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Impact of Particle Creation in Rastall Gravity †

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Abstract: We investigated the Friedmann–Lemaitre–Robertson–Walker (FLRW) cosmological models within the framework of Rastall gravity incorporating particle creation. The modified field equations for Rastall gravity are derived and exact solutions are obtained under various types of scale factors. The qualitative behaviour of our solutions depends on the Rastall coupling parameter $\psi = k\lambda$. Following Akarsu et al. (2020), we have restricted the Rastall coupling parameter $\psi(k=1)$ to the range $-0.0001 < \psi < 0.0007$ at 68% CL from CMB+BAO data. Further, we have discussed the distinct physical behavior of the derived models in detail.

Keywords: FLRW metric; Rastall gravity; particle creation

1. Introduction

Researchers have always been curious for understanding the universe from the past to the future scientifically. Einstein presented the general theory of relativity and gained considerable attention due to its success in building cosmological models. Nowadays, modifications of Einstein's gravity or extensions of Einstein's general theory of gravity are being studied to solve some of the problems presented by Einstein's general relativity to study cosmology. The ΛCDM model seems to be sufficient to describe the current scenario of the universe, although there are some unresolved issues. One of the critical ingredients of Einstein's theory of relativity is the covariant conservation of the energy-momentum. A number of modified theories of gravity have been proposed in the last few decades such as $f(T)$ gravity [1], $f(Q)$ gravity [2], $f(Q,T)$ gravity [3], R^2 gravity [4], $f(G)$ gravity [5], $f(R,G)$ gravity [6], $f(R)$ gravity [7] and $f(R,T)$ gravity [8]. In the present study we are interested in Rastall's gravity theory. Rastall's gravity theory was developed in 1972. Modified theories of gravity may or may not satisfy the conservation law of energy-momentum [9]. Thus one of the possible ways of extending general relativity is through relaxing the conservation law. In curved space time, the conservation law may or may not hold. In response to this, Rastall [9] proposed that the covariant divergence of the energy-momentum tensor might not be vanishing, but should be determined by the curvature of space-time through a coupling parameter, so that general relativity can be recovered at zero coupling. Recenty, Moraes and Santos proposed [10] the Lagrangian formalism of Rastall gravity by a non-minimal coupling between geometry and matter fields. Shabani and Ziaie [11] have developed the Lagrangian formulation for Rastall theory under the influence of perfect fluid matter content and linear equation of state in the framework of $f(R,T)$ gravity.

In this study, our primary focus is on the study of particle creation in Rastall gravity. Particle creation remains one of the most important unsolved problems in cosmology. Several cosmologists have discussed this phenomenon and its effects on the evolution of the universe. Also, they developed a cosmological model to discuss the thermodynamical

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aspects of the universe by using the mechanism of particle creation. A detailed exploration of the thermodynamics of particle creation with the change of specific entropy has been discussed [13–15]. Hamil et al. [16] discussed the mechanism of particle creation in the absence of a time-like singularity in the emergent universe scenario. Lyth et al. [17] discussed the cosmological consequences of the particle creation during the inflation era of the universe. Particle creation arises due to a change of space-time metric at the end of the inflationary era during the early universe [18]. The nature and origin of quantum fields are due to the back-reaction of particle creation by deriving the effective action of a scalar field [19]. The time dependence of particle creation is due to a quantized, massless, minimally coupled scalar field in two-dimensional flat space-time with an accelerating mirror [20]. Recently, Bishi and Lepse [21] studied the influence of the deceleration parameter with the particle creation mechanism. Following the above-stated research work based on Rastall gravity and the particle creation mechanism, we have been motivated to investigate the impact of particle creation in Rastall gravity by considering different types of scale factors $a(t)$.

2. Field equations

The modified field equation of the Rastall gravity are expressed as [12]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = k(T_{\mu\nu} - \lambda g_{\mu\nu}R). \tag{1}$$

It can be rewritten in the form

$$R_{\mu\nu} + (\psi - \frac{1}{2})g_{\mu\nu}R = kT_{\mu\nu}, \tag{2}$$

where k is the Rastall gravitational coupling constant and $\psi = k\lambda$ is the Rastall coupling parameter. Li et al. [22] have constrained the Rastall coupling parameter $\psi = 0.163 \pm 0.001(68\%CL)$ with the help of 118 galaxy–galaxy strong gravitational lensing systems. Further, using CMB+BAO data, Akarsu et al. [23] restricted $\psi = k\lambda$ to the range $-0.0001 < \psi < 0.0007$ (68% CL) for $k = 1$. The Rastall coupling parameter measures the deviation from general relativity ($\psi = 0$ i.e. $\lambda = 0$). Let us consider the FLRW metric

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{3}$$

where $\kappa = 0, +1$ and -1 represents the flat, closed and open universe, respectively.

