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Imprint of the Crystallization of Binary White Dwarfs on Gravitational-Wave Observations with LISA †

Loïc Perot and Nicolas Chamel *

Institute of Astronomy and Astrophysics, Université Libre de Bruxelles, CP 226, Boulevard du Triomphe, B-1050 Brussels, Belgium
* Correspondence: nicolas.chamel@ulb.be
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Abstract: Space-based gravitational-wave detectors such as the Laser Interferometer Space Antenna will allow to probe the interior of white dwarfs in binaries through the imprints of tidal effects on the gravitational-wave signal. In this study, we have computed the tidal deformability of white dwarfs in full general relativity taking into account the crystallization of their core. The elasticity of the core is found to systematically reduce the tidal deformability, especially for low-mass stars. Moreover, it is shown that the errors on the tidal deformability due to the use of the Newtonian theory can become important for massive white dwarfs. Finally, the orbital evolution of eccentric binaries is investigated. Measuring the precession rate of these systems could provide estimations of the individual masses. However, it is found that the neglect of crystallization could lead to very large errors.

Keywords: white dwarf; crystallization; tidal deformability; gravitational wave

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1. Introduction

The recent detection of gravitational waves (GWs) from compact binary systems by the LIGO-Virgo collaboration has opened a brand new era in the field of astronomy [1]. The upcoming space-based detector "Laser Interferometer Space Antenna" (LISA) [2], which is expected to be operative within the next decade, will allow to detect GW signals in the range 0.1 mHz to 1 Hz mainly supposing to come from white dwarf (WD) binaries. According to population synthesis models, more than 10,000 WD binaries are expected to be detected over the four-year mission of LISA [3–6]. Among these systems, eccentric binaries are targets of choice to study the internal constitution of WDs. Indeed, the precession of their periastron caused by tidal forces leads to a frequency splitting of the gravitational signal, which is proportional to the precession rate [7,8]. In turn, this precession rate, potentially measurable by LISA, depends on the tidal deformability parameter and therefore on the internal structure of the WDs. Information on the tidal effects could also be obtained for WD binaries with circular orbits from long-term monitoring of the GW signal [9–12].

Some of the WD binaries that will be observed by LISA will be aged of a few billions of years and could then be at least partially crystallized. Crystallization of the WD cores was predicted long ago [13–15] and found observational support from asteroseismological study of BPM 37093 [16]. Very recently, new strong observational evidences have been brought by the analysis of the GAIA data [17]. The crystallization of the WD cores may have an impact on their tidal deformability and should then be taken into account when analyzing the GW signal from old WD binaries. In spite of that, previous studies on the tidal deformations of WDs considered only purely perfect-fluid stars in Newtonian theory [18,19].

In this paper, we compute the tidal deformability of a WD taking into account the elasticity of the crystallized core in full general relativity. After briefly presenting the equation of state and the way to calculate tidal deformations in general relativity in Section 2, we present our numerical results in Section 3.

2. Structure and Deformability of White Dwarfs

2.1. Equation of State of White Dwarfs

The core of a WD with densities reaching ~10^6 g/cm^3 is made of fully ionized atomic nuclei in a charge compensating gas of relativistic electrons. Nuclei typically consist of light elements like carbon and oxygen, but also helium [20–22], neon or magnesium [23]. In this work, we assume for simplicity that the core is pure. Moreover, we consider that the WD is sufficiently aged that the core has cooled down and crystallized. The WD is then so cold that the thermal effects in the equation of state are neglected. The elastic properties of the crystallized core are described by a single elastic constant, the shear modulus, assuming that each element of matter at given density consists of an isotropic polycrystalline solid [24–26]. Finally, the surrounding layers of hydrogen or helium composing the atmosphere are ignored as they can only represent a maximum of ~1% of the total stellar mass to avoid a thermonuclear runaway.

2.2. Tidal Deformability Including Elasticity

WDs in a close orbit are tidally deformed by the mutual gravitational interactions. In the adiabatic approximation, the static external quadrupolar tidal field $\mathcal{E}_{ij}$ and the induced quadrupolar mass moment in the star Q_{ij} are related, to first order, by [27]

$$Q_{ij} = \frac{2}{3G} k_2 R^5 \mathcal{E}_{ij}, \tag{1}$$

where G is the universal gravitational constant, R is the stellar radius and k_2 is the Love number (also called apsidal motion constant) characterizing the tidal deformability of the star. The tidal parameter which is observable through GW detections is thus proportional to the combination $k_2 R^5$ (for a given mass).

The background structure of the star is described by the famous Tolman-Oppenheimer-Volkoff (TOV) equations [28,29], which relate the pressure, the mass-energy density and the gravitational mass inside the star, the radial coordinate being the independent variable. Integrating these equations from the center up to the surface and using the equation of state gives the mass M and the radius R of the star.

