

String-Inspired Correction to R^2 Inflation [†]

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Abstract: We study the Starobinsky-Bel-Robinson inflationary model in the slow-roll regime. In the framework of higher-curvature corrections to inflationary parameters, we estimate maximal possible value of dimensionless positive coupling constant β coming from M-theory.

Keywords: Starobinsky's scalaron; Starobinsky-Bel-Robinson modified gravity; inflation

1. Introduction

The Starobinsky-Bel-Robinson (SBR) modified gravity was proposed in [1,2] as an extension of the non-perturbative $(R + R^2)$ gravity [3] by the perturbative quantum correction inspired by M-theory. This quantum correction is given by the Bel-Robinson tensor squared [4,5].

As a result the four-dimensional cosmological SBR action can be presented in the form:

$$S_{SBR} = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g} \left[R + \frac{R^2}{6m^2} + \frac{\beta}{32m^6} (\mathcal{G}^2 - P_4^2) \right], \quad (1)$$

where

$$\mathcal{G} = {}^*R_{\mu\nu\lambda\rho} {}^*R^{\mu\nu\lambda\rho} = R^2 - 4R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}, \quad P_4 = {}^*R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}, \quad (2)$$

$${}^*R_{\mu\nu\lambda\rho} = \frac{1}{2}E_{\mu\nu\alpha\beta}R^{\alpha\beta}{}_{\lambda\rho}, \quad E_{\mu\nu\lambda\rho} = \sqrt{-g}\epsilon_{\mu\nu\lambda\rho}, \quad (3)$$

β is a dimensionless positive coupling constant. The Starobinsky-Bel-Robinson modified gravity is the model with higher-curvature terms. In [2], we obtained the equations of motion in the Starobinsky-Bel-Robinson model. We considered solutions to the model, their series expansions and slow-roll regime, inflationary parameters. Now we consider in detail how the expressions to inflationary parameters with the influence of higher-curvature corrections were obtained.

2. Starobinsky-Bel-Robinson Modified Gravity in the Friedmann Universe

The application of linearization procedure and variation principle leads to the system of equations of motions. In the spatially flat Friedmann Universe

$$ds^2 = -dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2),$$



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the P_4 term does *not* contribute to the equations of motion, that have the following form

$$\begin{aligned} & \left(R_{\rho\nu} - \frac{g_{\rho\nu}}{2}R\right)\left(1 + \frac{R}{3m^2}\right) + \frac{R^2}{12m^2}g_{\rho\nu} - \frac{1}{3m^2}\left(\frac{\nabla_\rho\nabla_\nu + \nabla_\nu\nabla_\rho}{2} - g_{\rho\nu}\square\right)R \\ & + \frac{\beta}{64m^6}\left[\mathcal{G}^2g_{\rho\nu} + 8\{(Rg_{\rho\nu} - 2R_{\rho\nu})\square\mathcal{G} - R\nabla_\rho\nabla_\nu\mathcal{G}\right. \\ & \left. + 2(R_\nu^\alpha\nabla_\alpha\nabla_\rho\mathcal{G} + R_\rho^\alpha\nabla_\alpha\nabla_\nu\mathcal{G}) - 2(g_{\rho\nu}R_{\alpha\beta} + R_{\alpha\rho\nu\beta})\nabla^\beta\nabla^\alpha\mathcal{G}\right] = 0. \end{aligned} \tag{4}$$

We consider the (0,0)-component of the system (4)

$$3H^2\left(1 + \frac{R}{3m^2}\right) - \frac{R^2}{12m^2} + \frac{H\dot{R}}{m^2} - \frac{\beta}{64m^6}\left[\mathcal{G}^2 - 48H^3\dot{\mathcal{G}}\right] = 0 \tag{5}$$

and the trace equation

$$R + \frac{1}{m^2}(\ddot{R} + 3H\dot{R}) - \frac{\beta}{16m^6}\left[\mathcal{G}^2 - 12(H^2\ddot{\mathcal{G}} + 2H\dot{H}\dot{\mathcal{G}} + 3H^3\dot{\mathcal{G}})\right] = 0, \tag{6}$$

where $H = \dot{a}/a$, the dot denotes the time derivative. Equation (6) can be obtained by summing of the first derivative of Equation (5) multiplied to H^{-1} and Equation (5) multiplied to 4. Therefore Equation (6) is a consequence of Equation (5). We rewrite Equation (5) in terms of the Hubble function $H(t)$ and its time derivatives

$$2(m^4 + 3\beta H^4)H\ddot{H} - (m^4 - 9\beta H^4)\dot{H}^2 + 6(m^4 + 3\beta H^4)H^2\dot{H} - 3\beta H^8 + m^6H^2 = 0. \tag{7}$$

using the relations

$$\dot{R} = 24\dot{H}H + 6\ddot{H}, \quad \dot{\mathcal{G}} = 48H(H^2 + \dot{H})\dot{H} + 24H^2(2H\dot{H} + \ddot{H}). \tag{8}$$