In the presence of creation of matter, the energy momentum tensor is given by

$$T_{ij} = (\rho + p + p_c)u_i u_j + (p + p_c)g_{ij} \tag{4}$$

where ρ and p are the energy density and pressure respectively, p_c is the creation pressure, u_i the fluid-four velocity vector such that $u^i u_i = -1$ and g_{ij} is the metric tensor. The trace of the energy momentum tensor is given as

$$T = \rho - 3(p + p_c) \tag{5}$$

Adiabatic particle production means particle as well as the entropy S (with entropy per particle [$\sigma = \frac{S}{N}$) being constant] have been produced in the space time. The creation pressure in the case of conserved specific entropy σ (that is, entropy per particle $\sigma = \frac{S}{N}$) is given by [24–26]

$$p_c = -\frac{(\rho + p)\Gamma}{3nH} \tag{6}$$

here $\Gamma = 3\eta Hn$ is the parameterization of the source function (See [27] and refs. therein). It determines whether particles are produced or annihilated. n refer to the particle number density and $0 \leq \eta \leq 1$ is a constant. The positive, negative and zero values of the source function represent particle production, particle annihilation and no particle production, respectively. With the help of the parameterization of the source function Γ, expression (6) leads to

$$p_c = -(\rho + p)\eta \tag{7}$$

The modified gravitational field equation (2) with the help of (3), (4) and (7) yields

$$\lambda_1 \dot{H} + \lambda_2 H^2 + \frac{\kappa}{a^2} = -\lambda_3 \rho,\ \dot{H} + \lambda_4 H^2 + \frac{\lambda_5 \kappa}{a^2} = \lambda_6[(1-\eta)p - \eta\rho] \tag{8}$$

where $\lambda_1 = \frac{2\psi}{2\psi-1}$, $\lambda_2 = \frac{4\psi-1}{2\psi-1}$, $\lambda_3 = \frac{k}{3(2\psi-1)}$, $\lambda_4 = \frac{3}{2}\lambda_2$, $\lambda_5 = \frac{6\psi-1}{6\psi-2}$, $\lambda_6 = \frac{k}{6\psi-2}$ From (8) we get the general expressions for energy density ρ, pressure p, and creation pressure p_c

$$\rho = -\frac{(\lambda_2 H^2 + \lambda_1 \dot{H})a^2 + \kappa}{\lambda_3 a^2},\ ;p = \frac{(\lambda_1\lambda_6\eta - \lambda_3)}{\lambda_3\lambda_6(\eta-1)}\dot{H} + \frac{\lambda_2\lambda_6\eta - \lambda_3\lambda_4}{\lambda_3\lambda_6(\eta-1)}H^2 + \frac{\kappa(\lambda_6\eta - \lambda_3\lambda_5)}{a^2(\lambda_3\lambda_6(\eta-1))} \tag{9}$$

$$p_c = -\eta\left[\frac{\lambda_1\lambda_6 - \lambda_3}{\lambda_3\lambda_6(\eta-1)}\dot{H} + \frac{\lambda_2\lambda_6 - \lambda_3\lambda_4}{\lambda_3\lambda_6(\eta-1)}H^2 + \frac{\kappa(\lambda_6 - \lambda_3\lambda_5)}{a^2\lambda_3\lambda_6(\eta-1)}\right] \tag{10}$$

3. Solution of the modified field equations

3.1. Model I

let us consider the scale factor of the form $a = \frac{-1}{t} + t^2$. The choice of this form of scale factor yields a time-dependent deceleration parameter. It is interesting to note that for $t > 1$, $\ddot{a} > 0$ and therefore the inflationary scenario of the universe can be observed for $t > 1$. The Hubble parameter and deceleration parameter q take the form

$$H = \frac{2t^3+1}{t(t^3-1)},\quad q = \frac{-\dot{H} + H^2}{H^2} = -2\frac{(t^3-1)^2}{(1+2t^3)^2} \tag{11}$$