To describe static tidal perturbations, one has to solve the perturbed Einstein field equations. For this purpose, one has to define a perturbed space-time metric

$$g_{\mu\nu} = g^0_{\mu\nu} + \delta g_{\mu\nu}, \tag{2}$$

with $g^0_{\mu\nu}$ the metric of a spherically symmetric space-time and $\delta g_{\mu\nu}$ the linear metric perturbation which is chosen to be reduced in the Regge-Wheeler gauge [30]. To take into account the existence of elasticity in the core, one has to add the shear constraint contribution $\delta\Pi_{\mu\nu}$ to the perfect-fluid one $\delta T_{\mu\nu}$ in the right-hand side of the perturbed Einstein equations [31,32]:

$$\delta G_{\mu\nu} = \frac{8\pi G}{c^4}\left(\delta T_{\mu\nu} + \delta\Pi_{\mu\nu}\right), \tag{3}$$

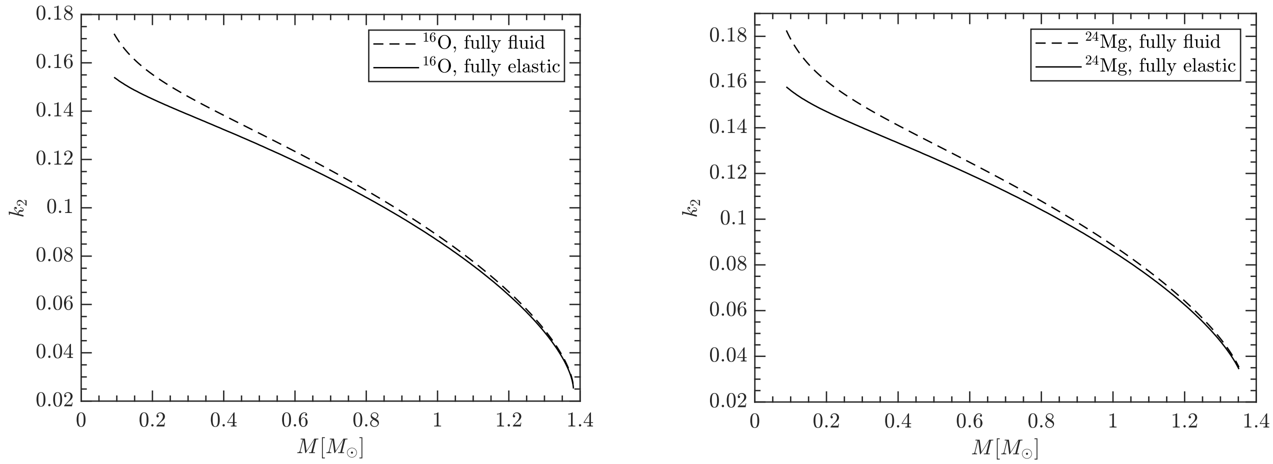
where c is the speed of light. Using the definition (2) in Equation (3) and using appropriate combinations of the different components leads to a system of six first-order ordinary differential equations. This system is to be solved from the stellar center using initial conditions from Taylor expansions of the different functions up to the surface where appropriate boundary conditions are imposed (see Ref. [33] for details). The matching of the interior solution with the exterior solution (in vacuum) at the stellar surface allows to obtain the tidal Love number k_2 [27]. Note that elasticity has no effect on the stellar radius and on the mass since it comes into play at the perturbed level only.

3. Results and Discussions

In this section, we present the numerical results obtained by solving the perturbation equations characterizing the tidal deformations together with the TOV equations.

3.1. Effect of the Crystallization on the Deformability

In Figure 1, we show k_2 as a function of the stellar gravitational mass M for a WD made of oxygen ^{16}O or magnesium ^{24}Mg, comparing entirely perfect-fluid and entirely solid stars. We notice that the presence of a crystallized core systematically reduces the tidal deformability, especially for low-mass stars. Moreover, the presence of heavy elements in the crystallized core reduces even more the tidal deformability, as we can see when comparing the two plots in Figure 1: these reductions amount to 3.3% and 4.2% for 0.6 $M_\odot$ oxygen or magnesium WDs, respectively. The deviations on the observable combination k_2R^5 are the same as on k_2 since the radius is unaffected by the presence of elasticity.



(**a**) (**b**)

Figure 1. Love number k_2 as a function of the gravitational mass M for a fully fluid or a fully elastic WD made of oxygen (**a**) or magnesium (**b**).

3.2. Comparison with Newtonian Results

In Figure 2, we show the errors on the stellar radius and the Love number incurred by the use of the Newtonian theory instead of general relativity for purely fluid stars, as it was commonly done in previous studies about WDs. We notice that the Newtonian theory systematically increases both the radius and the Love number. These deviations remain negligible for low mass stars up to ~1 $M_\odot$. However, for massive WDs, the deviations become increasingly important and reach ~10% both for R and k_2 at the Chandrasekhar limit. A direct consequence is that the observable combination k_2R^5 is even more affected: the errors on this parameter reach up to 100% close to the maximum mass. This enlightens the importance of considering general relativity at least for massive WDs.

