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Proceeding Paper

Nonsingular Bouncing Model in Closed and Open Universe †

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Abstract: We investigate the background dynamics of a class of model with noncanonical scalar field and matter both in FLRW closed and open spacetime. The detailed dynamical system analysis is carried out in bouncing scenario. Cosmological solutions satisfying the stability and bouncing conditions are obtained using the tools of dynamical system.

Keywords: Bouncing scenario; Noncanonical scalar field; Nonsingular bounce

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1. Introduction

The shortcomings of the standard model of cosmology address in inflation as well as bouncing scenario. The recent years have shown grand success for the inflationary paradigm supported by precision data. Though inflation solves most of the problems (horizon, flatness and entropy) of the standard model of cosmology, the issue with the initial singularity is not resolved under its domain. It is the alternate scenario, nonsingular bouncing model, that eradicates the singularity by constructing a universe which begins with a contracting phase and then bounces back to an expanding phase through a non zero minimum in the scale factor. Nonsingular bouncing models can be categorized into two types, matter bounce model [1] and Ekpyrotik models [2,3]. For a review on these models refer to [4–8]. Here let us not undermine the fact that the occurrence of singularity is an artifact of pushing the classical theory of gravity, General Relativity to the limit, the Planck region, where it no longer holds. We understand that in a true theory of quantum gravity the singularity would be mitigated because of the uncertainty principle. At the same time we emphasize that all the candidate theories of quantum gravity till date are tentative in nature as the complete theory of quantum gravity has not been discovered yet and hence the physics of Planck region is still unknown. It is in this spirit that one must keep exploring for the viable classical scenario, for the simple fact that away from the Planck region the universe looks classical. It is to be noted that in a classical bouncing scenario the singularity is avoided by construction and the universe bounces from a contracting phase to an expanding phase before reaching the Planck length. Therefore, until the final theory of quantum gravity is constructed, it is of equal importance to explore the classical dynamics due to non trivial Langrangian which can possibilly mimic a bouncing scenario. In recent past study of anisotropic bouncing scenario has been carried out by authors [9] and the necessary and sufficient conditions for a nonsingular bounce to occur in terms of the dynamical variables are derived. In this paper we consider a noncanonical scalar field with a general function of kinetic term $F(X)$, where $X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$. These theories are originally motivated to provide a large tensor to scalar perturbation in inflationary settings [10–12]. Dark energy with a general kinetic term $F(X)$ is modeled first in [13]. For other variants of models of dark energy in this context refer to [14]. Other works related to unifying dark matter, dark energy and/or inflation for noncanonical scalar field models are studied in [15–18]. In order to study the phase space in this model, we write the first order equations of motion in terms of dimensionless dynamical variables [19]. The motivation to use noncanonical scalar field as matter is to construct nonsingular bouncing models. The phase space analysis of a cosmological model with scalar field Lagrangian $F(X) - V(\phi)$ and matter for an FRW flat background is given in [20]. The condition for nonsingular bounce is also discussed in [20]. In order to explore the behaviour of curvature parameter near bounce in a nonsingular bouncing model we do a phase space analysis in an FRW closed and open universe. This can be easily extended to other nonsingular bouncing models.

We study the cosmology of a curved, closed and open, universe with a matter Lagrangian of the form $F(X) - V(\phi)$ and adhoc matter. In Section 2 we write the Einstein's equation in terms of dynamical variables suitable for the analysis of a bouncing scenario in FRW closed and open universe. Following this we find the fixed points and their stability in Section 3. It should be noted that one of the primary goals of this paper is to look for bouncing solution that goes to a stable fixed point at late-time. The importance of the Section 3 lies in the fact that it would give us the region of parameter space allowed by the stability criteria for stable fixed points. Thus, we intend to look for cosmological solutions whose values of parameters being picked, strictly, from the allowed region. Next, conditions for existence of nonsingular bouncing solution is derived in terms of dynamical variables in Section 4. We summaries our results in Section 5.

2. Einstein Equations in FRW Closed and Open Universe

The action for our model is given by

$$S = \int d^4x\sqrt{-g}[\frac{1}{2}R + F(X) - V(\phi) + L_m] \tag{1}$$

where L_m is the lagrangian of the matter field.

To see the behaviour of curvature parameter of the spacetime in a nonsingular bouncing scenario we work with a FRW closed and open universe. The line element of the same is given by:

$$ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2sin^2\theta d\phi^2]. \tag{2}$$

where $k = +1$ denotes closed and $k = -1$ denotes an open universe respectively.

The Hubble parameter H is defined as

$$H = \frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t}$$

In terms of Hubble parameter, the Einstein equations take the following form

$$\begin{aligned} \frac{\mathrm{d}H}{\mathrm{d}t} &= -H^2 - \frac{1}{6}(\rho + 3p)\,, \\ H^2 &= \frac{\rho}{3} - \frac{k}{a^2}\,, \end{aligned} \tag{3}$$

where $\rho = \rho_\phi + \rho_m$ and $p = p_\phi + p_m$.

Here the energy density ρ_ϕ and pressure p_ϕ of the scalar field is found to be

$$\begin{aligned} \rho_\phi &= 2XF_X - F + V\,, \\ p_\phi &= F(X) - V(\phi)\,, \end{aligned} \tag{4}$$

and ρ_m and p_m are the energy density and pressure due to the term L_m.

Substituting Equation (4) in first and third line of Equation (3), we get

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -H^2 - \frac{1}{6}(2XF_X - F + V + \rho_m + 3(F - V) + 3p_m) \tag{5}$$

$$H^2 = \frac{2XF_X - F}{3} + \frac{V}{3} - \frac{k}{a^2} + \frac{\rho_m}{3} \tag{6}$$

Here we further define few more variables which are useful for defining dimensionless dynamical variables. They are

$$\begin{aligned} \rho_k &= 2XF_X - F\,, \\ w_k &= \frac{F}{2XF_X - F}\,, \\ \sigma &= -\frac{1}{\sqrt{3}|\rho_k|}\frac{\mathrm{d}logV}{\mathrm{d}t}\,, \end{aligned} \tag{7}$$

where ρ_k is the kinetic part of the energy density ρ_ϕ, w_k is the ratio of kinetic part of the pressure p_ϕ to the ρ_k and σ is the auxiliary variable which depends on the variation of potential with time.