In this model the universe evolves with $-1 < q < 0$ and at the late times $q \to -0.5$. We observe that when $t \to \infty$, we get $H \to 0$. The physical quantities for this case are found as

$$\rho = -\left[\frac{\lambda_1 + \kappa t^4 - 2\lambda_1 t^3(4+t^3) + \lambda_2(1+2t^3)^2}{\lambda_3 t^2(t^3-1)^2}\right],\ p = \frac{\lambda_6\eta\left(\lambda_1 + \kappa t^4 - 2\lambda_1 t^3(4+t^3) + \lambda_2(1+2t^3)^2\right) - \lambda_3(1 + \lambda_4(1+2t^3)^2 + t^3(\kappa t\lambda_5 - 2t^3 - 8))}{\lambda_3\lambda_6(\eta-1)t^2(t^3-1)^2} \tag{12}$$

$$p_c = -\frac{\eta\left[(\lambda_1+\lambda_2)\lambda_6 + \lambda_6 t^3(\kappa t + 4\lambda_2(1+t^3) - 2\lambda_1(4+t^3)) - \lambda_3(1 + \lambda_4(1+2t^3)^2 + t^3(\kappa t\lambda_5 - 2t^3 - 8))\right]}{\lambda_3\lambda_6(\eta-1)t^2(t^3-1)} \tag{13}$$

3.2. Model II

Borrow [28] has studied the intermediate expansion law in cosmology for the first time. The intermediate form of scale factor gets an exponential function of the time as

$$a = \exp(mt^l) \tag{14}$$

where $m > 0$ and $0 < l < 1$ are constant. Ample amount of research work based on the intermediate scale factor in the isotropic and anisotropic metric background has been studied in various gravity theories [29–31]. It will be interesting to study the intermediate form of the scale factor in the framework of Rastall gravity with particle creation. The Hubble parameter and deceleration parameter are given by

$$H = mlt^{l-1}, \quad q = -1 - \frac{l-1}{mlt^{l}} \tag{15}$$

The physical quantities for this case are expressed as

$$\rho = -\frac{1}{\lambda_3}\left[e^{-2mt^{l}}\kappa + lmt^{l-2}(\lambda_1(l-1) + \lambda_2 lmt^{l})\right] \tag{16}$$

$$p = \frac{e^{-2mt^{l}}\Big(\lambda_6\eta(\kappa t^2 + e^{2mt^{l}}lmt^{l}(\lambda_1(l-1) + \lambda_2 lmt^{l})) - \lambda_3(e^{2mt^{l}}lmt^{l}(l-1+\lambda_4 lmt^{l}) + \kappa t^2\lambda_5)\Big)}{\lambda_3\lambda_6(\eta-1)t^2} \tag{17}$$

$$p_c = \frac{e^{-2mt^{l}}\eta\Big(e^{2mt^{l}}(\lambda_3 - \lambda_1\lambda_6)(l-1)lmt^{l} + e^{2mt^{l}}(\lambda_3\lambda_4 - \lambda_2\lambda_6)l^2m^2t^{2l} + \kappa t^2(\lambda_3\lambda_5 - \lambda_6)\Big)}{\lambda_3\lambda_6(\eta-1)t^2} \tag{18}$$

4. Discussions

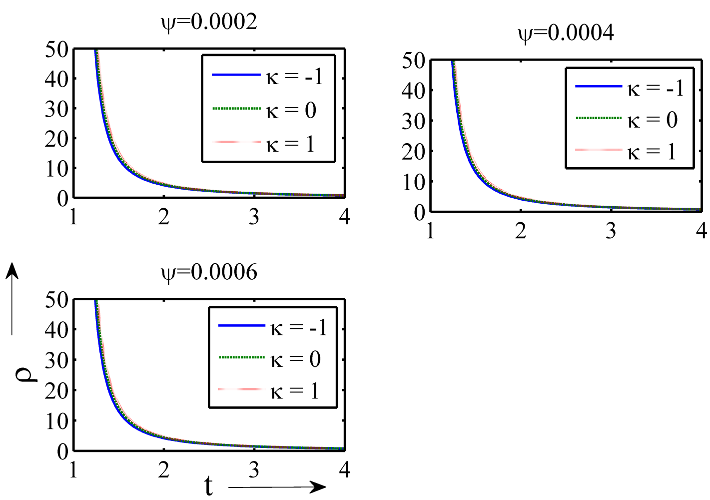

Figure 1. Profile of energy density against time for Model-I for different ψ

Figure 1 is the profile of the energy density ρ against cosmic time t for different values of $\psi(= 0.0002, 0.0004, 0.0006)$. The energy density is a decreasing and positive valued function of cosmic time t and $\rho \to 0$ when $t \to \infty$ for flat, open and closed universes. The pressure shows negative to positive behavior for different ψ. The profile of the creation pressure p_c shows a negative-positive-negative trend for different ψ values and different universes (flat, open and closed) (See Figure 2).

