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Thermodynamic Investigation of the QCD Phase Diagram with 2 + 1 Quark Flavors †

Hayet Sahi [1,2] **and Amal Ait El Djoudi** [2,*]

[1] Université M'hamed Bougara Boumerdès, Algeria; h.sahi@univ-boumerdes.dz
[2] Laboratoire de Physique des Particules et Physique Statistique, Ecole Normale Supérieure-Kouba, B.P.92, Vieux-Kouba, Algiers 16050, Algeria
* Correspondence: amal.aiteldjoudi@g.ens-kouba.dz

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Abstract: This work deals with the deconfinement phase transition from a hadronic gas (HG) phase consisting of massive pions, to a quark-gluon plasma (QGP) phase consisting of gluons, massless up and down quarks and massive strange quarks, in addition to their antiquarks. Based on the Bag and coexistence models, we study the variations of the pressure characterizing the HG and the QGP phases. For this latter, we calculate the partition function of the color-singlet QGP within the projection method, using a density of states containing the volume term only. We investigate the phase diagram of the strongly interacting matter, in the μ-T plane, in several cases: with massless pions in the HG phase then considernig their masses, and with 2 massless quarks then when adding massive strange quarks in the QGP phase.

Keywords: deconfinement phase transition; phase diagram; first order phase transition

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1. Introduction

Quantum chromodynamics (QCD) is the basic theory that describes the strong interactions, between the quarks and the gluons in the hadrons [1]. QCD predicts the asymptotic freedom of short-range quarks (distances <1 fm) inside the hadrons, in addition to the confinement property, which means that in the ordinary conditions, quarks and gluons are confined into hadrons. However, at high temperature and/or density, hadronic matter undergoes a phase transition: quarks and gluons become deconfined, i.e., free [2], in a new phase called the Quark-Gluon Plasma (QGP).The existence of this phase of matter was proposed in the mid-seventies, just ten years after the birth of the Quark Model of hadrons [3,4].

To illustrate the phase transition from the hadronic gas (HG) phase to the QGP phase, we will investigate the phase diagram of the strongly interacting matter, in the μ-T plane, based on the Gibbs criterion setting the mechanical equilibrium between the two phases at the transition, with massless and massive particles in the two phases.

2. Phase Diagram in the μ-T Plane

According to the Gibbs criterion, the HG and the QGP phases are in equilibrium when their pressures P, temperature T and chemical potential μ are equal, which reads:

$$P_{HG}(\mu_c, T_c) = P_{QGP}(\mu_c, T_c) \quad (1)$$

Thus, we have to calculate the pressure of both phases, using the thermodynamic relation:

$$P = T\frac{\partial \ln Z}{\partial V} \quad (2)$$

from the individual partition functions Z_{HG} and Z_{QGP} of the two phases.

For the HG phase, we consider an ideal gas of pions (π^+, π^- and π^o), which are the lightest bosons, and the obtained partition function reads:

$$Z_{HG}(T) = \exp\left(\frac{d_\pi V_{HG}}{2\pi^2 T}\int_0^{+\infty}\frac{k^4 dk}{\sqrt{k^2+m_\pi^2}\left(e^{\beta\sqrt{k^2+m_\pi^2}}-1\right)}\right) \quad (4)$$

where k is the momentum, d_π the pion degeneracy factor, m_π the pion mass and $\beta = \frac{1}{T}$.

The QGP being considered asan ideal gas of gluons, quarks and their antiquarks, and assuming non-interaction between its constituents, we can write Z_{QGP} as:

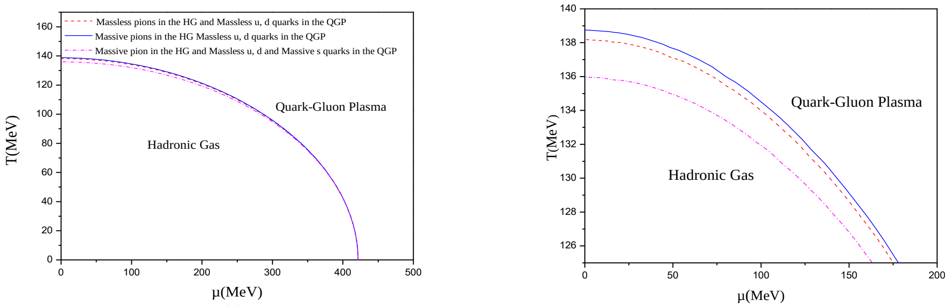
$$Z_{QGP}(\mu, T) = Z_{quark} Z_{gluon} Z_{vac} \quad (5)$$

