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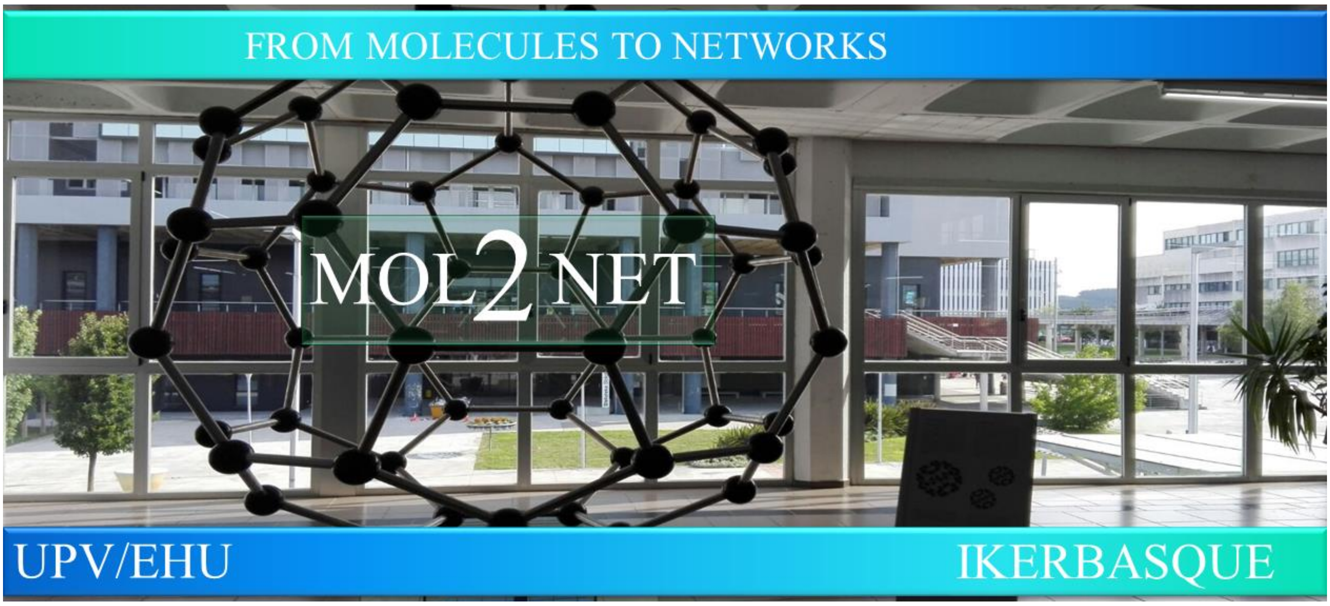

Applying the second index of Yager in a problem of routing and inventory with priority in the deliveries.

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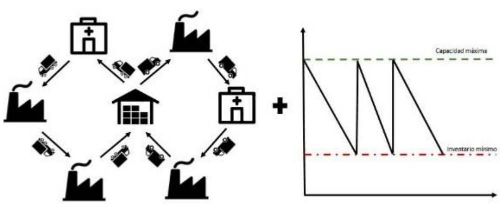
Graphical Abstract



Abstract.

In this paper the Inventory Routing Problem is addressed. The problem focuses on the distribution of liquid oxygen and considers two types of customers: industrial and medical facilities, the latter have priority in deliveries because of the quality of the product they demand. Also, the demand is considered uncertain. Here we presented a mathematical model where the demand is represented as a fuzzy number, and the objective is to minimize the total cost (inventory and routing costs) and seeks to create the optimal distribution routes considering *the demand and priority of the customers, as well as capacity of the trucks, among other important aspects. Finally, computational experiments were carried out using benchmark instances and the second index of Yager in order to analyze the effect of number of customers and levels of priority.*

Introduction

Logistics is essential for companies (Coelho et al., 2012), it optimizes the resources, looks to coordinate transportation, inventory management and material handling (Samia et al., 2017). The logistic cost is the most important economic indicator for a supply chain, especially transportation and inventory costs are the two main components of this indicator and have influence over distribution strategies (Drazen et al., 2012).

