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Supersymmetryc AdS Solitons, Ground States and Phase Transitions in Maximal Gauged Supergravity †

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Abstract: We review some recent soliton solutions in a class of four-dimensional supergravity theories. The latter can be obtained from black hole solutions by means of a double Wick rotation. For special values of the parameters, the new configurations can be embedded in the gauged maximal N = 8 theory and uplifted in the higher-dimensional D = 11 theory. We also consider BPS soliton solutions, preserving a certain fraction of supersymmetry.

Keywords: supergravity; supersymmetry; solitons; phase transitions

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1. Introduction

Solitons play a special role in classical physics as well as in quantum and string theory, determining a richer structure of the full non-perturbative regime. This different class of exact solutions can be obtained from a double Wick rotation of a former black hole configuration [1,2], the new solutions characterizing a regular spacetime configuration devoid of horizons.

In non-supersymmetric AdS gravity, solitons play a fundamental role as they can be treated as ground states for suitable field theories [2]. The negative mass of the AdS soliton has a natural interpretation as the Casimir energy of gauge theory living on the conformal boundary. In a non-susy version of the AdS/CFT conjecture, this would indicate that the soliton is the lowest energy solution with the chosen boundary conditions, leading to a different kind of positive energy conjecture [3].

BPS gravitational solitons preserving a certain fraction of supersymmetry can be also found, providing a privileged framework in studying the system evolution: the resulting dynamical equations are in fact typically first-order, as compared to the standard second order equations of motion.

In the following, we are going to review soliton solutions in AdS_4 with two Wilson lines, their phase structure and supersymmetric limits [4]. In particular, we will work in a dilaton truncation of a gauged N = 8 supergravity with vanishing axions, in the presence of non-trivial Fayet-Iliopoulos terms.

2. The Model

Let us consider the dilatonic sector of the STU model of the maximal SO(8)-gauged, N = 8 supergravity. When all the dilatons take the same value and after suitable identification of the vector fields, we obtain the well-known T^3 model with action

$$S=\frac{1}{\kappa}\int d^4x\sqrt{-g}\left(\frac{R}{2}-\frac{1}{2}(\partial\phi)^2+\frac{3}{L^2}\cosh\sqrt{\frac{2}{3}}\phi-\frac{1}{4}e^{3\sqrt{\frac{2}{3}}\phi}(F^1)^2-\frac{1}{4}e^{-\sqrt{\frac{2}{3}}\phi}(F^2)^2\right),$$

This T^3 model can be obtained from the general class of N = 2 theories, labeled by a parameter ν, that are studied in [5], by setting $\nu = -2$.

Hairy Soliton Solutions

Soliton solutions in the above model are obtained by means of a double Wick rotation

$$t\rightarrow i\varphi,\varphi\rightarrow it,Q_\Lambda\rightarrow iQ_\Lambda$$

of the planar black hole of [5] for $\nu = -2$, and explicitly read [4]

$$e^0=\sqrt{\Upsilon}\,dt,e^1=\sqrt{\Upsilon/f}\,\eta\,dx,e^2=\sqrt{\Upsilon f}\,d\varphi,e^2=\sqrt{\Upsilon f}\,dz,$$

$$\phi=\sqrt{\frac{3}{2}}\log x,A^1=Q_1(x^{-2}-x_0^{-2})d\varphi,A^2=Q_2(x^2-x_0^2)d\varphi,$$

$$\Upsilon(x)=\frac{4L^2x}{(x^2-x_0^2)^2\eta^2},f(x)=1+\frac{\eta^2(x^2-1)^3(3Q_1^2-x^2Q_2^2)}{6L^2x^2}.$$

The conformal boundary is located at $x = 1$, so that the above metric represents two different spacetimes, one for $x\in(0,1)$, and the other for x in the range $x\in(1,\infty)$, the solutions being physically distinguished by the sign of the dilaton field.

Soliton solutions are characterized by a φ-circle contracting in the interior of the geometry, at some position x_0 where $f(x_0)=0$. Regularity of the metric at x_0 also requires $\varphi\in[0,\Delta]$, with

$$\Delta^{-1}=\left|\frac{1}{4\pi\eta}\frac{df}{dx}\right|_{x=x_0}$$

while the solutions have magnetic fluxes at infinity given by

$$\Phi_M^1=\oint A^1=Q_1\Delta(1-x_0^{-2}),\ \Phi_M^2=\oint A^2=Q_2\Delta(1-x_0^2).$$

3. Results

In the following, we will briefly analyze BPS configurations and the phase structure of our solutions.

3.1. Supersymmetry

From an explicit calculation of the fermionic variations (Killing spinor equations), one can see that the solution preserves part of the supersymmetry when [4,5]

$$Q_1=-\frac{Q_2}{\sqrt{3}},$$

and, in this case, the metric function simplifies in

$$f(x)=1-\frac{(x^2-1)^4}{x^2}\frac{\eta^2Q_2^2}{6L^2}.$$

As one finds 4 chiral spinors in an N = 2 theory, the soliton solution turns out to be 1/2 BPS in N = 2, while it is 1/8 BPS with respect to the N = 8 theory.

