Imprint of the crystallization of binary white dwarfs on gravitational-wave observations with LISA

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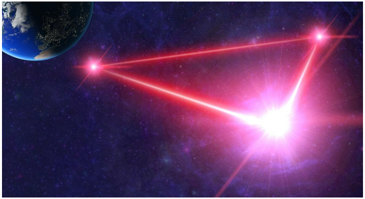
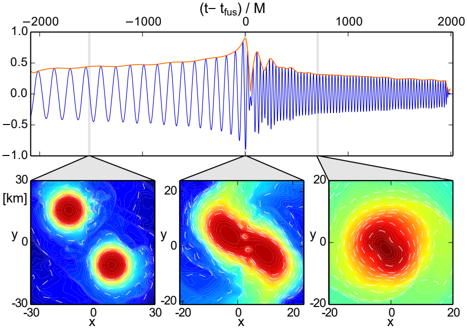
Motivations

- Future detection of up to 10^4 binary WDs with the space-based GW detector LISA
- Measurement of tidal effects will be possible during the inspiral phase, a few millions of years before the merger
- Previous studies considered only perfect-fluid stars in Newtonian theory
- Current strong observational evidences of crystallization of WD cores from GAIA



Inspiralling stars

- The stars are far away:
  - same dynamics as black holes with same masses
  - universal GW signal and calculation by the PN method
- The stars are getting closer:
  - tidal deformations
  - perturbative calculation and PN corrections to the GW signal



Bernuzzi et al., PRL **115**, 091101 (2015)

Tidal deformability of white dwarfs

- Assumptions:

  - static stars (non-rotating)
  - deformations remaining small in comparison to the star radius

- At linear order, the quadrupolar tidal field $\mathcal{E}_{ij}$ induces a quadrupolar moment $\mathcal{Q}_{ij}$:

$$\mathcal{Q}_{ij} = \frac{2}{3G} k_2 R^5 \mathcal{E}_{ij}$$

  with R the stellar radius and k_2 **the tidal Love number (also called apsidal motion constant)**

- Dimensionless measurable tidal parameter (correction of order 5 PN):

$$\Lambda_2 \sim k_2 R^5$$

  $\rightarrow$ the tidal deformability depends on the microphysics

Tidal deformability of white dwarfs

- The tidal deformability can be computed solving the perturbed Einstein equations
- Taking into account elasticity of crystallized matter:

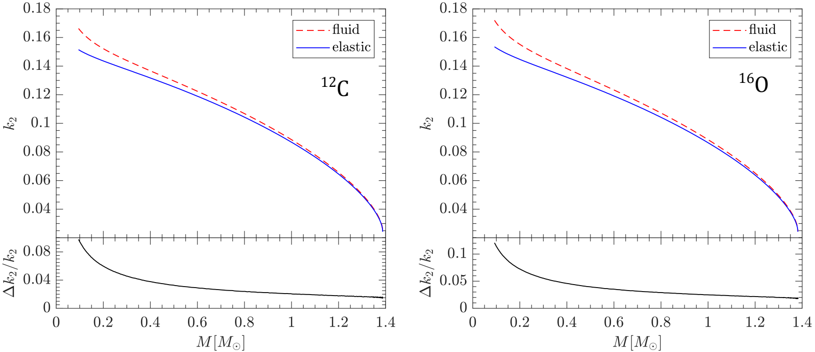
$$\delta G_{\mu\nu} = \frac{8\pi G}{c^4}(\delta T_{\mu\nu} + \delta\Pi_{\mu\nu})$$

with $\delta T_{\mu\nu}$ the perturbed perfect-fluid stress-energy tensor, and $\delta\Pi_{\mu\nu}$ the perturbed strain tensor

- The tensor $\delta\Pi_{\mu\nu}$ contains the shear modulus
- System of 6 ODEs to be solved in the elastic region with the appropriate boundary conditions at the center and at the surface

Impact of crystallization on the tidal deformability

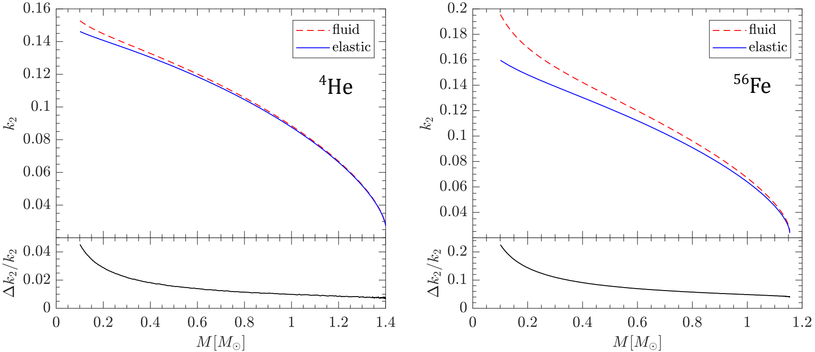
- Core consisting of fully ionized atomic nuclei coexisting with a relativistic gas of free electrons at $T = 0$ K
- Results in full general relativity comparing WDs entirely fluid or entirely elastic
- Non-negligible impact, especially for low mass stars: deviation of about 5 % for a typical $0.4M_\odot$ carbon or oxygen WD



Perot & Chamel, PRD **106**, 023012 (2022)