When decisions are taken in a sequential way the costs could increase, this is why it is important to make integrated decisions rather than sequential. When decisions are taken in an integrated way, the best policy for the entire system will be selected. Inventory and transportation systems are good examples of sequential decision making (Angel et al., 2014).

The inventory routing problem (IRP) is an extension of the vehicle routing problem (VRP) and has many applications (Moin and Salhi,2007). IRP integrates decisions of routing as well as inventory (Cordeau et al.,2007). The main decisions taken in this problem are: 1) which customers to visit?, 2) how much product to deliver? And 3) which route to employ? (Campbell et al., 1998). Therefore, the objective of an IRP is to find a balance between the inventory and routing costs to minimize the total costs that are incurred by these two segments in the supply chain (Milorad et al., 2014), (Drazen et al., 2012).

In real-world problems, some parameters are uncertain, the customer demands cannot be predicted accurately, but they can be described by distribution functions or several scenarios. Therefore, considering uncertainty helps managers to determine distribution and inventory policies more accurately (Erfaneh et al., 2019).

Demand is one of the most common parameters considered stochastic in real life (Campbell et al., 1998). There are two different approaches to deal with uncertainty, stochastic programming or fuzzy programming.

Some of the authors that have worked with stochastic programming are Bertazzi et al. (Bertazzi et al., 2013) focused on an IRP with stochastic demand and allow stock-outs. Luca et al. (Luca et al., 2015) proposed the demand from the customers as stochastic and the suppliers have a limited production capacity. Zhiwei et al. (Zhiwei et al., 2016) modeled the demand as a Poisson random variable. While Roldán et al. (Roldan et al., 2017) proposed to model the stochastic demands as continuous variables if their average is big enough. Moin and Halim (Moin and Halim, 2018) modeled the demand as a Markov decision process and an ant bee colony algorithm is implemented. Erfaneh et al. (Erfaneh et al., 2019) proposed a two-stage stochastic programming approach in order to explore the effects of uncertainty in demand.

Fuzzy programming is another approach to deal with uncertainty. Some authors that have worked with this approach are Ghasemkhani et al. (Ghasemkhani et al., 2019), they proposed a MILP that integrates the production with IRP, uncertain demand as a triangular fuzzy number, they considered multi-product, multi-period, heterogeneous fleets and time windows in the distribution. Also, Seed et al. (Saeed et al.,

2018) considered time windows and fuzzy demands, they simultaneously decide location for depots, terminal, routes and tours, helping to get better solutions. Elifcam (Elifcam, 2022) proposed a model to provide more efficient distribution and inventory planning to optimize the personal protective equipment for hospitals in a fuzzy environment. First, generates the cluster of each hospital, then an inventory-routing plan is generated. All demands and maximum inventory levels are uncertain. Hao Hu et al. (Hao et al., 2018) applied IRP for hazardous materials inventory and transportation among suppliers, manufacturers and retailers. The goal is to obtain the best balance between risk and cost under the assumption that the demands of retailers are triangular fuzzy variables.

This paper is focused on the distribution of liquid oxygen and the objective is to develop a fuzzy mathematical model to create distribution routes for a finite time horizon. We consider industrial and medical customers. This paper has two main characteristics: 1) to give priorities to medical customers (given a law of the Mexican government) and 2) to consider uncertainty of demand trough triangular fuzzy numbers implementing the second index of Yager. The paper is organized as follows, after this Introduction, we describe in detail the problem addressed here and the method used to solve the problem. Then, we present the mathematical model. After that, we show the results and discussion and finally, we present the conclusions of this research as well as the future work.

Problem description

The problem presented here is about inventory routing problem (IRP), the traditional characteristics include a single product decision or multiple products; one or multiple depots; different route structures as direct and multiple; one or multiple vehicles; homogeneous or heterogeneous fleet; the consideration of time windows; between others (Ávila Torres et al. 2020).

