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On Extensions of Starobinsky Model of Inflation †

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Abstract: We propose inflationary models that are one-parametric generalizations of the Starobinsky $R + R^2$ model. Using the conformal transformation, we obtain scalar field potentials in the Einstein frame that are one-parametric generalizations of the potential for the Starobinsky inflationary model. We restrict the form of the potentials by demanding that the corresponding function $F(R)$ is an elementary function. We obtain the inflationary parameters of the models proposed and show that the predictions of these models agree with current observational data.

Keywords: inflation; modified gravity; cosmology

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1. Introduction

Inflationary scenarios in the context of $F(R)$ gravity are actively studied [1–18].

The historically first $F(R)$ gravity inflationary model is the purely geometric $R + R^2$ model [1], described by the following action:

$$S_{\mathrm{Star.}}[g^J_{\mu\nu}] = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g^J}\left(R_J + \frac{1}{6m^2}R_J^2\right), \quad (1)$$

where the Ricci scalar R_J, the reduced Planck mass M_{Pl} and the inflaton mass m are introduced.

The Starobinsky inflationary model (1) is in a good agreement to the Planck measurements of the Cosmic Microwave Background (CMB) radiation [19,20]. The inflaton mass m is fixed by CMB measurements of the amplitude of scalar perturbations A_s. Note that the values of the scalar spectral index n_s and the tensor-to-scalar ratio r do not depend on m. This model is just the simplest model of inflation that has the maximal predictive power. At the present time, the value of tensor-to-scalar ratio r is not known, and probably will be observed by future experiments. If the observed value of the tensor-to-scalar ratio r be different from its value in the Starobinsky model, some corrections of this model will be required.

There are two pure geometric ways to generalize the Starobinsky model without adding scalar fields or other matter. One can either add string theory inspired terms [21–30] or construct new $F(R)$ gravity inflationary models, connected to the Starobinsky model [2–7,9,11,13–15,18]. Also, it is possible to combine these two ways, obtaining $f(R)$

models related to fundamental theories of gravity. For example, the supergravity models in some approximation can be considered as the $F(R)$ gravity models [21–24].

Adding of R^n term to the Starobinsky model with $n > 2$ does not allow to construct realistic inflationary models, because inflation demands a fine-tuning of initial values [2,14,15]. The talk is based on our paper [15], where a few new one-parameter generalizations of the Starobinsky $R + R^2$ inflationary model have been considered. In particular, it has been shown that the adding of $R^{3/2}$ term allows to construct viable inflationary model with the tensor-to-scalar ratio r in four times more than in the original Starobinsky model. At the same time, models with the $R^{3/2}$ term are ill-defined at $R \leqslant 0$, whereas the Starobinsky model is well defined and has no ghost for all $R > -3m^2$. Inflationary models proposed in [18] includes $(R + R_0)^{3/2}$ term, where $R_0 > 0$, they has all useful properties of the models with the $R^{3/2}$ term and well-defined for some negative R. We also proposed a new way of generalization of the Starobinsky model, based on suitable one-parametric generalizations of the corresponding scalar field potentials in the Einstein frame.

2. $F(R)$ Models and the Corresponding Scalar Potentials

The generic $F(R)$ gravity theories have the following action

$$S_F[g^J_{\mu\nu}] = \int d^4x\sqrt{-g^J}F(R_J) \tag{2}$$

with a differentiable function F.

To avoid graviton as a ghost and scalaron (inflaton) as a tachyon one should put the following conditions [31,32]:

$$\frac{dF}{dR_J} > 0 \quad \text{and} \quad \frac{d^2F}{dR_J^2} > 0\,, \tag{3}$$

that restrict possible values of parameters and R_J. In the Starobinsky model, the first condition in (3) is equivalent to $R_J > -3m^2$.

For any nonlinear function $F(R_J)$, action (2) can be rewritten as

$$S_J = \int d^4x\sqrt{-g^J}\left[F_{,\sigma}(R_J - \sigma) + F\right], \tag{4}$$

where a new scalar field σ has been introduced, and $F_{,\sigma} \equiv \frac{dF(\sigma)}{d\sigma}$. If $F_{,\sigma\sigma}(\sigma) \neq 0$, then eliminating σ via equation $R_J = \sigma$, one yields back action (2).

