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# Forecasts for ΛCDM and Dark Energy Models through Einstein Telescope Standard Sirens †

**Matteo Califano [1,*], Ivan de Martino [2], Daniele Vernieri [3] and Salvatore Capozziello [3]**

[1] Scuola Superiore Meridionale, Largo San Marcellino 10, I-80138 Napoli, Italy;
INFN Sezione di Napoli, Compl. Univ. Di Monte S. Angelo, Edificio G, Via Cinthia, I-80126 Napoli, Italy
[2] Departamento de Fisica Fundamental, Universidad de Salamanca, P. de la Merced S/N, Salamanca, Spain;
email1@gmail.com
[3] Dipartimento di Fisica, Università di Napoli "Federico II", Compl. Univ. di Monte S. Angelo, Edificio G, Via Cinthia, I-80126 Napoli, Italy;
Scuola Superiore Meridionale, Largo San Marcellino 10, I-80138 Napoli, Italy;
INFN Sezione di Napoli, Compl. Univ. Di Monte S. Angelo, Edificio G, Via Cinthia, I-80126 Napoli, Italy;
email2@gmail.com (D.V.); email3@gmail.com (S.C.)
* Correspondence: matteo.califano@unina.it

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**Abstract:** Gravitational Wave (GW) astronomy provides an independent way to estimate cosmological parameters. The detection of GWs from a coalescing binary allows a direct measurement of its luminosity distance, so these sources are referred to as "standard sirens" in analogy to standard candles. We investigate the impact for constraining cosmological models of the Einstein Telescope, a third-generation detector which will detect tens of thousands of binary neutron stars. We focus on the non-flat ΛCDM cosmology and some Dark Energy models that may resolve the so-called Hubble tension. To evaluate the accuracy down to which ET will constrain cosmological parameters, we consider two types of mock datasets depending on whether or not a short Gamma-Ray Burst is detected and associated with the gravitational wave event using the THESEUS satellite. Depending on the mock dataset, different statistical estimators are applied: one assumes that the redshift is known, and another marginalizes over it, taking a specific prior distribution.

**Keywords:** cosmological parameters; gravitational waves; neutron star mergers; Einstein Telescope

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## 1. Introduction

The observation of GWs from the coalescence of merging Binary Black Holes (BBH) [1,2] and Binary Neutron Stars (BNS) [3] gives an alternative tool to test General Relativity, relativistic astrophysics, and cosmology. We usually refer to GWs as "standard sirens" because, in analogy to the standard candles, they bring direct information on the luminosity distance of sources [4,5].

Contrary to most common electromagnetic (EM) distance measurements, the distance estimate with GWs is an absolute measurement. Hence, the standard sirens do not rely on the so-called cosmic distance ladder. Therefore, they are free from possible systematics arising from the calibration on other cosmic distance indicators.

In the Friedmann-Robertson-Walker cosmology, the most general form of the distance-redshift relation reads [6]

$$\mathrm{d_L(z)} = \frac{\mathrm{c}}{\mathrm{H_0}}\frac{1+\mathrm{z}}{\sqrt{\Omega_{\mathrm{k},0}}}\ \sinh\left[\sqrt{\Omega_{\mathrm{k},0}}\ \int_0^z \frac{dz'}{E(z')}\right], \tag{1}$$

where c is the speed of light, $\mathrm{H_0}$ is the Hubble constant, $\Omega_{\mathrm{k,0}}$ is the normalised energy density of the spatial curvature of the Universe, and E(z) is a function of redshift which in general depends on all the cosmological parameters that describe the background expansion of the Universe in any given cosmological model. The data $(\mathrm{d_L}, z)$ allow us to constrain the cosmological parameters in the distance-redshift relation. In particular, one can infer the Hubble constant $\mathrm{H_0}$ to the leading order, and beyond that the dark matter and dark energy fractions $\Omega_{\mathrm{m}}$, $\Omega_\Lambda$ of ΛCDM cosmology, or the dark-energy (DE) equation-of-state parameters.

Although GWs offer an alternative method to obtain distances in cosmology, they are not free of issues. In particular, the redshift parameter in the waveform is completely degenerate with the system masses. We can break the degeranacy extrapolating the information on the redshift from an electromagnetic signal. The main techniques are based on the statistical identification of the host galaxy of the GW source [4,7] or the seeking of electromagnetic emission following the GWs, such as short Gamma-Ray Burst (GRB) [3]. Another possibility relies on assuming the redshift probability distribution of GW events known from population synthesis simulations [8,9].