In this study, the considered IRP characteristics are: a single product which is liquid oxygen that must be delivered to two sectors as was mentioned before, industry and health; the distribution is made through one depot with multiple and heterogeneous vehicles of two sizes, large and medium capacity, and with the presence of priority customers from the health sector.

As we know, during the pandemic of coronavirus, oxygen was a very important gas for hospitals in order to treat sick people infected with the virus. Due to this, the oxygen experimented variations on the demand and the companies faced the problem of distribution when the oxygen demand increases for hospitals rather than decrease and, they must serve industrial companies too.

Here, we address the problem to create distribution routes looking out the demand as uncertain through triangular fuzzy numbers and give priority in deliveries (must serve first) to health sector customers due to a Mexican law that requires a certain level of quality of the oxygen in order to treat the patients, even this increases the total cost.

The uncertainty is addressed with triangular fuzzy numbers, which are defined as $\tilde{a} = (\underline{a}, a, \overline{a})$ where $\underline{a}$ and $\overline{a}$ represent the lower and upper limits. These levels can be established based on experience when there is no more reference of the values, or in case it applies, based on historical values.
In order to compare fuzzy numbers, we use the second index of Yager $Y_2(A) = \int_0^{hgt(A)} M(A_\alpha) d\alpha$ where $hgt(A) = 1 \; y \; M(A_\alpha) = \frac{a_\alpha^- + a_\alpha^+}{2}$ (Brunelli y Mezei, 2013).

Mathematical model

Assumptions

1. We assume a directed graph.
2. Each customer has a maximum capacity and a minimum inventory level required.

3. There is no production or distribution during period zero.
4. There is a set of vehicles with a given capacity.
5. The demand varies between -20% and +30% of the central value.
6. The variation in demand remains constant throughout the periods.

The model presented here is based on the formulations of Archetti et al. (Archetti et al., 2007) and (Archetti et al., 2014).
The objective function (1) minimizes the inventory cost at depot (h_0), the objective function also minimizes the inventory cost at customers (h_i) and minimizes the distribution cost c_{ij}.

$$\min \sum_{t\in T} h_0 \cdot I_{0t} + \sum_{i\in N'} \sum_{t\in T} h_i \cdot I_{it} + \sum_{k\in K} \sum_{i\in N'} \sum_{j\in N'} \sum_{t\in T} c_{ij} \cdot y_{ij}^{kt} \qquad (1)$$

Constraint (2) is about the inventory level at depot in current period I_{0t} that is equal to the inventory level at depot in previous period I_{0t-1} plus, the product available at depot at current period r_{0t} minus the total product delivered to all customers q_{it}^{k}.

$$I_{0t} = I_{0t-1} + \widetilde{r_{0t}} - \sum_{k\in K} \sum_{i\in N'} q_{it}^{k} \quad \forall t \in T\text{: } t > 0 \;\; (2)$$

Constraint (3), inventory level at customers (I_{it}) is equal to inventory level the customer had at previous period I_{it-1} minus the consumption of product at current period r_{it} plus, the total product delivered to the customer at current period with any route q_{it}^{k}.

$$I_{it} = I_{it-1} - \widetilde{r_{it}} + \sum_{k\in K} q_{it}^{k} \quad \forall i \in N' \; \forall t \in T\text{: } t > 0 \;\; (3)$$

In order to guarantee the correct operation of the tank, the inventory level at customer must be greater or equal to a minimum inventory level I_i^{min} as expressed in constraint (4).

$$I_{it} \geq I_i^{min} \quad \forall i \in N \; \forall t \in T \;\; (4)$$

The constraints (5-7) are related to the order-up-to level policy. This set of constraints indicates that every time a customer is visited then they must fill the tank to its maximum capacity.