If $F_{,\sigma} > 0$, then the Weyl transformation of the metric $g_{\mu\nu} = \frac{2F_{,\sigma}(\sigma)}{M_{Pl}^2}g^J_{\mu\nu}$ allows one to get the following action in the Einstein frame [33]:

$$S_E = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - \frac{h(\sigma)}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_E(\sigma)\right], \tag{5}$$

where

$$h(\sigma) = \frac{3M_{Pl}^2}{2F_{,\sigma}^2}F_{,\sigma\sigma}^2 \quad \text{and} \quad V_E(\sigma) = M_{Pl}^4\frac{F_{,\sigma}\sigma - F}{4F_{,\sigma}^2}\,. \tag{6}$$

It is easy to see that the following field transformation

$$\phi = \sqrt{\frac{3}{2}}M_{Pl}\ln\left[\frac{2}{M_{Pl}^2}F_{,\sigma}\right]. \tag{7}$$

gives the action S_E with the standard scalar field ϕ:

$$S_E = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_E(\sigma(\phi))\right]. \tag{8}$$

It has been shown in [15], that the non-canonical dimensionless field

$$y \equiv \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_{Pl}}\right) = \frac{M_{Pl}^2}{2F_{,\sigma}} \tag{9}$$

is useful for considering of generalization of the Starobinsky inflationary model, because it is small during inflation.

For the Starobinsky model, the potential is

$$V_{\text{Star.}}(y) = V_0(1-y)^2\,, \quad \text{where} \quad V_0 = \frac{3}{4}m^2M_{Pl}^2\ . \tag{10}$$

If $V(y)$ is given, then the corresponding function $F(R_J)$ can be found in the parametric form (see [15] for detail):

$$R_J(y) = \frac{2}{M_{Pl}^2}\left(2\frac{V}{y} - V_{,y}\right), \qquad F(y) = \frac{V}{y^2} - \frac{V_{,y}}{y}\,. \tag{11}$$

Equation (9) can be obtained as a consequence of Eqs. (11). The first stability condition $F_{,\sigma} > 0$ is equivalent to $y > 0$. Using Eqs. (9) and (11), we get the second stability condition in terms of y:

$$F_{,\sigma\sigma} = \frac{M_{Pl}^4}{4\left(2yV_{,y} - 2V - y^2V_{,yy}\right)} > 0. \tag{12}$$

3. One-Parametric Generalizations of $V_{\text{Star.}}(y)$ and the Corresponding $F(R)$ Models

In Ref. [15], we proposed a new method for construction of $F(R)$ inflationary models based on the use of the potential $V(y)$.

The field y is small during slow-roll inflation, so potential $V(y)$ can be written as follows

$$V(y) = V_0\left[1 - 2y + \mathcal{O}\left(y^2\right)\right]\,, \tag{13}$$

where only the first two terms are essential for the CMB observables.

The potential

$$V(y) = V_0\left[1 - 2y + y^2\omega(y)\right], \tag{14}$$

with an arbitrary analytic function $\omega(y)$ that order 1 does not essential change the inflationary parameters predicted by the Starobinsky model. The Starobinsky model appears at $\omega(y) = 1$.

Equations (11) that defined $F(R_J)$ are given by

$$R_J = 3m^2\left(\frac{1}{y} - 1 - \frac{1}{2}y^2\frac{d\omega}{dy}\right)\ , \tag{15}$$

$$F = V_0\left(\frac{1}{y^2} - \omega - y\frac{d\omega}{dy}\right)\,. \tag{16}$$

If ω is an arbitrary constant, then

$$F(R_J) = F_{Star.}(R_J) - \Lambda\,, \tag{17}$$

where $\Lambda = V_0(1-\omega)$ is a cosmological constant.

A new model corresponds to

$$\omega(y) = \omega_0 + \omega_1 y\,, \tag{18}$$

where $\omega_0 \leqslant 1$ and $\omega_1 > 0$ are constants. The constant ω_1 should be positive for the potential V bounded from below. The inequality $\omega_0 \leqslant 1$ is needed for positivity of a cosmological constant, see Equation (17).