Figure 3 portrays the variation of the energy density ρ with time t for different ψ. Here we observe that the energy density ρ is a positive and decreasing function of cosmic time t with respect to different values of ψ for all the universes (flat, open and closed). At the initial phase, the pressure takes positive values, and after that it takes negative values for different values of ψ. In all the universes (flat, open and closed), the particle creation pressure is an increasing function of cosmic time and approaches zero with the evolution of time for different ψ (See Figure 4).

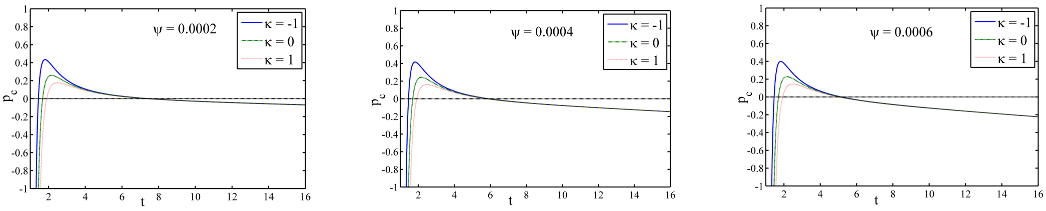

Figure 2. Profile of particle creation for Model-I against time for different ψ

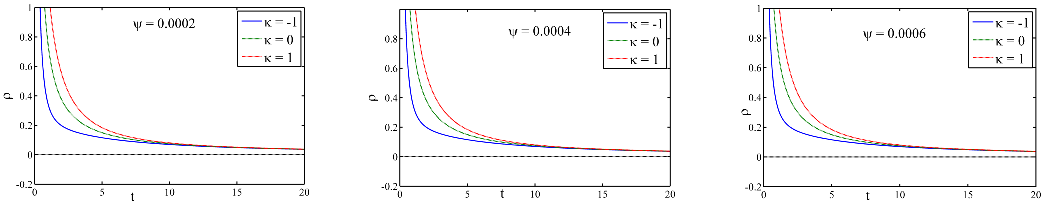

Figure 3. Profile of energy density for Model-II against time for different ψ

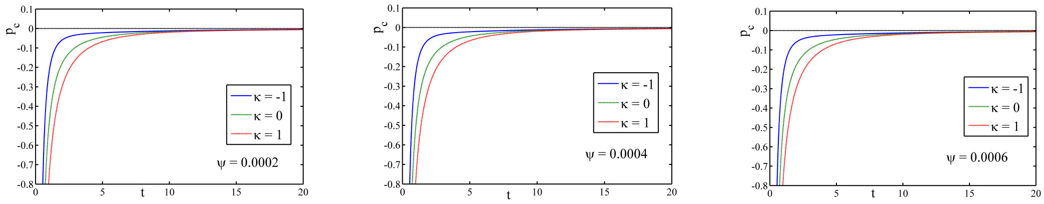

Figure 4. Profile of particle creation pressure for Model-II against time for different ψ

5. Conclusions

In this manuscript we have examined particle creation in the context of Rastall gravity. Particle creation mechanisms in the considered modified gravity models permit us to understand particle production annihilation in the universe. Rastall gravity is a non-conservative theory and an extension of general relativity. The key element of this theory is that non-vacuum solutions are dependent on the Rastall coupling parameter and are significantly different from their corresponding solutions in general relativity. We derived and solved the Rastall gravity field equations under two different scale factors. The deceleration parameter q portrays negative behaviour for all the models which indicates the universe's accelerated expansion. Here, we can observe that all the models have positive energy density ρ. Furthermore, we get positive to negative, and negative to positive behavior of the pressure p. If the energy density is positive, the associated negative pressure will drive the accelerated expansion of the universe. This means that all the models indicates accelerated expansion of the universe. Also, for the particle creation pressure, its presence or absence is indicated by zero or negative particle creation pressure p_c, respectively. As a result, particle creation p_c occurs in all the models for different values of the Rastall coupling parameter $\psi (= 0.0002, 0.0004, 0.0006)$ with $k = 1, 0, -1$.

6. Patents

No patents result from the work reported in this manuscript.

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