$$\sum_{k\in K} q_{it}^{k} \geq U_{i \sum_{k\in K} z_{it}^{k}} - I_{it-1} \quad \forall i \in N' \; \forall t \in T\text{: } t > 0 \;\; (5)$$

$$\sum_{k\in K} q_{it}^{k} \leq U_i - I_{it-1} \quad \forall i \in N' \; \forall t \in T\text{: } t > 0 \;\; (6)$$

$$q_{it}^{k} \leq U_i \cdot z_{it}^{k} \quad \forall k \in K \;\; \forall i \in N' \; \forall t \in T\text{: } t > 0 \;\; (7)$$

Constraint (8) guarantee that the truck capacity (Q) will not be exceeded.

$$\sum_{i\in N'} q_{it}^{k} \leq Q \cdot z_{0t}^{k} \quad \forall k \in K \; T\text{: } t > 0 \;\; (8)$$

Each customer will be visited just by one truck as expressed in constraint (9).

$$\sum_{k \in K} z_{it}^{k} \leq 1 \quad \forall i \in N' \ \forall t \in T\!: \mathrm{t} > 0 \ \ (9)$$

The flow constraint (10) indicates that if the customer was visited then for every arc entering a customer, then must be one arc leaving the same customer.

$$\sum_{j \in N: j \neq i} y_{ij}^{kt} + \sum_{j \in N: j \neq i} y_{ji}^{kt} = 2 \cdot z_{it}^{k} \quad \forall i \in N,\ \forall k \in K,\ \forall t \in T \ \ (10)$$

The subtour elimination constraints are represented in (11-13) with Miller-Tucker Zemlin constraints

$$u_{0ik}^{t} + \sum_{j \in V\ j \neq i} u_{jik}^{t} - \sum_{j \in N' j \neq i} u_{ijk}^{t} = q_{ik}^{t}, \quad \forall i \in N', k \in K, t \in T \ \ (11)$$

$$u_{ijk}^{t} \ \geq \ q_{jk}^{t} - \ Q_k\left(1 - y_{ij}^{kt}\right), \quad \forall i \in N, \forall\, j \in N',\ \forall k \in K,\ \forall t \in T \ (12)$$

$$u_{ijk}^{t} - \ Q_k\left(y_{ij}^{kt}\right), \quad \forall i \in N, \forall\, j \in N',\ \forall k \in K,\ \forall t \in T \ (13)$$

Constraint (14) allocates at the beginning of a route to those customers with priority deliveries (P_j).

$$\sum_{j \in N: i \neq j} y_{ji}^{kt} \cdot P_j \geq P_j \cdot z_{it}^{k} \quad \forall i \in N',\ \forall k \in K,\ \forall t \in T\!: \mathrm{t} > 0 \ \ (14)$$

The non-negativity and integrality of variables are represented with constraints (15-17)

$$\mathrm{z}_{it}^{k} \ \in \{0{,}1\} \quad \forall i \ \in N, \quad \forall k \ \in K, \quad \forall t \ \in T \quad (15)$$

$$q_{it}^{k} \geq 0 \quad \forall i \in N',\ \forall k \in K,\ \forall t \in T \ \ (16)$$

$$\mathrm{y}_{0j}^{kt} \ \in \{0{,}1{,}2\} \quad \forall j \ \in N', \quad \forall k \ \in K, \quad \forall t \ \in T \quad (17)$$

Results and Discussion *(optional)*

To show the application and the performance of 2nd Yager index to address the fuzziness in the demand, we selected a set of instances of 5,10,15 and 20 customers from a benchmark of (Archetti et al., 2007). Each set is composed of five instances that we adapted in order to include percentages of health sector customers which should be prioritized, the levels established for the quantitative experiment were 0%,25%,50% and 75% of the total customers.

The demand of the benchmark instances was considered as the central value of the demand, then the lower and upper values of the triangular fuzzy number were stated as -20% and +30% of the central value, respectively to simulate the variation of the real problem. We consider two types of distribution vehicles and a planning time of three periods. For the instances of 20 customers, we established a time limit of 3600 seconds.