Impact of crystallization on the tidal deformability

- Core consisting of fully ionized atomic nuclei coexisting with a relativistic gas of free electrons at $T = 0$ K
- Results in full general relativity comparing WDs entirely fluid or entirely elastic
- Increasingly impact in the presence of heavy elements: deviation of 2 % and 12 % for $0.3M_{\odot}$ helium and iron WDs



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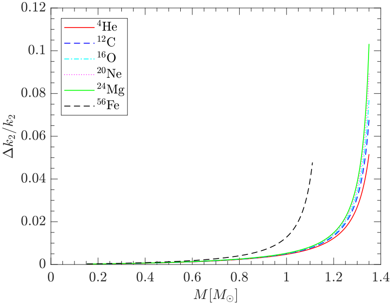
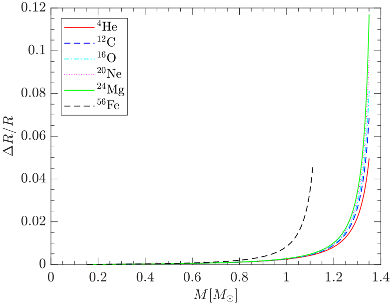
- Comparison between Newtonian and fully relativistic results for k_2 and R
- General relativity systematically reduces k_2 and R
- For low mass WDs, relativistic corrections remain negligible in comparison with corrections due to elasticity, but become larger when approaching the Chandrasekhar limit (up to 10% near the maximum mass)





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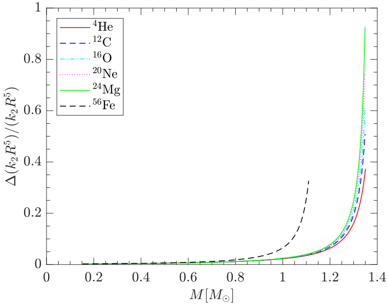
Newtonian vs relativistic results

- Comparison between Newtonian and fully relativistic results for the observable combination $k_2 R^5$
- The errors due to the neglect of relativistic corrections remain negligible pour low mass WDs, but become very large near the maximum mass (up to 100% !)



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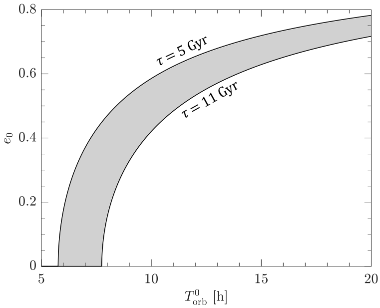
- The precession of the periastron of eccentric binaries leads to the frequency splitting of the GW signal
- The frequency splitting is proportional to the precession rate $\dot{\gamma}$
- The precession rate is the sum of 3 contributions:

$$\dot{\gamma} = \dot{\gamma}_{\mathrm{GR}} + \sum_{\mathrm{i}=1,2} \dot{\gamma}_{\mathrm{rot,i}} + \dot{\gamma}_{\mathrm{tid,i}} \,,$$

  where $\dot{\gamma}_{\mathrm{GR}}$ comes from general relativity, $\dot{\gamma}_{\mathrm{rot,i}}$ from the rotation and $\dot{\gamma}_{\mathrm{tid,i}}$ from the tidal effects in each star
- $\dot{\gamma}_{\mathrm{GR}}$ depends only on the total mass **whereas** $\dot{\gamma}_{\mathrm{rot,i}}$ **and** $\dot{\gamma}_{\mathrm{tid,i}}$ **depend on the individual masses and the parameters** $(k_2 R^5)_{\mathrm{i}}$ **!**

Eccentric binaries

- For the tidal effects to be measurable, $\sum_{i=1,2} \dot{\gamma}_{\rm rot,i} + \dot{\gamma}_{\rm tid,i}$ has to represent at least ~ 10 % of $\dot{\gamma}$ (observational uncertainties) $\rightarrow$ this is achieved a few millions of years before the merger
- By computing the binary parameters evolution, we show it is possible to currently find such binaries in the LISA sensitivity band and aged of at least ~ 5 Gyr (estimated time for the core to crystallize)



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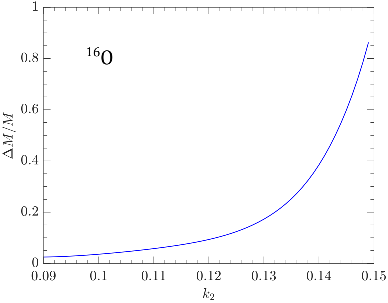
Eccentric binaries

- Measuring $\sum_{\mathrm{i}=1,2} \dot{\gamma}_{\mathrm{rot,i}} + \dot{\gamma}_{\mathrm{tid,i}}$ provides a way to deduce the individual masses of the WDs
- We need a relation between M_i and $(k_2 R^5)_\mathrm{i}$
- Neglecting elasticity of the crystallized core can lead to dramatic errors on the inferred masses, especially for low mass WDs!



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Conclusions

- Crystallization has the biggest impact on k_2 for low mass WDs with heavy elements $\rightarrow$ way to probe existence of iron WDs?
- Relativistic corrections on $k_2 R^5$ are negligible for low mass WDs but become very important when approaching the maximum mass
- Neglecting crystallization when trying to deduce individual masses from eccentric binaries can lead to dramatic errors on the masses, especially for low mass stars

$\rightarrow$ More details and results in:

https://doi.org/10.1103/PhysRevD.106.023012