Neglecting the interaction between scalar field and matter, continuity equation for ρ_ϕ in terms of dimensionless time variable N $(dN = Hdt)$, is

$$\frac{\mathrm{d}}{\mathrm{d}N}(2XF_X - F + V) + 6XF_X = 0. \tag{8}$$

Now we define a set of dimensionless dynamical variables which is suitable for nonsingular bounce models. Relevance of these variables that they remain finite during the entire evolution across bounce. The dynamical variables are

$$\tilde{x} = \frac{\sqrt{3}H}{\sqrt{|\rho_k|}}\,, \tilde{y} = \frac{\sqrt{|V|}}{\sqrt{|\rho_k|}}sign(V)\,, \tilde{z} = \frac{\sqrt{|k|}}{\sqrt{|\rho_k}}sign(\tilde{z})\,, \tilde{\Omega}_m = \frac{\rho_m}{|\rho_k|}. \tag{9}$$

Here $sign(\tilde{z}) \equiv sign(k)$ denotes FRW closed universe for $+1$ and open for -1. Using Equations (3), (8) and (9) and parameters defined in Equation (7), the evolution equations of $\tilde{x}$, $\tilde{y}$ and $\tilde{z}$ are written as,

$$\begin{aligned} \frac{\mathrm{d}\tilde{x}}{\mathrm{d}\tilde{N}} &= -\frac{3}{2}\left[(w_k - w_m)sign(\rho_k) + (1 + w_m)(\tilde{x}^2 - \tilde{y}|\tilde{y}|) + \frac{\tilde{z}|\tilde{z}|}{3}(1 + 3w_m)\right] \\ &+ \frac{3}{2}\tilde{x}[(w_k + 1)\tilde{x} - \sigma\tilde{y}|\tilde{y}|sign(\rho_k)], \\ \frac{\mathrm{d}\tilde{y}}{\mathrm{d}\tilde{N}} &= \frac{3}{2}\tilde{y}[-\sigma + (w_k + 1)\tilde{x} - \sigma\tilde{y}|\tilde{y}|sign(\rho_k)], \\ \frac{\mathrm{d}\tilde{z}}{\mathrm{d}\tilde{N}} &= -\tilde{z}\tilde{x} + \frac{3}{2}\tilde{z}(\tilde{x}(1 + w_k) - \tilde{y}|\tilde{y}|sign(\rho_k)), \\ \frac{\mathrm{d}\tilde{\Omega}_m}{\mathrm{d}\tilde{N}} &= -3(1 + w_m)\tilde{x}\tilde{\Omega}_m - \tilde{\Omega}_m[3\sigma\tilde{y}|\tilde{y}|sign(\rho_k) - 3\tilde{x}(1 + w_k)]\,, \end{aligned} \tag{10}$$

where $d\tilde{N} = \sqrt{\frac{|\rho_k|}{3}}dt$ and the constraint equation relating dynamical variables is

$$\tilde{x}^2 - \tilde{y}|\tilde{y}| + \tilde{z}|\tilde{z}| - \tilde{\Omega}_m = 1 \times sign(\rho_k). \tag{11}$$

The equation for parameter σ becomes [20]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tilde{N}} = -3\sigma^2(\Gamma - 1) + \frac{3\sigma(2\Xi(w_k+1) + w_k - 1)}{2(2\sigma+1)(w_k+1)}\left[(w_k+1)\tilde{x} - \sigma\tilde{y}^2\right] \tag{12}$$

where $\Xi = \frac{XF_{XX}}{F_X}$ and $\Gamma = \frac{VV_{\phi\phi}}{V_\phi}$.

For our model we have taken power law form for $F(X) = F_0 X^\eta$, where F_0 is a constant. For this form of $F(X)$, $w_k = \frac{1}{2\eta-1}$ and $\Xi = \eta - 1$.

Potential $V(\phi)$ is taken as $V(\phi) = V_0 e^{-c\phi}$, where V_0 and c are constants with positive values. For this choice of $V(\phi)$, Γ becomes unity.

In the next section, we do a fixed point analysis of dynamical equations for $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ and σ. The evolution of $\tilde{\Omega}_m$ is determined from the constraint Equation (11).

3. Fixed Point Analysis

In this section, we do a fixed point analysis of our system of dynamical equation in order to extract the qualitative information about the nature of solution. Fixed points are calculated by taking the first derivative of the dynamical variables to be zero. The stability of a fixed point is determined from the behavior of a small perturbation around that fixed point.

We get the set of fixed points $\tilde{x}_c$, $\tilde{y}_c$, $\tilde{z}_c$ and σ_c by solving the following set of equations simultaneously (where the subscript c denotes fixed points). Now, if we define the slopes of the dynamical variables $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ and σ as $f(\tilde{x},\tilde{y},\tilde{z},\sigma)$, $g(\tilde{x},\tilde{y},\tilde{z},\sigma)$, $h(\tilde{x},\tilde{y},\tilde{z},\sigma)$ and $i(\tilde{x},\tilde{y},\tilde{z},\sigma)$. The set of equations we need to solve to obtain the fixed point is

$$\begin{aligned}
f(\tilde{x},\tilde{y},\tilde{z},\sigma) &\equiv \frac{\mathrm{d}\tilde{x}}{\mathrm{d}\tilde{N}} = 0\,, \\
g(\tilde{x},\tilde{y},\tilde{z},\sigma) &\equiv \frac{\mathrm{d}\tilde{y}}{\mathrm{d}\tilde{N}} = 0\,, \\
h(\tilde{x},\tilde{y},\tilde{z},\sigma) &\equiv \frac{\mathrm{d}\tilde{z}}{\mathrm{d}\tilde{N}} = 0\,, \\
i(\tilde{x},\tilde{y},\tilde{z},\sigma) &\equiv \frac{\mathrm{d}\sigma}{\mathrm{d}\tilde{N}} = 0
\end{aligned} \tag{13}$$

where,

$$\begin{aligned}
f(\tilde{x},\tilde{y},\tilde{z},\sigma) &\equiv -\frac{3}{2}[(w_k - w_m)(sign\rho_k) + (1+w_m)(\tilde{x}^2 - \tilde{y}|\tilde{y}|) + (1+3w_m)\frac{\tilde{z}|\tilde{z}|}{3}] \\
&+ \frac{3}{2}\tilde{x}[(w_k+1)\tilde{x} - \sigma\tilde{y}|\tilde{y}|sign(\rho_k)]\,, \\
g(\tilde{x},\tilde{y},\tilde{z},\sigma) &\equiv \frac{3}{2}\tilde{y}[-\sigma + (w_k+1)\tilde{x} - \sigma\tilde{y}|\tilde{y}|sign(\rho_k)]\,, \\
h(\tilde{x},\tilde{y},\tilde{z},\sigma) &\equiv -3\tilde{z}\tilde{x} + 3\tilde{z}\tilde{x}(1+w_k) - 3\tilde{z}\tilde{y}|\tilde{y}|sign(\rho_k)\,, \\
i(\tilde{x},\tilde{y},\tilde{z},\sigma) &\equiv \frac{3}{2}\frac{[2\Xi(w_k+1) + (w_k-1)]}{2(2\sigma+1)(w_k+1)}[(w_k+1)\tilde{x} - \sigma\tilde{y}^2].
\end{aligned} \tag{14}$$

The corresponding fixed point for $\tilde{\Omega}_m$ can be found using the constraint Equation (11).