Customers	Priority (%)	Min of Gap	Max of Gap	Min of Ex. Time	Max of Ex. Time
5	0	0.00	0.00	0.49	1.00
	25	0.00	0.00	0.43	1.30
	50	0.00	0.00	0.41	1.30
	75	0.00	0.00	0.38	0.79

10	0	0.00	0.00	3.63	308.45
	25	0.00	0.00	2.10	97.62
	50	0.00	0.00	3.03	34.40
	75	0.00	0.00	2.91	103.07
15	0	0.00	0.09	824.95	3615.27
	25	0.00	0.04	272.63	3605.47
	50	0.00	0.03	114.90	3604.62
	75	0.00	0.00	87.66	3555.27
20	0	0.08	0.22	3610.40	3619.66
	25	0.00	0.22	506.40	3625.56
	50	0.00	0.14	1114.34	3610.39
	75	0.00	0.11	1311.69	3607.18

Table 1: Gap and execution time by set of instances

In Table 1, we show the minimum and maximum of the gap and execution time by set of instances for the different levels of priority. For the instances with 5 customers, we see the biggest execution time was obtained for 25% and 50% of priority. Then, in instances with 10 customers, the biggest execution time was obtained for 0% of priority and the minimum of the maximum execution time was obtained for 50% of priority. For instances of 15, we can observe that in some cases we obtained an optimal solution in less than 850 seconds. In relation to the gap, we observe that the instances of 5 and 10 customers were solved easily to optimality. For instances with 15 customers, we got in most of the cases a gap less than 0.05 unless the instances with 0% of priority, in that way, we can say that the most difficult scenario to solve is when the instance does not have priority. As we limit the execution time for the instances with 20 customers, the gaps obtained for these instances are greater than 10%, and the biggest gaps were obtained for the cases with 0% and 25% of priority. We would like to point out that the gaps obtained for the instances with 20 customers are much bigger than the gaps obtained for the instances with 15 customers, this shows the complexity of the problem.

To show the effect of priority in the design of the routes, Figure 1 presents the solution for a case of 15 customers, three periods and no priority customers, and Figure 2 presents the same case with 25% priority customers. The priority customers were those denoted with the numbers 1, 3, 6.
In both figures the period of the routes, the vehicle (V#), the customers (white nodes) and the depot or starting point of the routes (gray node with D denoting Depot) are indicated. The total delivery (quantity in gray) is also indicated. Vehicles V1 and V2 have the same capacity, larger than that of vehicles V3 and V4. These two vehicles also have the same capacity as each other. Therefore, vehicles 3 and 4 can be used for a route with lower capacity.

First, Figure 1 shows that all customers are visited, and only customer 8 is visited twice, in period 1 and period 2. Then, in Figure 2, customer 8 is visited only once in period 2 with a total delivery of 158, which adds up to that required if delivered separately in both periods.
The solution with 25% of priority customers also includes the visit of all customers.

In Figure 1, the route for period 1 allows to deliver a total of 350 units of product, while the routes for period 2 deliver in the four vehicles: 370, 368, 170 and 123 units; being a total of 1031 units for period 2. In comparison, in Figure 2, in period 1, the only route allows to deliver 356 product units, and in period 2 routes deliver 370, 368, 129 and 158 product units, with a total of 1025 for period 2.
Certainly, the solution for the case with 25% priority customers remains similar to that obtained for 0% priority customers. This is, the routes for period 2 with vehicle 1 and vehicle 2 are the same in both solutions only inverted; and the route for period 1 (Figure 2) includes visiting the 80% of customers included on the route for the solution in Figure 2.
Summarizing, the solution provided will be adjusted to consider the health sector customers, but in some cases, when possible, to deal with this consideration, the solution will involve the inverted route. The last, is natural when the problem is symmetric to avoid additional costs.