3.2. Phase Structure

Fixed Charge Boundary Condition

From the boundary point of view, it is natural to parameterize the solutions in terms of the boundary data we hold fixed. In particular, we can consider fixed charges, holding fixed the ratios $Q_1/\eta, Q_2/\eta$ and the period Δ.

Let us then consider the free energy of the solution in the fixed charge framework, where we also define

$$q_{1,2} = \frac{\Delta^2}{4\pi^2 L}\frac{Q_{1,2}}{\eta}.$$

The free energy of the hairy solution in this framework reads [4]

$$\frac{F_\phi}{V} = \pm\frac{2}{3\eta\kappa}\left(3Q_1^2 - Q_2^2\right) \pm \frac{2Q_1^2}{\eta\kappa}\left(1 - x_0^{-2}\right) \mp \frac{2Q_2^2}{\eta\kappa}\left(1 - x_0^2\right),$$

where the minus (plus) is for the solutions with $x < 1$ ($x > 1$) and where $V = \beta\Delta\Delta_z$, β being the inverse temperature and $\Delta_z = \int dz$.

We find supersymmetric configurations when $q_2 = -\sqrt{3}q_2$. On the other hand, the Einstein-Maxwell solutions of [6], with $\phi = 0$ and a single vector with charge Q, also satisfy the same boundary conditions, their parameters being related to ours by

$$q_1 = \frac{\Delta^2}{4\pi^2 L}\frac{Q}{\sqrt{8}\, L^2}.$$

In the pure Einstein-Maxwell solution, the free energy reads [6]

$$\frac{F_{EM}}{\Delta\Delta_z} = \frac{2\pi^3 L^2}{\Delta^3\kappa}\xi^2(5 - 4\xi),$$

with $q_1^2 = 2^{-7}\xi^3(4 - 3\xi)$. Using now the expression of free energy of the AdS soliton $G_0 = -\frac{32}{27}\frac{\pi^3 L^2}{\Delta^3\kappa}\Delta\Delta_z$ [2] as a convenient normalization, we find for the supersymmetric hairy solution [4]

$$\frac{F_\phi}{|G_0|} = \frac{27}{\sqrt{2}}|q_1|.$$

We show in Figure 1 the ratio $\frac{F}{|G_0|}$ as a function of q_1 for $q_1 = -\sqrt{3}q_1$. We can see that there exists a branch of non-susy Einstein-Maxwell solutions with lower free energy than the supersymmetric hairy solution. This seems surprising, as we would expect the susy solutions to saturate a BPS bound, forbidding the existence of solutions with lower energy.

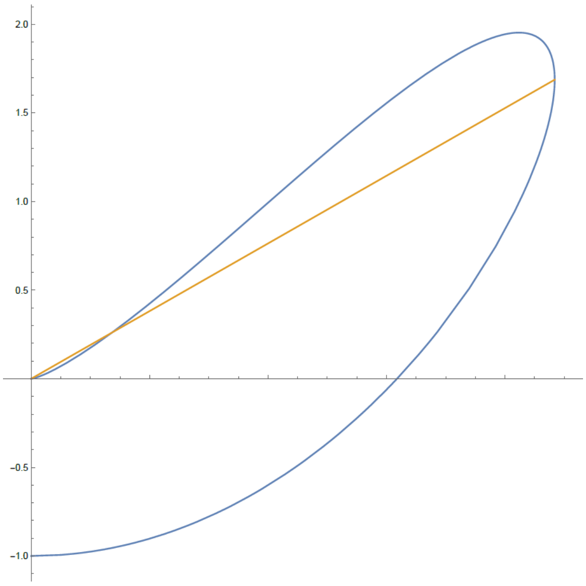

Figure 1. The rescaled free energy $F/|G_0|$ as a function of q1 on the supersymmetric shell $q_1 = -\sqrt{3}q_1$. The yellow line represents the hairy supersymmetric solitons, while the non-supersymmetric pure Einstein-Maxwell solitons are shown in blue [4].

The above result, however, is not in contradiction with the positive energy theorem [7,8], which implies that the energy of a supersymmetry-preserving solution is lower than the energy of any other solution satisfying the same boundary conditions. In this regard, a necessary condition for the positive energy theorem to apply is the existence, for the non-susy solution, of an asymptotic Killing spinor which coincides, up to $O(1/r^2)$ terms at radial infinity, with the Killing spinor of the susy one. Since the latter has antiperiodic boundary conditions at infinity, in order for the positive energy theorem to apply, the non-susy solutions should admit an asymptotic Killing spinor with the same properties at the boundary: this is the case if the charges at infinity have specific values, for which the energy of the non-supersymmetric solution exceeds that of the supersymmetric one. Then, there is no contradiction with the positive energy theorem if we include among the boundary conditions those applying to the asymptotic Killing spinors [4].

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