The stability of the fixed points can be examined from the evolution of perturbations around fixed points. Now, if $(\tilde{x}_c,\tilde{y}_c,\tilde{z}_c,\sigma_c)$ is a fixed point and $\delta\tilde{x} = \tilde{x} - \tilde{x}_c, \delta\tilde{y} = \tilde{y} - \tilde{y}_c$, $\delta\tilde{z} = \tilde{z} - \tilde{z}_c$ and $\delta\sigma = \sigma - \sigma_c$ be the respective perturbation around it, then the evolution of the perturbation is determined by

$$
\begin{aligned}
\delta\dot{\tilde{x}} &= \dot{\tilde{x}} = f(\tilde{x}_c + \delta\tilde{x}, \tilde{y}_c + \delta\tilde{y}, \tilde{z}_c + \delta\tilde{z}, \sigma + \delta\sigma)\,, \\
\delta\dot{\tilde{y}} &= \dot{\tilde{y}} = g(\tilde{x}_c + \delta\tilde{x}, \tilde{y}_c + \delta\tilde{y}, \tilde{z}_c + \delta\tilde{z}, \sigma + \delta\sigma)\,, \\
\delta\dot{\tilde{z}} &= \dot{\tilde{z}} = h(\tilde{x}_c + \delta\tilde{x}, \tilde{y}_c + \delta\tilde{y}, \tilde{z}_c + \delta\tilde{z}, \sigma + \delta\sigma)\,, \\
\delta\dot{\sigma} &= \dot{\sigma} = h(\tilde{x}_c + \delta\tilde{x}, \tilde{y}_c + \delta\tilde{y}, \tilde{z}_c + \delta\tilde{z}, \sigma + \delta\sigma)
\end{aligned}
\tag{15}
$$

The evolution equations,upto first order, for these perturbations are

$$
\begin{pmatrix} \delta\dot{\tilde{x}} \\ \delta\dot{\tilde{y}} \\ \delta\dot{\tilde{z}} \\ \delta\dot{\sigma} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \delta\tilde{x} \\ \delta\tilde{y} \\ \delta\tilde{z} \\ \delta\sigma \end{pmatrix}
\tag{16}
$$

where the matrix is

$$
\mathbf{A} = \begin{pmatrix}
\frac{\partial f}{\partial \tilde{x}} & \frac{\partial f}{\partial \tilde{y}} & \frac{\partial f}{\partial \tilde{z}} & \frac{\partial f}{\partial \sigma} \\
\frac{\partial g}{\partial \tilde{x}} & \frac{\partial g}{\partial \tilde{y}} & \frac{\partial g}{\partial \tilde{z}} & \frac{\partial g}{\partial \sigma} \\
\frac{\partial h}{\partial \tilde{x}} & \frac{\partial h}{\partial \tilde{y}} & \frac{\partial h}{\partial \tilde{z}} & \frac{\partial h}{\partial \sigma} \\
\frac{\partial i}{\partial \tilde{x}} & \frac{\partial i}{\partial \tilde{y}} & \frac{\partial i}{\partial \tilde{z}} & \frac{\partial i}{\partial \sigma}
\end{pmatrix}
\tag{17}
$$

is the Jacobian matrix and is evaluated at the fixed point $(\tilde{x}_c, \tilde{y}_c, \tilde{z}_c, \sigma_c)$ and hence each entry of $\mathbf{A}$ is a number. The solution of the system of equations can be found by diagonalizing the matrix $\mathbf{A}$. A non trivial solution exists only when the determinant $|\mathbf{A} - \check{}\mathbf{I}|$ is zero. Thus, solving this equation in λ we would get all the eigen values of the system corresponding to each fixed points.

We have two cases: one with positive kinetic term, $sign(\rho_k) = +ve$ and other one with negative kinetic term, $sign(\rho_k) = -ve$..

3.1. Closed Universe

3.1.1. Case I, with $sign\rho_k = +ve$

In this case we study the fixed points for all possible values of parameters in an FRW closed universe. The fixed point $(0,0,0,0)$ is obtained for $w_k = w_m$ signifying all the dynamical variables $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ and σ, going to zero at late times. It is a nonhyperbolic fixed point as the eigen value of $\mathbf{A}$ for this is $(0,0,0,0)$. It's stability cannot be decided from our first order analysis of perturbations. From now onwards eigenvalues would mean eigenvalues of matrix $\mathbf{A}$.

The second fixed point $(1,0,0,0)$ denotes a late time kinetic dominated universe with other dynamical variables $\tilde{y}$, $\tilde{z}$ and σ becoming zero. In this case eigenvalues are $(\frac{3(w_k+1)}{2}, -1 + \frac{3}{2}(1+w_k), \frac{3}{2}(-1 + w_k + \frac{(1-w_k)(1+w_k)}{w_k}), 3(w_k - w_m))$. This is a stable fixed point for the region of parameter space shown in the Figure 1.

The next stable fixed point in this subsection is $(-1,0,0,0)$ with eigenvalue $(1 + \frac{3}{2}(-1-w_k), \frac{3}{2}(-1-w_k), \frac{3(-1-w_k)(-1+w_k+\frac{(1-w_k)(1+w_k)}{w_k})}{2(1+w_k)}, \frac{3}{2}(-1-w_k) - \frac{3}{2}(1+w_k) + 3(1+w_m))$ shows again a late time kinetic dominated phase but with a negative value of Hubble parameter H signifying a contracting universe. This fixed point is found to be stable for the region of parameter space shown in Figure 2. The point $(-1,0,0,0)$ may not be important from bouncing point of view, as we need the universe to transit to an expanding phase to be discussed in Section 4.

The remaining fixed points, in this section, being $(0, 0, \sqrt{3}\frac{\sqrt{-w_k+w_m}}{\sqrt{1+3w_m}}, 0)$ for $w_m > w_k$ and $w_m > -\frac{1}{3}$, $(0, \frac{\sqrt{w_k-w_m}}{\sqrt{1+w_m}}, 0, 0)$ with $w_k > w_m$ and $w_m > -1$, with eigen values
$(0, 0, -\sqrt{\frac{3}{2}}\sqrt{w_k + 3w_k^2 - w_m - 3w_k w_m}, \sqrt{\frac{3}{2}}\sqrt{w_k + 2w_k^2 - w_m - 3w_k w_m}),$

$(0, 0, -\frac{3}{2}\sqrt{\frac{w_k+w_k^2-w_m-w_kw_m}{2}}, \frac{3}{\sqrt{2}}\sqrt{w_k+w_k^2-w_m-w_kw_m})$ are also nonhyperbolic points. The stability of such fixed points goes beyond the linear stabilty analysis. All the fixed points and their stability conditions are noted in Table 1.