Nowadays, the LIGO/Virgo/KAGRA collaboration best estimation of Hubble constant is $\mathrm{H_0} = 68^{+6}_{-8}\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ at 68% of confidence level with the statistical identification of the host galaxy [7]. However, so far the GWs do not help solve the so-called Hubble tension because the accuracy is still too high, and the estimations agree with both the late-time and the early-time measurements [10–13].

Nevertheless, the next generation of GW detectors, e.g., the Einstein Telescope (ET), will offer the possibility to achieve an accuracy on the Hubble constant below 1% [14]. Here, we will focus on the simulation ET standard sirens. Moreover, we assume that the redshift of the coicendent short GRB will be detected using the Transient High Energy Sources and Early Universe Surveyor (THESEUS)) [15–17]. We forecast the accuracy of cosmolofical parameters for a non-flat ΛCDM and a set of DE models introduced to solve the Hubble tension [18,19]. We consider the following parametrizations of the E(z) function:

- non-flat ΛCDM, with the E(z) function defined by [6]

$$\mathrm{E^2(z)} = \Omega_{\mathrm{m,0}}(1+\mathrm{z})^3 + \Omega_{\mathrm{k,0}}(1+\mathrm{z})^2 + \Omega_{\Lambda,0}\,; \tag{2}$$

- non-flat $\omega$CDM, with the E(z) function defined by [20]

$$\mathrm{E^2(z)} = \Omega_{\mathrm{m,0}}(1+\mathrm{z})^3 + \Omega_{\mathrm{k,0}}(1+\mathrm{z})^2 + \Omega_{\Lambda,0}(1+\mathrm{z})^{3(1+\omega_{\mathrm{DE}})}\,; \tag{3}$$

- Interacting DE, [21–25]

$$\mathrm{E^2(z)} = \Omega_{\mathrm{m,0}}(1+\mathrm{z})^3 + \Omega_{\Lambda,0}\left[(1+\mathrm{z})^{3(1+\omega_{\mathrm{DE}}^{\mathrm{eff}})} + \frac{\xi}{3\omega_{\mathrm{DE}}^{\mathrm{eff}}}\left(1-(1+\mathrm{z})^{3\omega_{\mathrm{DE}}^{\mathrm{eff}}}\right)(1+\mathrm{z})^3\right], \tag{4}$$

where $\omega_{\mathrm{DE}}^{\mathrm{eff}} = \omega_{DE} + \xi/3$ and $\xi$ is the coupling constant;

- Time-Varying Gravitational Constant, [26]

$$\mathrm{E^2(z)} = \Omega_{\mathrm{m,0}}(1+\mathrm{z})^{(3-\delta_{\mathrm{G}})} + \Omega_{\Lambda,0}(1+\mathrm{z})^{\delta_{\mathrm{G}}\frac{\Omega_{\mathrm{m,0}}}{\Omega_{\Lambda,0}}}\,, \tag{5}$$

with $\delta_{\mathrm{G}}$ the parametrization of Gravitational Constant evolution;

- Emergent DE, [27–29]

$$\mathrm{E^2(z)} = \Omega_{\mathrm{m,0}}(1+\mathrm{z})^3 + \Omega_{\Lambda,0}\left[\frac{1-\tanh\left(\Delta\log_{10}\left(\frac{1+\mathrm{z}}{1+\mathrm{z_t}}\right)\right)}{1+\tanh(\Delta\log_{10}(1+\mathrm{z_t}))}\right], \tag{6}$$

where is $\Delta$ a free parameter and $\mathrm{z_t}$ is the epoch where the matter energy density and the DE density are equal.

In the following sections, we briefly summarize the procedure used to build up the mock data catalog (Section 2) and the statistical analysis techniques (Section 3). Finally, in the Section 2, we discuss our results.

## 2. Mock Data Generation

Following the procedure illustrated in [30,31], we simulate the GWs events to forecast the precision down to which ET will be able to constrain the cosmological parameters. We want to consider only the BNS mergers because we could detect their EM counterpart. To generate the synthetic dataset, we assume as fiducial cosmological model a $\Lambda$CDM with best-fit values given by [13], $H_0 = 67.66$ km $s^{-1}$ $Mpc^{-1}$, $\Omega_{m,0} = 0.31$, $\Omega_{\Lambda,0} = 0.69$ and $\Omega_{k,0} = 0.0$. Then, we extract the redshift of the source from a probability distribution, p(z), defined from the Star Formation Rate (SFR) and the time delay distribution. The function p(z) is

$$p(z) = \mathcal{N}\frac{R_m(z)}{1+z}\frac{dV(z)}{dz}, \tag{7}$$

where $\mathcal{N}$ is a normalization factor, dV(z)/dz is the comoving volume element, and $R_m(z)$ is the merger rate per unit of volume in the source frame. We can parametrize the rate $R_m(z)$ in terms of the SFR $R_f(z)$ [32], and the time delay distribution $P(t_d) \propto t_d^{-1}$ as suggested by population synthesis models [33].