Equation (15) leads to the depressed cubic equation

$$y^3 + \frac{2}{\omega_1}\left(1 + \frac{R_J}{3m^2}\right)y - \frac{2}{\omega_1} = 0\,. \tag{19}$$

Equation (19) has a negative discriminant and, therefore, only one real root.

Equation (16) yields the following explicit $F(R_J)$ function:

$$\begin{aligned}
\frac{F}{V_0} &= \frac{1}{y^2} - \omega_0 - 2\omega_1 y \\
&= \omega_1^{2/3}\left[\left(1 + \sqrt{1 + \frac{8}{27\omega_1}\left(1 + \frac{R_J}{3m^2}\right)^3}\right)^{1/3} + \left(1 - \sqrt{1 + \frac{8}{27\omega_1}\left(1 + \frac{R_J}{3m^2}\right)^3}\right)^{1/3}\right]^{-2} \\
&- 2\omega_1^{2/3}\left[\left(1 + \sqrt{1 + \frac{8}{27\omega_1}\left(1 + \frac{R_J}{3m^2}\right)^3}\right)^{1/3} + \left(1 - \sqrt{1 + \frac{8}{27\omega_1}\left(1 + \frac{R_J}{3m^2}\right)^3}\right)^{1/3}\right] \\
&- \omega_1^{2/3}\left[\left(1 + \sqrt{1 + \frac{8}{27\omega_1}}\right)^{1/3} + \left(1 - \sqrt{1 + \frac{8}{27\omega_1}}\right)^{1/3}\right]^{-2} \\
&+ 2\omega_1^{2/3}\left[\left(1 + \sqrt{1 + \frac{8}{27\omega_1}}\right)^{1/3} + \left(1 - \sqrt{1 + \frac{8}{27\omega_1}}\right)^{1/3}\right].
\end{aligned}$$

The limit $\omega_1 \to 0$ is smooth and gives back the Starobinsky model (1).

From Eq. (12), we get that

$$F_{,\sigma\sigma}(y) = \frac{M_{Pl}^2}{6m^2(1 + \omega_1 y^3)}\,, \tag{20}$$

so, the considered $F(R)$ gravity model satisfies the conditions (3) at $\omega_1 \geqslant 0$.

Our approach suggests another one-parametric deformation of the Starobinsky potential (10):

$$V(y) = V_0\left(1 - y - \zeta y^2\right)^2, \tag{21}$$

where the parameter $\zeta \geqslant 0$. This potential can be realized in supergravity [34], while the potential (10) is recovered at $\zeta = 0$. This possibility has been investigated in Ref. [15] as well.

4. Conclusions

The accelerated expansion of the early universe, inflation, has been described in $F(R)$ gravity. The Starobinsky model of inflation [1], which was proposed more than 40 years ago, is in good agreement with the current observational data of the cosmic microwave background radiation [19,20]. New $F(R)$ inflationary models can be constructed as expansions of the Starobinsky model that smoothly connected to it. Such models will be in good agreement with the current observational data if the additional parameter is small enough (for the Starobinsky model its value is equal to zero).

The main purpose of our paper [15] was to explore inflationary models that are one-parametric generalization of the Starobinsky model and can be regarded as $F(R)$ models. We investigate two ways to modify the Starobinsky model. We can either initially modify $F(R)$ function or the corresponding scalar field potential $V_E(\phi)$. When we use the second way it is suitable to rewrite the potential in terms of dimensionless variable y that is small during inflation.

One of restrictions of the inflaton scalar potential is the condition the corresponding $F(R)$ should be an elementary function. More restrictions of the potential arise when one demands their minimal embedding into supergravity [22,24,35]. For instance, the R^3 term is excluded in supergravity, whereas $R^{3/2}$ and $(R+R_0)^{3/2}$ terms arise in certain versions of the chiral $F(\mathcal{R})$ supergravity [22]. The potential (21) is extendable in the minimal supergravity framework that requires the scalar potential to be a real function squared.