Table 1. Stablity Analysis of fixed points for closed universe with $sign(\rho_k) = +ve$.

Fixed Points $(x_c, y_c, z_c, \sigma_c)$	**Stability Conditions**
$(0, 0, 0, 0)$ for $w_k = w_m$	Can't decide
$(1, 0, 0, 0)$	Stable (see Figure 1)
$(-1, 0, 0, 0)$	Stable (see Figure 2)
$(0, 0, \sqrt{3}\frac{\sqrt{-w_k+w_m}}{\sqrt{1+3w_m}}, 0)$ with $w_m > w_k$ and $w_m > -\frac{1}{3}$	Can't decide
$(0, \frac{\sqrt{w_k-w_m}}{\sqrt{1+w_m}}, 0, 0)$ with $w_k > w_m$ and $w_m > -1$	Can't decide

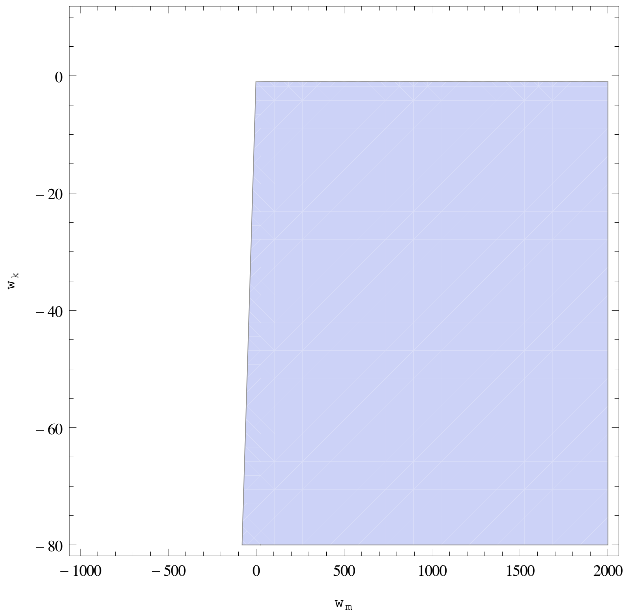

Figure 1. Allowed region of parameter space for the fixed point $(1, 0, 0, 0)$ in closed universe.

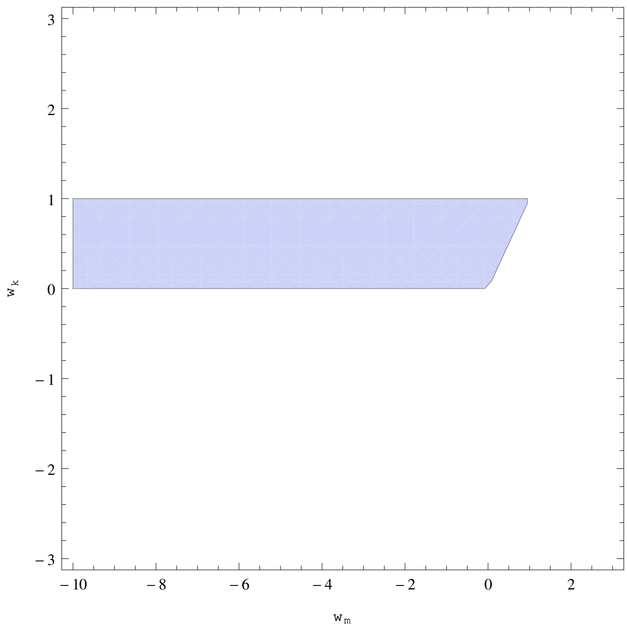

Figure 2. Allowed region of parameter space for the fixed point $(-1, 0, 0, 0)$ in closed universe.

3.1.2. Case II, $sign(\rho_k) = -ve$

In this section, we state the results of stability analysis of our dynamical variables for the negative sign of kinetic energy density. The fixed points are found to be $(0,0,0,0)$, $(0,0,-\sqrt{3}\sqrt{\frac{w_k-w_m}{1+3w_m}},0)$ and $(0,\sqrt{\frac{-w_k+w_m}{1+w_m}},0,0)$ with eigen values $(0,0,0,0)$, $(0,0,-\sqrt{\frac{3}{2}}\sqrt{-w_k-3w_k^2+w_m+3w_kw_m}, \sqrt{\frac{3}{2}}\sqrt{-w_k-3w_k^2+w_m+3w_kw_m})$ and $(-3,0,0\frac{rw_k-w_k^2+w_m+w_kw_m}{\sqrt{2}},3\frac{\sqrt{w_k-w_k^2+w_m+w_kw_m}}{\sqrt{2}})$ respectively. All these fixed points are non-hyperbolic and tabulated in Table 2.

Table 2. Stablity Analysis of fixed points for closed universe with $sign(\rho_k) = -ve$

Fixed Points $(x_c, y_c, z_c, \sigma_c)$	**Stability Conditions**
$(0,0,0,0)$ for $w_k = w_m$	Can't decide
$(0,0,-\sqrt{3}\sqrt{\frac{w_k-w_m}{1+3w_m}},0)$	Can't decide
$(0,\sqrt{\frac{-w_k+w_m}{1+w_m}},0,0)$	Can't decide