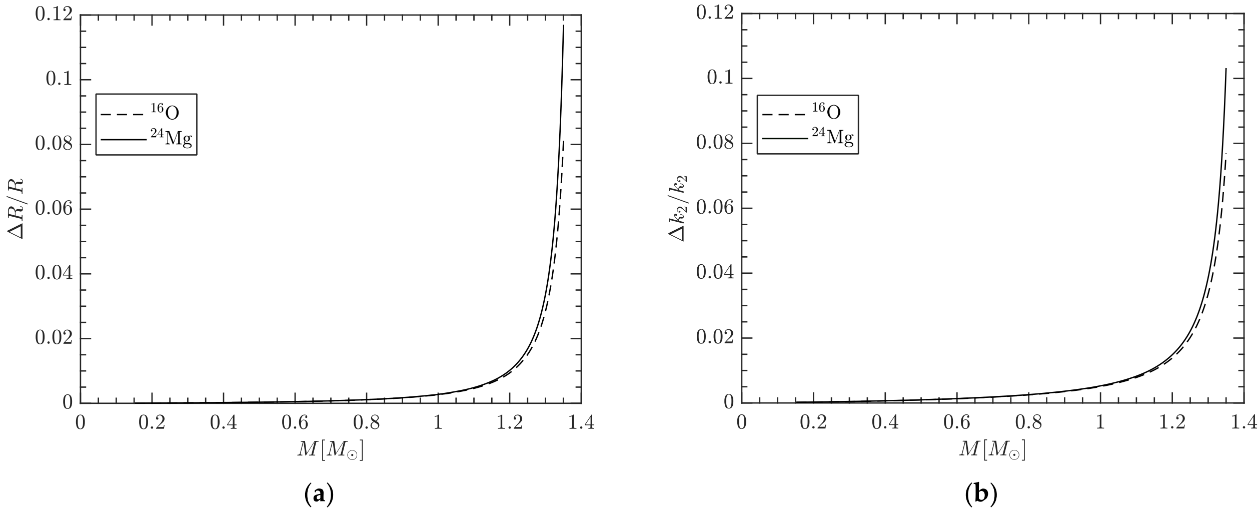


Figure 2. Relative errors incurred by the use of Newtonian theory instead of general relativity on the stellar radius R (**a**) and the Love number k_2 (**b**) as a function of the gravitational mass M, for oxygen or magnesium WDs.

3.3. Eccentric Binaries

In this last part, we focus on eccentric binary WDs. The precession of the periastron leads to a frequency splitting of the GW signal proportional to the precession rate $\dot{\gamma}$ (the dotted notation denotes a time derivative) which is the sum of three contributions [7,8]:

$$\dot{\gamma} = \dot{\gamma_{\rm GR}} + \sum_{i=1,2} \left(\dot{\gamma_{\rm tid,i}} + \dot{\gamma_{\rm rot,i}}\right), \tag{4}$$

with $\dot{\gamma_{\rm GR}}$ the contribution from general relativity and $\dot{\gamma_{\rm tid,i}}$, $\dot{\gamma_{\rm rot,i}}$ the contributions from the tidal effects and the rotation of the two stars (the index i identifying each star), respectively. While the general relativity contribution depends only on the eccentricity and the total mass of the binary, the two others from tidal effects and rotation also depend on the individual masses and the observable tidal deformability parameters $(k_2R^5)_i$ of each star and then on their internal constitution.

By solving the equations governing the orbital evolution of a binary system and using Equation (4), we have shown that there exists eccentric WDs with crystallized cores and close enough to merger for the tidal and rotational contributions to be measurable by LISA. Detailed results can be found in Ref. [33]. From such measurements, it will be potentially possible to infer the individual masses. However, we have found that neglecting the elasticity of the crystallized core could lead to dramatic errors: for a given value of k_2R^5, the relative error on the mass is about 10% for a 0.6 M$_\odot$ oxygen WD and can even reach ~80% for a very low-mass star of the same composition.

4. Conclusions

We have investigated the role of the crystallization of the core of binary WDs on their tidal deformability in the framework of full general relativity. Such old WDs could be potentially observed by the space-based GW detector LISA.

We have found that the elasticity of the solid core systematically reduces the tidal deformability especially for low-mass stars. Moreover, the presence of heavy elements in the core increases this effect.

Comparing Newtonian results with our general-relativistic ones for fluid stars, we have found that the deviations on the radius and the tidal deformability incurred by the use of the Newtonian theory remain negligible for low-mass stars but become increasingly important close to the maximum mass.

Focusing on eccentric binaries, we have shown that the measure of the precession rate of the periastron of such systems with LISA could potentially provide information on the tidal deformability parameters or the individual masses. We have found that neglecting the elasticity of the crystallized WD core can lead to dramatic errors on the inferred masses.

Finally, in our study we have considered cold WDs that have entirely crystallized. However, during the inspiral phase, the outermost layers of the star could be melt due to tidal heating especially close to merger. This issue could be investigated by running thermal evolution simulations of WD binaries.

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