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References

1. Starobinsky, A.A. A new type of isotropic cosmological models without singularity. *Phys. Lett. B* **1980**, *91*, 99–102. https://doi.org/10.1016/0370-2693(80)90670-X.
2. Barrow, J.D.; Cotsakis, S. Inflation and the Conformal Structure of Higher Order Gravity Theories. *Phys. Lett. B* **1988**, *214*, 515–518. https://doi.org/10.1016/0370-2693(88)90110-4.
3. Berkin, A.L.; Maeda, K.i. Effects of R**3 and R box R terms on R**2 inflation. *Phys. Lett. B* **1990**, *245*, 348–354. https://doi.org/10.1016/0370-2693(90)90657-R.
4. Saidov, T.; Zhuk, A. Bouncing inflation in nonlinear R^2+R^4 gravitational model. *Phys. Rev. D* **2010**, *81*, 124002, https://doi.org/10.1103/PhysRevD.81.124002.
5. Huang, Q.G. A polynomial f(R) inflation model. *JCAP* **2014**, *02*, 035. https://doi.org/10.1088/1475-7516/2014/02/035.
6. Sebastiani, L.; Cognola, G.; Myrzakulov, R.; Odintsov, S.D.; Zerbini, S. Nearly Starobinsky inflation from modified gravity. *Phys. Rev. D* **2014**, *89*, 023518. https://doi.org/10.1103/PhysRevD.89.023518.
7. Motohashi, H. Consistency relation for R^p inflation. *Phys. Rev. D* **2015**, *91*, 064016. https://doi.org/10.1103/PhysRevD.91.064016.
8. Broy, B.J.; Pedro, F.G.; Westphal, A. Disentangling the $f(R)$—Duality. *JCAP* **2015**, *03*, 029. https://doi.org/10.1088/1475-7516/2015/03/029.
9. Bamba, K.; Odintsov, S.D. Inflationary cosmology in modified gravity theories. *Symmetry* **2015**, *7*, 220–240. https://doi.org/10.3390/sym7010220.
10. Odintsov, S.D.; Oikonomou, V.K. Unimodular Mimetic $F(R)$ Inflation. *Astrophys. Space Sci.* **2016**, *361*, 236. https://doi.org/10.1007/s10509-016-2826-9.
11. Miranda, T.; Fabris, J.C.; Piattella, O.F. Reconstructing a $f(R)$ theory from the α-Attractors. *JCAP* **2017**, *09*, 041. https://doi.org/10.1088/1475-7516/2017/09/041.
12. Motohashi, H.; Starobinsky, A.A. $f(R)$ constant-roll inflation. *Eur. Phys. J. C* **2017**, *77*, 538. https://doi.org/10.1140/epjc/s10052-017-5109-x.
13. Cheong, D.Y.; Lee, H.M.; Park, S.C. Beyond the Starobinsky model for inflation. *Phys. Lett. B* **2020**, *805*, 135453. https://doi.org/10.1016/j.physletb.2020.135453.
14. Rodrigues-da Silva, G.; Bezerra-Sobrinho, J.; Medeiros, L.G. Higher-order extension of Starobinsky inflation: Initial conditions, slow-roll regime, and reheating phase. *Phys. Rev. D* **2022**, *105*, 063504. https://doi.org/10.1103/PhysRevD.105.063504.
15. Ivanov, V.R.; Ketov, S.V.; Pozdeeva, E.O.; Vernov, S.Y. Analytic extensions of Starobinsky model of inflation. *JCAP* **2022**, *03*, 058. https://doi.org/10.1088/1475-7516/2022/03/058.
16. Odintsov, S.D.; Oikonomou, V.K. Running of the spectral index and inflationary dynamics of F(R) gravity. *Phys. Lett. B* **2022**, *833*, 137353. https://doi.org/10.1016/j.physletb.2022.137353.
17. Modak, T.; Röver, L.; Schäfer, B.M.; Schosser, B.; Plehn, T. Cornering Extended Starobinsky Inflation with CMB and SKA. *arXiv* **2022**, arXiv:astro-ph.CO/2210.05698.
18. Pozdeeva, E.O.; Vernov, S.Y. $F(R)$ gravity inflationary model with $(R+R_0)^{3/2}$ term. *arXiv* **2022**, arXiv:gr-qc/2211.10988.
19. Akrami, Y.; Arroja, F.; Ashdown, M.; Aumont, J.; Baccigalupi, C.; Ballardini, M.; Banday, A.; Barreiro, R.; Bartolo, N.; Basak, S.; et al. Planck 2018 results. X. Constraints on inflation. *Astron. Astrophys.* **2020**, *641*, A10. https://doi.org/10.1051/0004-6361/201833887.
20. Ade, P.A.R.; Ahmed, Z.; Amiri, M.; Barkats, D.; Thakur, R.; Bischoff, C.A.; Beck, D.; Bock, J.J.; Boenish, H.; Bullock, E.; et al. Improved Constraints on Primordial Gravitational Waves using Planck, WMAP, and BICEP/Keck Observations through the 2018 Observing Season. *Phys. Rev. Lett.* **2021**, *127*, 151301. https://doi.org/10.1103/PhysRevLett.127.151301.
21. Ketov, S.V. Chaotic inflation in F(R) supergravity. *Phys. Lett. B* **2010**, *692*, 272–276. https://doi.org/10.1016/j.physletb.2010.07.045.
22. Ketov, S.V.; Starobinsky, A.A. Embedding $(R+R^2)$-Inflation into Supergravity. *Phys. Rev. D* **2011**, *83*, 063512. https://doi.org/10.1103/PhysRevD.83.063512.