where: $Z_{vac} = \exp\left(\frac{-BV_{QGP}}{T}\right)$, accounts for the real vacuum pressure B exerted on the perturbative vacuum, *B* being the bag constant. The calculation ofZ_{quark} and Z_{gluon} gives:

$$Z_{q\bar{q}}(\mu, T) = \exp\left(\frac{d_Q V_{QGP}}{6\pi^2 T}\int_0^{+\infty}\frac{k^4 dk}{\sqrt{k^2+m_q^2}\left(1+e^{\beta(\sqrt{k^2+m_q^2}\pm\mu)}\right)}\right) \quad (6)$$

$$Z_{gluon} = \exp\left(\frac{d_G V_{QGP}\pi^2}{90}T^3\right) \quad (7)$$

Setting the condition$\boldsymbol{P_{HG} = P_{QGP}}$,with $B^{1/4} = 200$ MeV, weobtain the phase diagram in the $\boldsymbol{\mu - T}$ plane shown in Figure 1, characterized by a critical line separating the HG and the QGP phases and giving at each point the transition parameters $\boldsymbol{\mu_c, T_c}$,in all the studied cases of massless and massive particles. When $\boldsymbol{T < T_c}$, the HG is favored, and when $\boldsymbol{T > T_c}$, the QGP is favored. There are two extreme cases: The first at $\mu = 0$, obtainable in URHIC, and the second at T = 0, which could be reached in the core of certain neutron stars. It can also be noted by comparing the red dashed curve (with massless pions in the HG phase) and the solid blue curve (with massive pions), in Figure 1b, that there is a slight difference in T_c at $\mu = 0$, meaning that the pion mass has to be considered for accurate investigation of the transition temperature, while the small mass of u and d quarks can simply be neglected. Also, by examining the dashed-dotted magenta line, obtained by considering massive s quarks additionally to massless u and d quarks in the QGP phase, a clear mismatch appears compared to the two other curves, showing that the number of flavors considerably affects the transition parameters, and particularly the T_c value at $\mu = 0$.



(a) (b)

Figure 1. (**a**) Phase diagram in the $\mu - T$ plane, with $B^{1/4} = 200$ MeV, for both cases of massless pions (red dashed line) and massive pions (blue solid line) in the HG phase with massless u and d quarks in the QGP phase, with the third case of massive pions in the HG and massive strange quarks additionally to massless u and d quarks in the QGP (magenta dashed-dotted line). (**b**) Magnification of the phase diagram in the region of high temperatures and low and intermediate chemical potentials, for the same cases as in (**a**).

3. Phase Transition to a Color-Singlet QGP

To implement the color-singletness constraint into the quantum statistical description of the QGP, we use the group theoretical projection formulated by Turko and Redlich [5]. The projected partition function of a QGP on the color-singlet SU(3) representation, in a volume V, at temperature T and chemical potential μ reads:

$$Z_{QGP}(\mu, T, V_{QGP}) = \frac{4}{9\pi^2} e^{-\frac{BV_{QGP}}{T}} \int_{-\pi}^{+\pi} \int_{-\pi}^{+\pi} d\varphi d\psi M(\varphi, \psi) e^{(g_1+g_2+g_3)V_{QGP}} \tag{9}$$

$$M(\varphi, \psi) = \left(\mathrm{Sin}\left[\frac{1}{2}\left(\psi + \frac{\varphi}{2}\right)\right] \mathrm{Sin}\left[\frac{\varphi}{2}\right] \mathrm{Sin}\left[\frac{1}{2}\left(\psi - \frac{\varphi}{2}\right)\right]\right)^2 \tag{10}$$

g_1,$g_2$and g_3 result for massless u and d quarks, massive s quarks and massless gluons respectively,

$$g_1 = \frac{d_Q \pi^2}{6} T^3 \sum_{q=r,b,g} \left[\frac{7}{30} - \left(\frac{\theta_q - i\beta\mu}{\pi}\right)^2 + \frac{1}{2}\left(\frac{\theta_q - i\beta\mu}{\pi}\right)^4\right] \tag{11}$$

$$g_2 = d_Q \frac{(m_s T)^{\frac{3}{2}}}{\sqrt{2}\pi^{\frac{3}{2}}} T^3 \sum_{q=r,b,g} \left[1 - \frac{(\beta\mu - i\theta_q)^2}{2!} + \frac{(\beta\mu - i\theta_q)^4}{4!}\right] e^{-\frac{m_s}{T}} \tag{12}$$

$$g_3 = \frac{d_G \pi^2}{12} T^3 \sum_{G=1}^{4} \left[-\frac{7}{30} + \left(\frac{\theta_G - \pi}{\pi}\right)^2 - \frac{1}{2}\left(\frac{\theta_G - \pi}{\pi}\right)^4\right] \tag{13}$$