It is important to note that these solutions are obtained considering the fuzziness in the rate of consumption of demand by period.

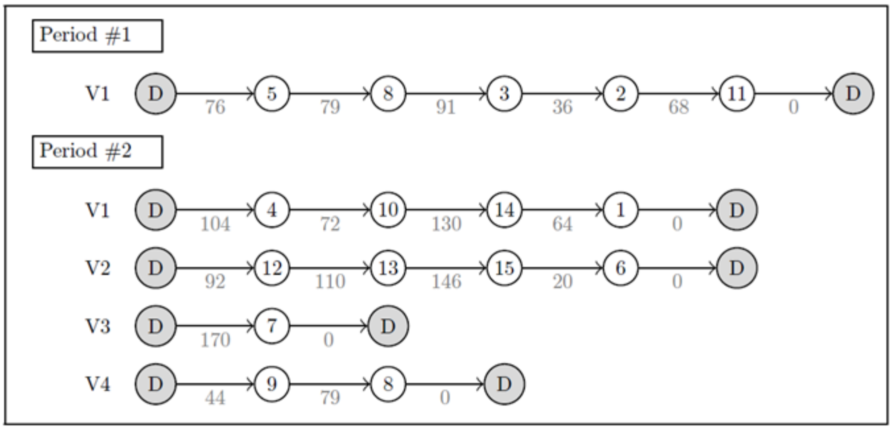


Figure 1. Inventory routing problem solution with 0% of priority customers (instance w/15 customers and 3 periods).

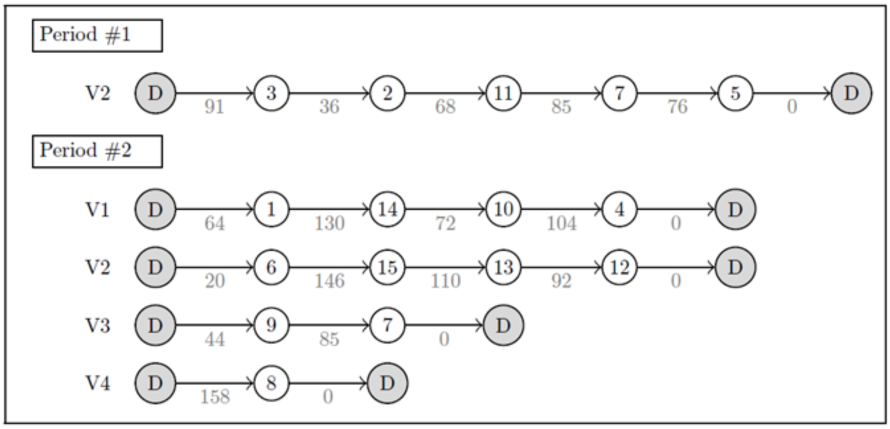


Figure 2. Inventory routing problem solution with 25% of priority customers (instance w/15 customers and 3 periods).

In terms of cost, the solution with 25% of priority customers increase a 2.07% respect the total cost of solution with 0% priority customers. But in cases when the health sector customers are first served in the routes without priorities (when priorities are ignored), the cost of the priority case is at most the one obtained in the original case.

Conclusions *(optional)*

The implemented model considers uncertainty in demand and gives priority to a set of customers. Triangular fuzzy numbers were used in order to represent the fuzziness of demand and the second index of Yager was the selected method to compare those numbers. One important characteristic to implement this method is to define the fluctuation in fuzzy parameters that depends on the level of knowledge of the decision maker or based on historical values.

The effect in cost derived from the consideration of priority customers will depend on which customers are prioritized, as it can result in the same route when no changes are needed or in a completely different solution.

To point out, the priority of customers in deliveries is a feature not commonly studied, but still being a feature in many real problems. While incorporating time windows is an option for problems where customers have time restrictions or specifications.

As future work, it will still be necessary to study and quantify the computational scalability of the solutions times for various levels of priority, variation and other parameters belonging to IRP.

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