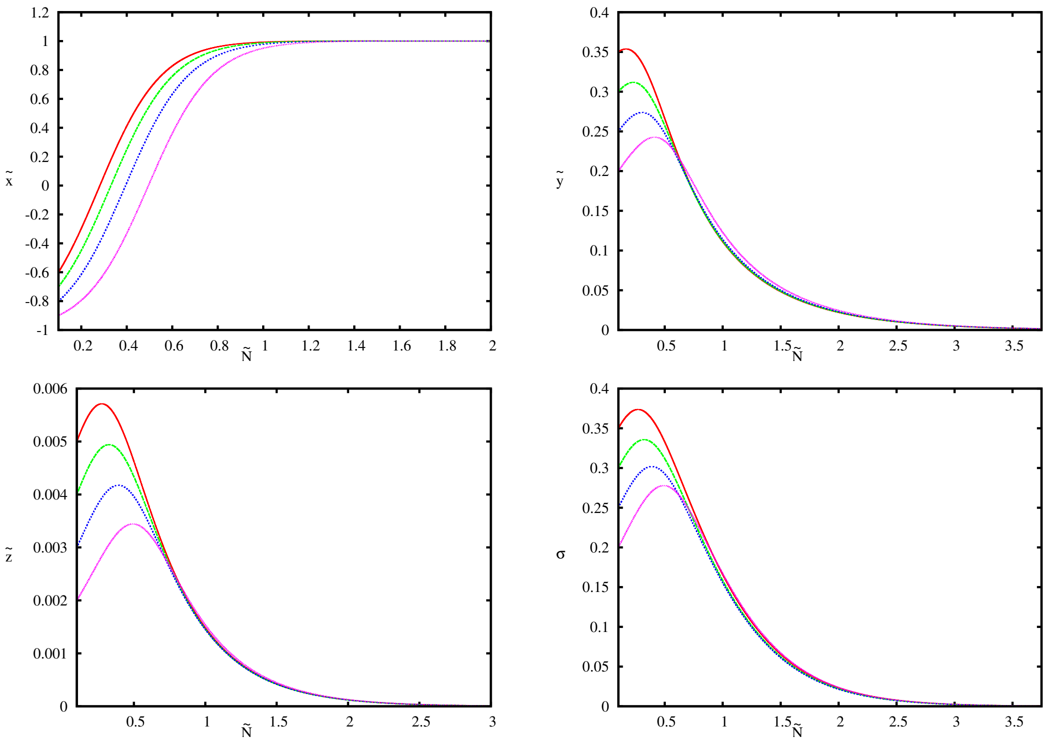

Figure 3. Evolution of the dynamical variables $\tilde{x}$ (**top left**), $\tilde{y}$ (**top right**), $\tilde{z}$ (**bottom left**) and σ (**bottom right**) for the fixed point $(\tilde{x}_c, \tilde{y}_c, \tilde{z}_c, \sigma_c) = (1,0,0,0)$ with the values of parameters $sign(\tilde{z}) = +ve$, $w_k = -2.0$, $w_m = 1/3$ and $sign(\rho_k) = +ve$ for different initial conditions.

3.2. Open Universe

3.2.1. Case I, with $sign\rho_k = +ve$

In this case we study the fixed points for all possible values of parameters in an FRW open universe. The fixed point $(0,0,0,0)$ is obtained for $w_k = w_m$ signifying all the dynamical variables $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ and σ, going to zero at late times. It is a nonhyperbolic fixed point as the eigen value of **A** for this is $(0,0,0,0)$. It's stability cannot be decided from our first order analysis of perturbations.

The second fixed point $(1,0,0,0)$ denotes a late time kinetic dominated universe with other dynamical variables $\tilde{y}$, $\tilde{z}$ and σ becoming zero. In this case eigenvalues are

$(\frac{3(w_k+1)}{2}, -1+\frac{3}{2}(1+w_k), \frac{3}{2}(-1+w_k+\frac{(1-w_k)(1+w_k)}{w_k}), 3(w_k-w_m))$. This is a stable fixed point for the region of parameter space shown in the Figure 4.

The next stable fixed point in this subsection is $(-1,0,0,0)$ with eigenvalue $(1+\frac{3}{2}(-1-w_k), \frac{3}{2}(-1-w_k), \frac{3(-1-w_k)(-1+w_k+\frac{(1-w_k)(1+w_k)}{w_k})}{2(1+w_k)}, \frac{3}{2}(-1+w_k)-\frac{3}{2}(1+w_k)+3(1+w_m))$ shows again a late time kinetic dominated phase but with a negative value of Hubble parameter H signifying a contracting universe. This fixed point is found to be stable for the region of parameter space shown in Figure 5. The point $(-1,0,0,0)$ may not be important for bouncing point of view, as again, we need the universe to transit to an expanding phase to be discussed in Section 4.

The remaining two fixed points being $(0,0,-\frac{\sqrt{3}\sqrt{w_k-w_m}}{\sqrt{1+3w_m}},0)$ with $w_k > w_m$ and $w_m > -\frac{1}{3}$ and $(0,\frac{\sqrt{w_k-w_m}}{\sqrt{1+w_m}},0,0)$ with $w_k > w_m$ and $w_m > -1$ with eigen values $(0,0,-\sqrt{\frac{3}{2}}\sqrt{w_k+3w_k^2-w_m-3w_kw_m}, \sqrt{\frac{3}{2}}\sqrt{w_k+3w_k^2-w_m-3w_kw_m})$ and $(0,0,-\frac{3}{\sqrt{2}}\sqrt{w_k+w_k^2-w_m-w_kw_m}, \frac{3}{\sqrt{2}}\sqrt{w_k+w_k^2-w_m-w_kw_m})$ are also nonhyperbolic points. The stability of such fixed points goes beyond the linear stabilty analysis. All the fixed points and their stability are noted in Table 3.

Table 3. Stablity Analysis of fixed points for open universe with $sign(\rho_k) = +ve$.

Fixed Points $(x_c, y_c, z_c, \sigma_c)$	**Stability Conditions**
$(0,0,0,0)$ for $w_k = w_m$	Can't decide
$(1,0,0,0)$	Stable (see Figure 4)
$(-1,0,0,0)$	Stable (see Figure 5)
$(0,0,-\frac{\sqrt{3}\sqrt{w_k-w_m}}{\sqrt{1+3w_m}},0)$ with $w_k > w_m$ and $w_m > -\frac{1}{3}$	Can't decide
$(0,\frac{\sqrt{w_k-w_m}}{\sqrt{1+w_m}},0,0)$ with $w_k > w_m$ and $w_m > -1$	Can't decide

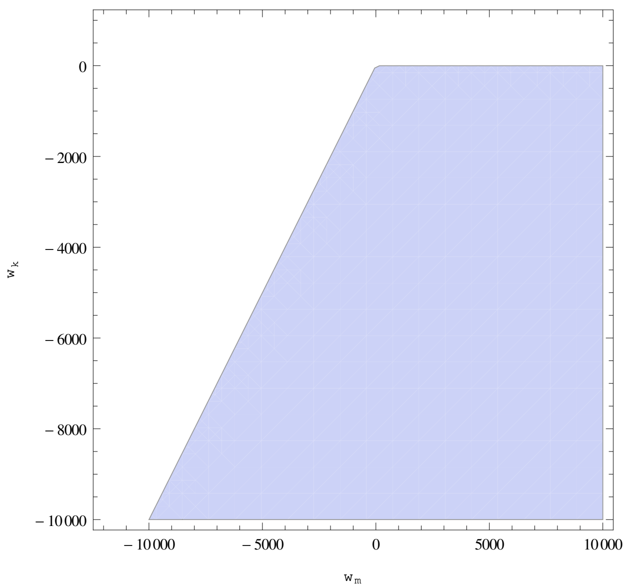

Figure 4. Allowed region of parameter space for the fixed point $(1,0,0,0)$ in open universe.