We apply the slow-roll approximation $|\dot{H}| \ll |H\dot{H}|$ and $|\ddot{H}| \ll H^2$ to Equation (7):

$$6(m^4 + 3\beta H^4)\dot{H} - 3\beta H^6 + m^6 = 0 \tag{9}$$

and get the solution up to first order corrections

$$H(t) \approx \frac{m^2(t_0 - t)}{6} - \beta\left(\frac{m}{6}\right)^6(t_0 - t)^5\left[\frac{m^2}{14}(t_0 - t)^2 + \frac{18}{5}\right] + \mathcal{O}(\beta^2). \tag{10}$$

3. Inflationary Parameters

In [6], the approach for studying slow-roll inflation in the models with higher-curvature terms has been proposed. In the Friedmann Universe, the equations of motions can be presented such as:

$$F(H^2) = 2\alpha l^2(\psi(\psi - H^2) - H\dot{\psi}), \quad 12\psi = R \tag{11}$$

$$\dot{H}F'(H^2) = -\alpha l^2(\ddot{\psi} - H\dot{\psi} + 2\dot{H}\psi) \tag{12}$$

where α, l are the model constants, $\alpha l^2 = 2/m^2$. Equation (12) is direct time differential of Equation (11). The function $F(H^2)$ has the following polynomial structure

$$F(H^2) = H^2 + l^{-2} \sum_{n=3}^{\infty} (-1)^n \lambda_n (l \cdot H)^{2n} \tag{13}$$

In the case of slow-roll regime, the inflationary parameters can be determined by the function $F(H^2)$.

Equation (5) is equivalent to (11) with

$$F(H^2) = H^2 - \frac{\beta}{192m^6} (48H^3\dot{\mathcal{G}} - \mathcal{G}^2). \tag{14}$$

In the slow-roll regime, the expression (14) can be simplified

$$48H^3\dot{\mathcal{G}} - \mathcal{G}^2 \approx 24^2H^6(6\dot{H} - H^2).$$

With help of solution (10) we get $\beta\dot{H} \approx -\beta m^2/6$ and rewrite Equation (5) such as:

$$R\left(\frac{R}{12} - H^2\right) - H\dot{R} = 3m^2F(H^2), \quad F(H^2) = \left(H^2 - \frac{3\beta H^8}{m^6} + \frac{18\beta H^6\dot{H}}{m^6}\right). \tag{15}$$

To apply comparative analysis we rewrite the function $F(H^2)$ in the form

$$F(H^2) = H^2 - \lambda_3(l^2)^2(H^2)^3 + \lambda_4(l^2)^3(H^2)^4 \tag{16}$$

where

$$\lambda_3 = \frac{3\beta\alpha^2}{4}, \quad \lambda_4 = -\frac{3\beta\alpha^2}{8} \tag{17}$$

In [6], the expressions for the spectral index n_s and the tensor-to-scalar ratio r were obtained using coefficients λ_3 and λ_4 as follows

$$n_s = 1 - \frac{2}{N} - \frac{32\lambda_3N}{27\alpha^2} + \frac{4\lambda_4N^2}{3\alpha^3}, \quad r = \frac{12}{N^2} - \frac{16\beta}{9\alpha^2} + \frac{16\lambda_4N}{3\alpha^3}. \tag{18}$$

Thus, Equation (17) leads to the inflationary parameters:

$$n_s = 1 - \frac{2}{N} - \frac{8\beta N}{9} - \frac{\beta N^2}{2}, \quad r = \frac{12}{N^2} - \frac{16}{3}\beta - 2\beta N \tag{19}$$

The numerical estimation of spectral index [2] leads to the following interval for value of β :

$$0 \leq \beta \leq 3.9 \cdot 10^{-6} \tag{20}$$

If β is from the interval (20), then values of the tensor-to-scalar ratio and the amplitude of scalar perturbations do not contradict modern observations [7,8].

4. Conclusions

The SBR modified gravity was suggested as part of the gravitational low-energy effective action in four space-time dimensions, originating from non-perturbative superstring theory (or M-theory) in higher spacetime dimensions, in the presence of the mass terms for dilaton and axion when their kinetic terms are ignored [1,2]. In [2], we study different solutions of the model using series representations. The attractor solutions are most interesting in studying of slow-roll inflationary scenarios. In [2], we analyzed inflationary scenario in the Starobinsky-Bel-Robinson modified gravity. Here we wrote just some details applied to get formulas of inflationary parameters in the frameworks of gravity models with higher-curvature terms [6].

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