23. Ketov, S.V.; Tsujikawa, S. Consistency of inflation and preheating in F(R) supergravity. *Phys. Rev. D* **2012**, *86*, 023529. https://doi.org/10.1103/PhysRevD.86.023529.
24. Farakos, F.; Kehagias, A.; Riotto, A. On the Starobinsky Model of Inflation from Supergravity. *Nucl. Phys. B* **2013**, *876*, 187–200. https://doi.org/10.1016/j.nuclphysb.2013.08.005.
25. Koshelev, A.S.; Modesto, L.; Rachwal, L.; Starobinsky, A.A. Occurrence of exact R^2 inflation in non-local UV-complete gravity. *JHEP* **2016**, *11*, 067. https://doi.org/10.1007/JHEP11(2016)067.
26. Koshelev, A.S.; Sravan Kumar, K.; Mazumdar, A.; Starobinsky, A.A. Non-Gaussianities and tensor-to-scalar ratio in non-local R^2-like inflation. *JHEP* **2020**, *06*, 152. https://doi.org/10.1007/JHEP06(2020)152.
27. Ketov, S.V. Starobinsky–Bel–Robinson Gravity. *Universe* **2022**, *8*, 351. https://doi.org/10.3390/universe8070351.
28. Koshelev, A.S.; Kumar, K.S.; Starobinsky, A.A. Generalized non-local R^2-like inflation. *arXiv* **2022**, arXiv:hep-th/2209.02515.
29. Rodrigues-da Silva, G.; Medeiros, L.G. Second-order corrections to Starobinsky inflation. *arXiv* **2022**, arXiv:arXiv:astro-ph.CO/2207.02103.
30. Ketov, S.V.; Pozdeeva, E.O.; Vernov, S.Y. On the superstring-inspired quantum correction to the Starobinsky model of inflation. *JCAP* **2022**, *12*, 032. https://doi.org/10.1088/1475-7516/2022/12/032.
31. Starobinsky, A.A. Disappearing cosmological constant in f(R) gravity. *JETP Lett.* **2007**, *86*, 157–163. https://doi.org/10.1134/S0021364007150027.
32. Appleby, S.A.; Battye, R.A.; Starobinsky, A.A. Curing singularities in cosmological evolution of F(R) gravity. *JCAP* **2010**, *06*, 005. https://doi.org/10.1088/1475-7516/2010/06/005.
33. Maeda, K.i. Towards the Einstein-Hilbert Action via Conformal Transformation. *Phys. Rev. D* **1989**, *39*, 3159. https://doi.org/10.1103/PhysRevD.39.3159.
34. Ketov, S.V. On the equivalence of Starobinsky and Higgs inflationary models in gravity and supergravity. *J. Phys. A* **2020**, *53*, 084001. https://doi.org/10.1088/1751-8121/ab6a33.
35. Ferrara, S.; Kallosh, R.; Linde, A.; Porrati, M. Minimal Supergravity Models of Inflation. *Phys. Rev. D* **2013**, *88*, 085038. https://doi.org/10.1103/PhysRevD.88.085038.