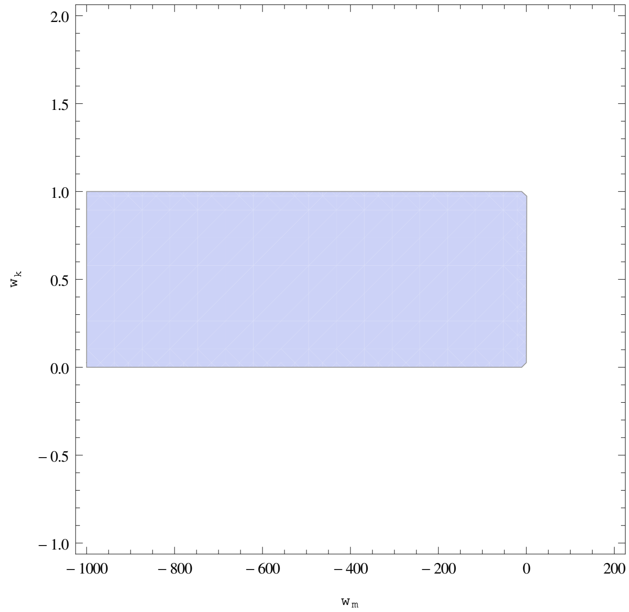

Figure 5. Allowed region of parameter space for the fixed point $(-1,0,0,0)$ in open universe.

3.2.2. Case II, $sign(\rho_k) = -ve$

In this section, we state the results of stability analysis of our dynamical variables for the negative sign of kinetic energy density. The fixed points are found to be $(0,0,0,0)$, $(0,0,-\sqrt{\frac{3(-w_k+w_m)}{1+3w_m}},0)$ with $w_m > w_k$ and $w_m > -\frac{1}{3}$, and $(0,\sqrt{\frac{-w_k+w_m}{1+w_m}},0,0)$ with $w_m > w_k$ and $w_m > -1$ with eigen values $(0,0,0,0)$, $(0,0,-\sqrt{\frac{3}{2}}\sqrt{-w_k-3w_k^2+w_m+3w_kw_m}, \sqrt{\frac{3}{2}}\sqrt{-w_k-3w_k^2+w_m+3w_kw_m})$ and $(0,0,-\frac{3\sqrt{-w_k-w_k^2+w_m+w_kw_m}}{\sqrt{2}},\frac{3\sqrt{-w_k-w_k^2+w_m+w_kw_m}}{\sqrt{2}})$ respectively. All these fixed points are nonhyperbolic and tabulated in Table 4.

Table 4. Stablity Analysis of fixed points for open universe with $sign(\rho_k) = -ve$.

Fixed Points $(x_c, y_c, z_c, \sigma_c)$	**Stability Conditions**
$(0,0,0,0)$ for $w_k = w_m$	Can't decide
$(0,0,-\sqrt{\frac{3(-w_k+w_m)}{1+3w_m}},0)$ with $w_m > w_k$ and $w_m > -\frac{1}{3}$	Can't decide
$(0,\sqrt{\frac{-w_k+w_m}{1+w_m}},0,0)$ with $w_m > w_k$ and $w_m > -1$	Can't decide

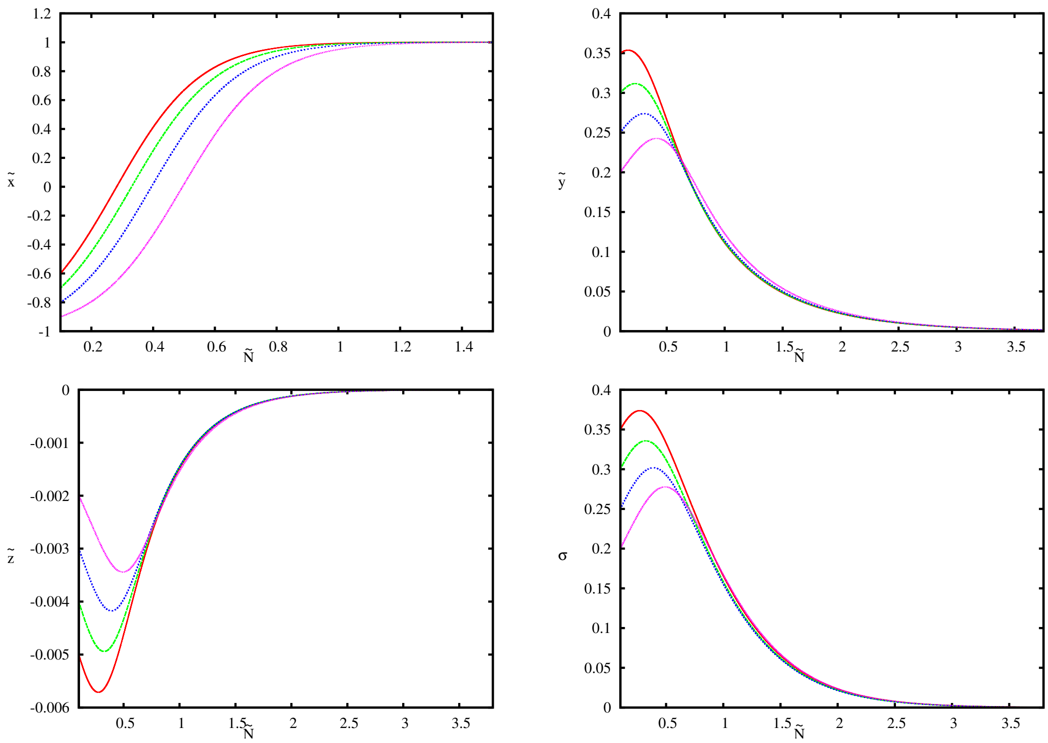

Figure 6. Evolution of the dynamical variables $\tilde{x}$ (**top left**), $\tilde{y}$ (**top right**), $\tilde{z}$ (**bottom left**) and σ (**bottom right**) for the fixed point $(\tilde{x}_c, \tilde{y}_c, \tilde{z}_c, \sigma_c) = (1, 0, 0, 0)$ with the values of parameters $sign(\tilde{z}) = -ve$, $w_k = -2.0$, $w_m = 1/3$ and $sign(\rho_k) = +ve$ for different initial conditions.

