Physical Picture of Electron Spin

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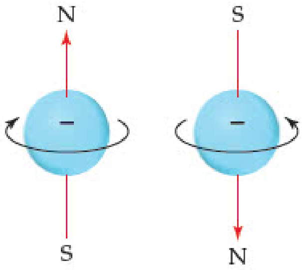
Problem with electron spinning

- Magnitude of spin $\sim \frac{\hbar}{2}$
- Assume electron radius of $r_e \sim 10^{-15}$ m
- The electron angular momentum $\sim m_e v r_e$
- In order for $m_e v r_e \sim \frac{\hbar}{2}$ need $v \sim 100c$!
- Pauli pontificates that spin has no classical counterpart.



According to Pauli this picture is not correct

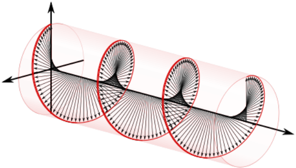
And spin is a rotation

- In a “famous” 1986 American Journal of Physics article * Ohanian showed electrons do rotate
- Takes Pauli, text book writers, and physics pedagogy to task.
- Accepted by experts but not textbook writers, YouTubers
- Spin is rotation of **field** mass-energy
- This is exactly the same as classical mechanics.

H. Ohanian, “ What is Spin?”, American Journal of Physics, Vol. **54**, *500 (1986).*

Photon spin example

- Momentum density $T^{0i} = P^i = \left(\vec{E} \times \vec{B}\right)^i = \left(\vec{E} \times (\nabla \times \vec{A})\right)^i$.
- Now $\left(\vec{E} \times (\nabla \times \vec{A})\right) = E^i \nabla A^i - (\vec{E} \cdot \nabla)\vec{A}$
- The total angular momentum is $\vec{J} = \int \vec{x} \times (E^i \nabla A^i) d^3x - \int \vec{x} \times (\vec{E} \cdot \nabla)\vec{A} d^3x = \vec{L} + \vec{S}$.
- For circular polarized E&M wave last term gives $\pm\hbar$

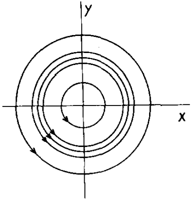


Dirac equation – momentum density

- Dirac momentum density $\vec{G} = \frac{\hbar}{4i}\left(\Psi^\dagger \nabla \Psi - \Psi^\dagger \vec{\alpha} \partial_t \Psi\right) + h.c.$

- Using Dirac equation and commutation of $\vec{\alpha}$'s we get $\vec{G} = \frac{\hbar}{2i}\left(\Psi^\dagger \nabla \Psi - (\nabla \Psi^\dagger)\Psi\right) + \frac{\hbar}{4} \nabla \times \left(\Psi^\dagger \vec{\sigma} \Psi\right) = \vec{L} + \vec{S}.$

- Take Gaussian wave packet $\Psi = (\pi d^2)^{-3/4} e^{-r^2/2d^2}(1,0,0,0)$
  Gives $\vec{G} = \frac{\hbar e^{-r^2/d^2}}{2d^2}\left(\frac{1}{\pi d^2}\right)^{3/2}(-y\hat{\mathbf{x}} + x\hat{\mathbf{y}}) \propto \hat{\varphi}.$



Electron Spin

- Now in general the total angular momentum is $\vec{J} = \int \vec{x} \times \vec{G} d^3x$

  $= \frac{\hbar}{2i} \int \vec{x} \times [\Psi^\dagger \nabla \Psi - (\nabla \Psi^\dagger)\Psi] d^3x + \frac{\hbar}{4} \int \vec{x} \times [\nabla \times (\Psi^\dagger \vec{\sigma})\Psi] d^3x$

- For the Gaussian wavepacket the first term is zero

- Second term (via double cross product and integration by parts)

  $\frac{\hbar}{2} \int \Psi^\dagger \vec{\sigma} \Psi d^3x = \vec{S} \rightarrow \vec{S}_{operator} = \frac{\hbar}{2} \vec{\sigma}.$

- Thus spin **is** rotation of the field mass-energy of Ψ same as for the photon field A_μ same as for a classical object.

Conclusion

- Electron spin is just like every other angular momentum – a rotation of energy-mass.
- Spin is *intrinsic/inherent* in the wave field, but it is not *internal*
- To paraphrase Galileo " And yet it rotates".