In the following, we study the variations of the pressures P_{HG} and P_{QGP} with temperatures T, at $\mu = 0$, for $B^{1/4} = 200$ MeV, in different cases of massless and massive particles in both HG and QGP phases. Figure 2 shows the curve of the HG pressure, for massless pions (red dotted line) and massive pions (blue dotted line) as well as that of the QGP for massless u and d quarks only (dashed lines), then when adding the massive s quarks (solid lines), for a small and a large volume in both last cases.

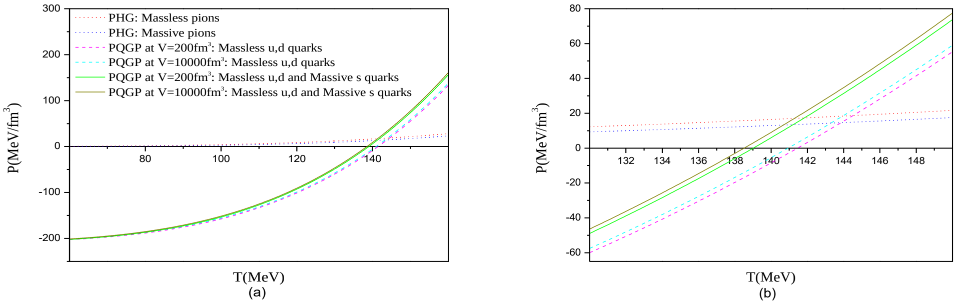


Figure 2. (**a**) Variations of the pressure of the HG and a color-singlet QGP with temperature at $\mu = 0$, for $B^{1/4}$ = 200 MeV, for massless and massive particles in both phases; (**b**) Magnification of the pressure curves at left (in (**a**)) in the region of high temperatures.

In Figure 2, we studied the effect of the particle masses for small and large volume of the QGP system on the deconfinement phase transition temperature T_c at μ = 0, when considering the color-singletness requirement for the QGP phase, by the extraction of the value of T_c from the intersection point of the curves P_{HG} and P_{QGP}. The obtained values of T_c in the different studied cases are shown in Table 1, and we can easily see that as the volume increases, T_c decreases until the volume reaches 10,000 fm^3, where T_c begins to stabilize. We can also notice that T_c (massless particles) > T_c (massive particles).

Table 1. The critical temperature Tc for different volumes, within a color-singlet QGP.

V (fm^3)	200	1000	10000
Tc(MeV) for Massless pions (HG) and Massless u, d quarks (QGP)	144.507	143.947	143.941
Tc(MeV) for Massive pions (HG) and Massless u, d quarks (QGP)	143.859	143.305	143.299
Tc(MeV) for Massless pions (HG) and Massless u, d and massive s quarks (QGP)	141.828	141.291	141.285
Tc(MeV) for Massive pions (HG) and Massless u, d and massive s quarks (QGP)	141.216	140.685	140,679

4. Conclusions

This work shows the influence of the mass of particles in both HG and QGP phases on the phase diagram in the $\mu - \mathrm{T}$ plane, as we found that the (small) mass of the up and down quarks does not affect the study, while accounting for the mass of pions does affect the critical parameters and especially the critical temperatures at small μ. Also, considering massive strange quarks additionally to u and d quarks has a clear effect on the phase diagram. The same influence is also investigated, when the color-singletness condition is considered for the QGP phase, for different volumes of the system.

References

1. Akram, F. Basics of Quantum Chromodynamics. In *Proceedings of the 5th School on LHC Physics, Islamabad, Pakistan, 15–26 August 2016*; NCP: Islamabad, Pakistan, 2016.
2. Ramadas, S. QGP in Quark Stars. Ph.D. Thesis, University of Calicut, Kerala, India, April 2014.
3. Pasechnik, R.; Šumbera, M. Phenomenological Review on Quark-Gluon Plasma: Concepts vs. Observations. *arXiv* **2016**, arXiv:1611.01533v3.
4. Apolinário, L.; Milhano, J.G.; Salamand, G.P.; Salgado, C.A. Probing the Time Structure of the Quark-Gluon Plasma with Top Quarks. *Phys. Rev. Lett.* **2018**, *120*, 232301.
5. Redlich, K.; Turko, L. Quantum gases with internal symmetry. *Phys. Lett. B* **1981**, *104*, 153.