4. Bouncing Scenario

Now we obtain the conditions for non singular bounce to occur and also show the evolution of dynamical variables numerically. A nonsingular bounce is attained whenever the universe passes from a contracting phase to an expanding phase through a minimum value of the average scale factor $a(t)$, but not zero. Mathematically, it satisfies

$$(H)_b \equiv \frac{1}{a_b(t)} \left(\frac{\mathrm{d}a(t)}{\mathrm{d}t} \right)_b = 0, \tag{18}$$

where subscript b denotes value of the variable at the bounce, and

$$\left(\frac{\mathrm{d}^2 a(t)}{\mathrm{d}t^2} \right)_b > 0 \tag{19}$$

for minimum to occur. This implies

$$\left(\frac{\mathrm{d}H}{\mathrm{d}t} \right)_b = \left(\frac{\ddot{a}}{a} \right)_b - \left(\frac{\dot{a}}{a} \right)_b^2 > 0 \tag{20}$$

Now, writing the above conditions in terms of dynamical variables for bouncing, we get $\tilde{x}_b = 0$ and $\left(\frac{\mathrm{d}\tilde{x}}{\mathrm{d}\tilde{N}} \right)_b > 0$ which translate to the following equation

$$\left(\frac{\mathrm{d}\tilde{x}}{\mathrm{d}\tilde{N}} \right)_b = -\frac{3}{2} \left[(w_k - w_m)(sign\rho_k) + (1 + w_m)(-\tilde{y}|\tilde{y}|) + \frac{(1 + 3w_m)}{3} \tilde{z}|\tilde{z}| \right] > 0. \tag{21}$$

This implies

$$\Big(\tilde{y}|\tilde{y}|(1 + w_m) - \tilde{z}|\tilde{z}|(1 + 3w_m) \Big)_b > 1 \times sign(\rho_k)(w_k - w_m). \tag{22}$$

At the bounce we then obtain the constraint equation among dynamical variable as

$$\left(\tilde{x}^2 - \tilde{y}|\tilde{y}| + \tilde{z}|\tilde{z}| - \tilde{\Omega}_m\right)_b = -\tilde{y}|\tilde{y}| + \tilde{z}|\tilde{z}| - \tilde{\Omega}_m = 1 \times sign(\rho_k). \tag{23}$$

Now, for different negative initial conditions of $\tilde{x}$ (contracting phase), Figure 3 (top left) and Figure 6 (top left) for closed and open universe respectively, show its transition to positive values (expanding phase) crossing zero (bounce). The bouncing is guaranteed by the positivity of the slope of $\tilde{x}$ as shown in Figure 7 (left plot for closed and right plot for open). Thus, top left of Figures 3, 6 and 7, together, do indeed represent stable bouncing scenario in FRW closed and open universe. This is obtained by setting the values of equation of state parameters $w_k = -2$ ($\eta = 1/4$), $w_m = 1/3$ and $sign(\rho_k) = +ve$ and $sign(y) = +ve$. The evolution of other dynamical variables can be seen in Figures 3 and 6, which show their asymptotic evolution to the respective fixed points for the same choice of parameters.

It can be seen that the fixed point $(\tilde{x}_c, \tilde{y}_c, \tilde{z}_c, \sigma_c) = (1, 0, 0, 0)$ does give rise to a stable bouncing universe as it satisfies Equations (22) and (23) for open ($sign(\tilde{z})$=-ve) and closed ($sign(\tilde{z})$=+ve) universe. From this analysis, we conclude that finally after the bounce our universe at late times is driven by kinetic energy density in both the cases. The other fixed point $(-1, 0, 0, 0)$, though stable, can not give rise to a bouncing scenario as it ends up with a negative value of Hubble parameter, H, signifying a late time contracting phase.

Also, we show the behaviour of curvature parameter, $\tilde{z}$, in this nonsingular bouncing set up. The curvature parameter increases initially in the contracting phase reaching an extremum at the bounce and then decreases to zero in the expanding phase as shwon in Figure 8 for both open and closed universe. Thus, the curvature parameter remains finite at the bounce as expected in a nonsingular bouncing scenario and at late time universe becomes flat irrespective of whether we start initially with closed or open.This may be useful for building realistic models.

The comparison between bouncing solutions for open and closed is done in Figure 9. It has been found that bouncing occurs earlier in the case of open than in the closed universe as shown in left hand side of Figure 9 for the same set of initial conditions and parameters. Also, it is noted that, though, the solutions differ appreciably near the bounce, they approach to the same value at late time owing to zero value of the curvature parameter. The nonsingular bounce happens only for negative values of $\tilde{\Omega}_m$ with our choice of parameters as shown in Figure 9 (right) for both open and closed universe.

Finally we show the effect of different values of η on the behavior of bouncing solutions in Figure 10 for both closed and open universe. All the plots are generated for the same set of initial conditions and the same set of parameters $w_m = 1/3$, $sign(\rho_k) = +ve$ but with three different values of parameters $\eta = 1/4, 1/6$ and $1/8$. It has been observed that the value of η has a direct impact on the occurence of bouncing point. Indeed, the position of bouncing point is delayed as we decrease the value of η for both closed and open universe as shown in Figure 10 (top left and top right). The bottom left and bottom right of Figure 10 indicate the effect of η on the curvature parameter for both closed and open cases respectively. It has been found that the magnitude of maximum value of $\tilde{z}$, at the bounce, decreases as we decreases the value of η.

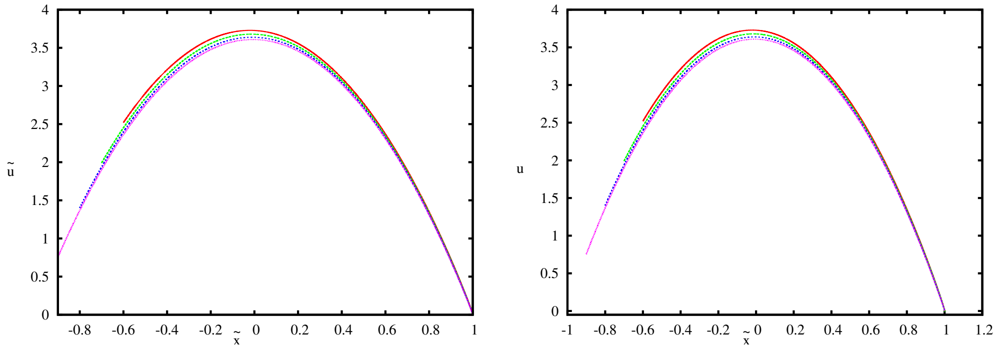

Figure 7. $u \equiv \frac{d\tilde{x}}{d\tilde{N}}$ vs $\tilde{x}$ for closed (**left**) and for open (**right**) with $w_k = -2.0$, $w_m = 1/3$ and $sign(\rho_k) = +ve$.

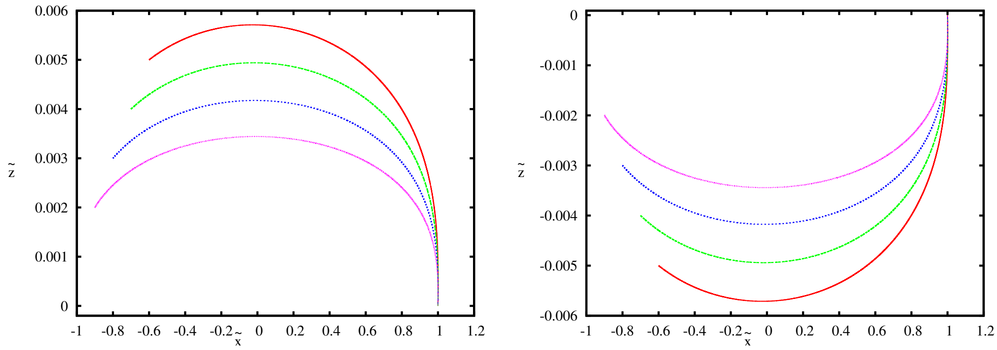

Figure 8. $\tilde{z}$ vs $\tilde{x}$ for closed (**left**) and for open (**right**) with $w_k = -2.0$, $w_m = 1/3$ and $sign(\rho_k) = +ve$.

