo, M.; Abbasi, M.; Joglekar, Y.N.; Murch, K. Quantum state tomography across the exceptional point in a single dissipative qubit. *Nat. Phys.* **2019**, *15*, 1232–1236.
9. Rice, P.R.; Brecha, R.J. Cavity induced transparency. *Opt. Commun.* **1996**, *126*, 230–235.
10. Cardimona, D.A.; Alsing, P.M.; Mozer, H.; Rhodes, C. Interference effects in a three-level atom in a cavity beyond the weak-field approximation. *Phys. Rev. A* **2009**, *79*, 063817. https://doi.org/10.1103/PhysRevA.79.063817.