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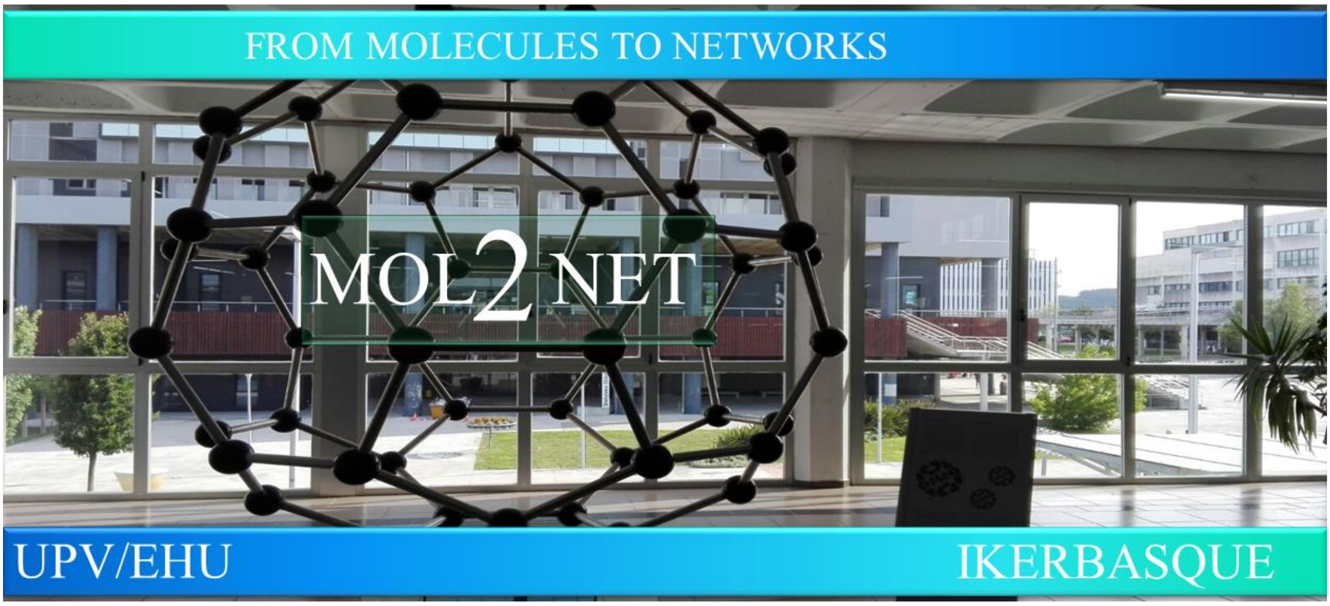

Theoretical location of optima via analysis techniques

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Graphical Abstract	**Abstract.**
	In Hausdorff locally convex topological vector spaces, the Krein-Milman theorem allow us to describe compact convex sets as the closed convex hull of their extreme points. This coupled with measure-theoretic inequalities from the theory of probability, gives us a theoretical proof of where the optima of certain programs can be located. We aim to extend this results to abstract convexities in more general settings.

Introduction

In linear programming, a very useful and well known fact is [2], when minimizing over a polyhedron P, if P has at least one extreme point and if there exists an optimal solution, then there exists an optimal solution which is an extreme point of P. Bauer' s maximum principle [1] extends this result: it states that if K is a nonempty compact convex set in some Hausdorff locally convex topological vector space, a real-valued upper semi continuous function defined on K attains its maximum over K at some extreme point of K. It is thus of interest to see in what other settings can the optimal solutions of an optimization problem can be narrowed. In the following, we explore ideas related to this, which is work in progress from the author.

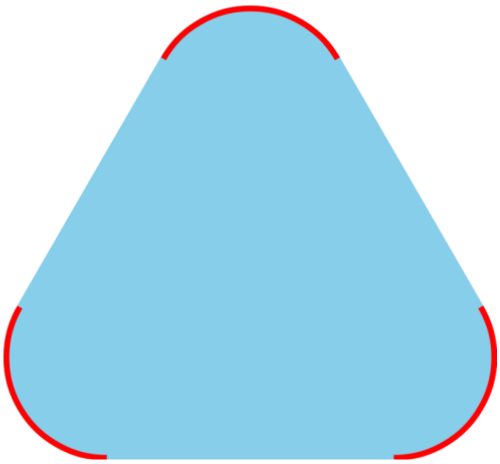

Figure 1. Extreme points (red) of convex set (blue).

Materials and Methods

For simplicity, suppose one aims to maximize a continuous function f over a compact, convex set K. If on a finite dimensional space, a result by Minkowski [5] says that K equals the convex hull of its exterior points, i.e. K = conv[ext(K)]. This allows for any k in K to be written as the expected value of a random variable X which takes values in ext(K). By Jensen' s inequality [3], f(EX) ≤ E[f(X)], and in turn, E[f(X)] is less or equal than the supremum of f over ext(K). Thus by the aforementioned Minkowski result, for any k in K, f(k) is bounded by the supremum of f over ext(K). While Bauer' s principle would then follow if ext(K) is closed, as is the case for the feasible set of linear programs, this proof sketch points towards ways that concentration inequalities from probability theory (such as Jensen´s) may aid in the theoretical location of optima.

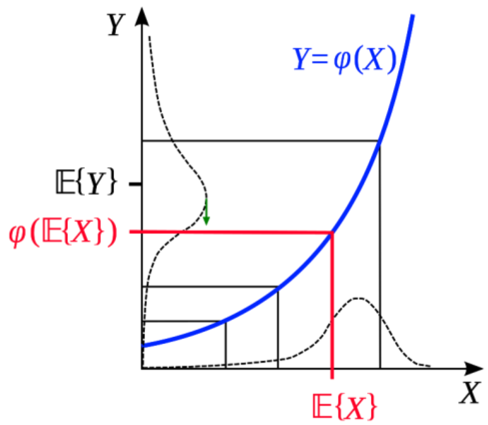

Figure 2. Graphical representation of Jensen's inequality.

Results and Discussion

One can find abstract description of the combinatorial properties that characterize convexity[4]. This can be used to define convex sets in arbitrary spaces, particularly in discrete settings such as graphs, simplicial complexes, and hypergraphs. Given how prevalent convexity ideas are in optimization theory, and the importance of optimizing over discrete sets, a research direction the author is pursuing is: can the results above be generalized to these discrete (abstract) convexities? Possible approaches involve the "translation" of the proof sketch in the Methods section to a discrete setting. In doing this, one would need to define what a convex function is in terms of abstract convex sets. Tentatively, quasi-convexity may provide enough material to work with. I.e., one can see if $\{x : f(x) < t\}$ is convex for any real-value t.

References

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