Then, using the latest power spectral density of ET, we simulate the detector response to estimate the number and the parameters of GW events. Hence, we select the events above a given of the Signal-to-Noise Ratio (SNR). We adopt a SNR threshold equal 9. Finally, we add a Gaussian noise component, $\mathcal{N}\left(d_L^{fid}, \sigma_{d_L}\right)$ to our estimations of the luminosity distances $d_L^{fid}$. based on the fiducial cosmological model. The variance counts for different sources of uncertainties:

$$\sigma_{d_L} = \sqrt{\sigma_{inst}^2 + \sigma_{lens}^2 + \sigma_{pec}^2} \tag{8}$$

The first term is the most relevant due to the instrumental part. At leading order, $\sigma_{inst}$ is strictly related to the SNR through the relation $\sigma_{inst} = 2d_L/SNR$ [34]. The second and the last ones are related to some extra contributions in the noise due to the observational features. We consider the lensing [35] and the peculiar velocity of the host galaxy contribution [36]. Setting a duty cycle for ET equal to 80%, we build our mock catalogs containing GWs events for one, five, and ten years of observational runs. We estimate a rate of $0.5 \times 10^4$ events per year.

Since the number of combined events is strictly affected by the features of the satellite, we have to set the duty cycle of the THESEUS satellite to 80%[15] and the sky coverage to 1/2. Furthermore, since the THESEUS satellite can localize a source within five arcminutes of its central field of view, we record only 1/3 of the total number of combined events in the realistic case [15,37]. We find a rate of 10 combined events per year.

## 3. Analysis and Results

We analyze each mock catalog using an MCMC algorithm. We consider both events with a detected electromagnetic counterpart (Bright Sirens) and those without the direct redshift information (Dark Sirens). When we know the redshift from the detection of GRB, the single event likelihood is [9,38]

$$p(d_i \,|\Theta) = \frac{\int p(d_i|D_L)p_{pop}(D_L\,|z\,,\ \Theta)p(z,z_i)\,dzd\,D_L}{\int p_{det}(D_L)p_{pop}(D_L\,|z\,,\ \Theta)p(z,z_i)\,dzd\,D_L}. \tag{9}$$

where $p(z, z_i) = \delta(z - z_i)$ with $z_i$ the redshift associated to the GRB, $\Theta$ is the set of cosmological parameters, and $p_{pop}(D_L\,|z\,, \Theta) = \delta\left(D_L - d_L^{th}(z, \Theta)\right)$ . Furthermore, the denominator is a normalization factor that takes into account the selection effects [38]. To study the Dark Sirens case, we assume to know redshift prior information related to the distribution p(z), and then we marginalize over this distribution [8,9]. In this case, the likelihood is

$$p(\mathrm{d_i}|\,\Theta\,) = \int_0^{z_{\max}} p\left(\mathrm{d_i}\middle|\mathrm{d_L^{th}}(\mathrm{z_i},\Theta)\right) p_{\mathrm{obs}}(\mathrm{z_i}|\Theta)\mathrm{dz_i}, \tag{10}$$

where the probability prior distribution of the redshift, $p_{obs}(z_i|\Theta)$, is obtained from the observed events and already includes detector selection effects [8].

In Table 1, we report the results obtained after ten observation years for all the models considered and for the Bright and Dark Sirens, respectively.

**Table 1.** The median value and the 68% confidence level of the posterior distributions of the parameters of our models for SNR equal 9 and ten years of observations, as obtained from the MCMC analyses carried out on mock catalog collecting the Bright and Dark Sirens, respectively.