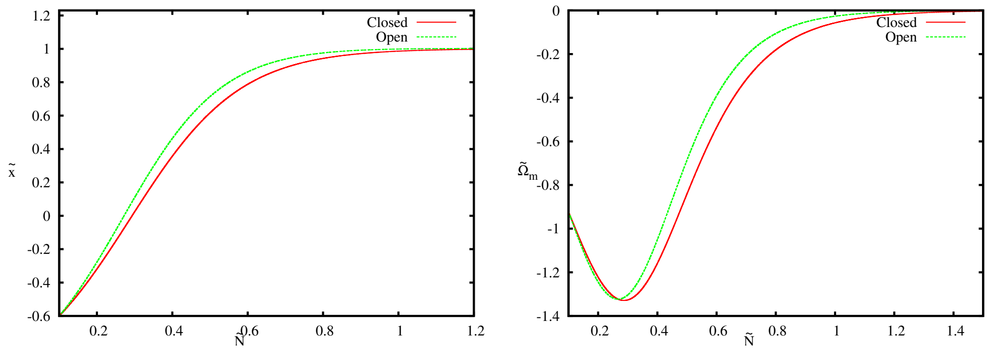

Figure 9. Comparision between closed and open for $\tilde{x}$ vs $\tilde{N}$ (**left**) and $\tilde{\Omega}_m$ vs $\tilde{N}$ (**right**) with $w_k = -2.0$, $w_m = 1/3$ and $sign(\rho_k) = +ve$.

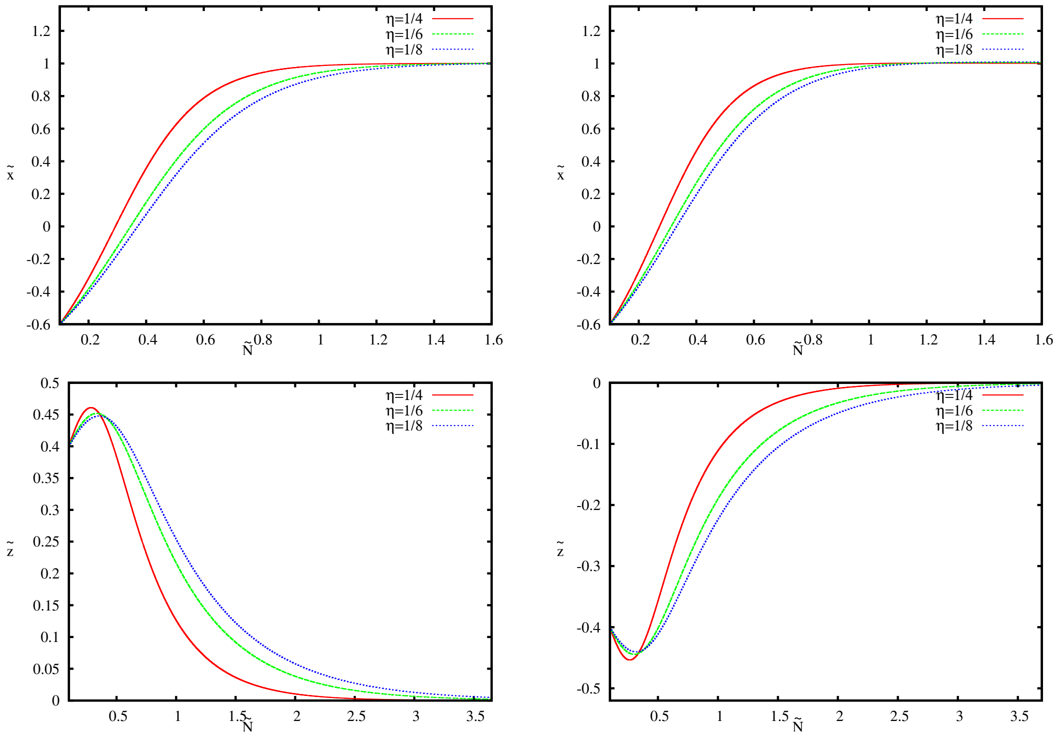

Figure 10. Evolution $\tilde{x}$ for closed (**top left**) and open (**top right**); $\tilde{z}$ for closed (**bottom left**) and open (**bottom right**) with values $\eta = 1/4$,1/6, and $1/8$, $w_m = 1/3$ and $sign(\rho_k) = +ve$.

5. Conclusions

A cosmological scenario with a noncanonical scalar field and matter is explored in this work. Using dynamical equations for a set of dimensionless dynamical variables, we find all the fixed point for the two cases with positive and negative kinetic energy density term in FRW closed and open universe. Allowed region of parameter spaces for stability of fixed points are shown for both cases. The necessary and sufficient conditions for a nonsingular bounce are obtained in terms of the dynamical variables. Thus, stable bouncing solutions are obtained satisfying nonsingular bouncing conditions and stability criteria. This is achieved for the negative energy density of matter, $\tilde{\Omega}_m$, with equation of state parameter $w_m = 1/3$ in both closed and open universe. In addition to this, the finitude of curvature parameter at the bounce is obtained as expected in a nonsingular bouncing scenario and universe becomes flat at late time irrespective of whether we start with a closed or open one. Finally, the effect of the parameter η on the behaviour of bouncing solution is noted. It is seen that the point of occurence of bounce is delayed as we decrease the value of η and the magnitude of value of curvature parameter at the bounce decreases with η for both open and closed universe.

We restrict our analysis to a positive sign of potential. It is straightforward to extend our analysis for a negative potential by changing the parameter $sign(y)$ to -1.

As mentioned above this work is in the classical regime. And it is obvious that there are two ways to solve the initial singularity: Either by invoking modification in the matter sector or by modifying the gravity sector which includes modified theories of gravity. However Non of these modifications deal with the physics at the Planck region. In the popularly known inflationary paradigm which predicts the formation of structure, still treats the spacetime to be classical and obviously this can not be correct at the high curvature limit.

This is interesting and a matter of paramount importance to include quantum effects. In the next seires of work we propose to formulate the present model in the framework of loop quantum cosmology. The loop quantum cosmology in the case of minimally coupled scalar field is well studied. Our future work would involve in studying the effective dynamics due non-trivial Lagrangian like the present case. This would be followed by a detailed dynamical system analysis of the phase space.

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