| | | | | |
|---|---|---|---|---|
| | | **Non-flat ΛCDM** | | |
| | $H_0$ | $\Omega_{k,0}$ | $\Omega_{\Lambda,0}$ | **-** |
| Bright Sirens | $67.49^{+0.70}_{-0.87}$ | $-0.11^{+0.16}_{-0.15}$ | $0.74^{+0.12}_{-0.15}$ | - |
| Dark Sirens | $67.68^{+0.04}_{-0.03}$ | $0.00^{+0.01}_{-0.01}$ | $0.69^{+0.01}_{-0.01}$ | - |
| | | **Non-flat ωCDM** | | |
| | $H_0$ | $\Omega_{k,0}$ | $\Omega_{\Lambda,0}$ | $\boldsymbol{\omega_{DE}}$ |
| Bright Sirens | $67.49^{+0.70}_{-0.87}$ | $-0.05^{+0.19}_{-0.17}$ | $0.66^{+0.20}_{-0.16}$ | $-1.35^{+0.84}_{-0.98}$ |
| Dark Sirens | $67.68^{+0.06}_{-0.05}$ | $-0.01^{+0.02}_{-0.02}$ | $0.68^{+0.03}_{-0.03}$ | $-0.95^{+0.09}_{-0.11}$ |
| | | **Interacting Dark Energy** | | |
| | $H_0$ | $\Omega_{m,0}$ | $\xi$ | **-** |
| Bright Sirens | $67.55^{+1.02}_{-1.03}$ | $0.24^{+0.13}_{-0.14}$ | $-0.76^{+0.83}_{-0.92}$ | - |
| Dark Sirens | $67.70^{+0.05}_{-0.05}$ | $0.32^{+0.01}_{-0.01}$ | $-0.02^{+0.06}_{-0.06}$ | - |
| | | **Time-varying Gravitational Constant** | | |
| | $H_0$ | $\Omega_{m,0}$ | $\delta_G$ | **-** |
| Bright Sirens | $67.81^{+0.97}_{-0.93}$ | $0.29^{+0.10}_{-0.07}$ | $-0.26^{+0.42}_{-0.46}$ | - |
| Dark Sirens | $67.65^{+0.04}_{-0.04}$ | $0.31^{+0.01}_{-0.01}$ | $-0.02^{+0.02}_{-0.02}$ | - |
| | | **Emergent Dark Energy** | | |
| | $H_0$ | $\Omega_{m,0}$ | $\Delta$ | **-** |
| Bright Sirens | $67.51^{+0.81}_{-0.92}$ | $0.36^{+0.05}_{-0.06}$ | $0.21^{+0.89}_{-0.83}$ | - |
| Dark Sirens | $67.66^{+0.03}_{-0.03}$ | $0.310^{+0.002}_{-0.002}$ | $0.00^{+0.01}_{-0.01}$ | - |

It is worth stressing that we always recover our fiducial cosmological model within the 68% Confidence Interval. Independently of the model used in the statistical analysis, we obtain an accuracy of ~1% with bright sirens and reach ~ 0.1% with dark sirens. This accuracy will be competitive with respect to the other cosmological probes to solve the Hubble tension [39]. However, when we consider the constraints on the additional parameters, in the non-flat ωCDM and interacting DE models, the parameters $\omega_{DE}$ and $\xi$ will be constrained with an accuracy worse than current bounds [22,24,40]. In the case of the time-varying gravitational constant model, the bound on the parameter $\delta_G$ is one order of magnitude higher than current constraints [22]. Whereas, we show that ET will also be able to improve the bounds in the emergent DE model. In particular, we have an improvement of a factor of on the additional cosmological parameter Δ with respect to the current analysis [28]. For a more detailed comparison see [30].

## 4. Discussion and Conclusions

We used mock catalogs of GW events from BNSs to test the capabilities of ET on constraing the ΛCDM cosmological model and provide insight into the dark energy models. Namely, we investigate the non-flat ΛCDM, the non-flat ωCDM the interacting dark energy, the emergent dark energy and the time-varying gravitational constant models. The third generation GW detector promises to constrain the Hubble constant with subpercent accuracy [15], offering a possible solution to the Hubble tension.

We built the mock catalogs containing GW events considering one, five, and ten years of observational runs, and SNR thresholds equal 9. Additionally, starting from each of those three mock catalogs, we extracted a mock catalog of GW events with an associated GRB detected using the THESEUS satellite.

In the analysis, we distinguish the catalogs depending on whether the redshift information comes from the GRB (Bright Sirens) or the BNS merger rate (Dark Sirens). We assume the rate is *a priori* known to follow the SFR. Although realistically, the redshift evolution of the merger rate will be uncertain, prior knowledge of the SFR from other astrophysical observations will provide valuable information for standard siren analyses.

Our results show the huge capability of ET to solve the Hubble tension independently by the theoretical framework chosen, but also point out that, to strongly constrain the DE models we have considered, ET will need to be complemented with other datasets. The ET standard sirens will represent an alternative approach to constrain the cosmological parameters and the DE models; moreover, they will be affected by different systematics compared to the analyses based on classical electromagnetic standard candles.

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