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The Regular Black Hole by Gravitational Decoupling †

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Abstract: In this paper, we consider the gravitational decoupling method to obtain a hairy regular black hole which corresponds to the Hayward model. We modify the hairy Schwarzschild solution to obtain the regular Kretschmann scalar. The energy-momentum of a new model is considered, and we show that there is an energy exchange between its parts.

Keywords: gravitational decoupling; regular black hole; hairy black hole

1. Introduction

The gravitational collapse of massive stars can end up in a black hole or naked singularity formation. According to the famous singularity theorem [1] the singularity must form during the gravitational collapse if the weak energy condition is held. However, singularities are generally regarded as indicating the breakdown of the theory, and one needs to modify the theory in this region by considering the quantum effects. Quantum field theory, applied to the stationary black holes, leads to the Hawking radiation [2], which requires the negative ingoing energy flux. This fact means that a black hole evaporates up to the singularity. For this reason, a regular (non-singular) black hole is considered [3]. Hayward has considered the formation and evaporation of regular black holes [4]. Another famous fact regarding black holes is the no-hair theorem, i.e. a black hole can have only three charges - mass, electric charge and angular momentum. However, it was shown that a black hole can have soft hair [5]. Recently, it was understood that one can obtain a hairy black hole by using the gravitational decoupling method [6,7]. In this paper, we use the gravitational decoupling method and Hayward's regular black hole model to obtain a hairy regular black hole. This paper is organized as follows: in sec. II we briefly describe the gravitational decoupling. In sec. III we introduce the hairy regular black hole. Sec. IV is the conclusion, including the discussion of the formation and evaporating hairy regular vaidya black hole. The system of units $c = 8\pi G = 1$ will be used throughout the paper. Also, we use the signature $\{-,+,+,+\}$.

2. The gravitational decoupling

It is well-known that obtaining the analytical solution of the Einstein equations, in most cases, is a really hard task. We know that we can obtain an analytical solution of the spherically symmetric spacetime in the case of the perfect fluid as the gravitational source. However, if we consider the more realistic case when the perfect fluid is coupled with other matter, then it is nearly impossible to obtain the analytical solution. In papers [8–10] it was shown, by means of the Minimal Geometric Deformation (MGD) [11,12], that we can

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decouple of the gravitational sources. For example, one can write the energy-momentum tensor T_{ik} as:

$$T_{ik} = \tilde{T}_{ik} + \alpha\Theta_{ik}\,. \tag{1}$$

Where $\tilde{T}_{ik}$ is the energy-momentum tensor of the perfect fluid, α is the coupling constant to the energy-momentum tensor Θ_{ik}. Moreover, there is only gravitational interaction between two sources, i.e.:

$$T^{ik}_{;k} = 0 \rightarrow \tilde{T}^{ik}_{;k} = \alpha\Theta^{ik}_{;k} = 0\,. \tag{2}$$

This fact allows us to think about Θ_{ik} as a possible dark matter candidate. However, there can be energy exchange between two sources:

$$\tilde{T}^{ik}_{;k} = -\alpha\Theta^{ik}_{;k}\,. \tag{3}$$

The Einstein tensor G_{ik}, which corresponds to the energy-momentum tensor (1), can be decoupled in the following way:

$$G_{ik} = \tilde{G}_{ik} + \hat{G}_{ik}\,. \tag{4}$$

Where one has two separated Einstein equations to solve:

$$\begin{aligned} \tilde{G}_{ik} &= \tilde{T}_{ik} \\ \hat{G}_{ik} &= \alpha\Theta_{ik}\,. \end{aligned} \tag{5}$$

By solving (5) separately, one obtains the solutions $\tilde{g}_{ik}$ and $\hat{g}_{ik}$, the combination of which gives the solution to the Einstein equation:

$$G_{ik} = T_{ik}\,. \tag{6}$$

By using this method, Ovale [6] has obtained hairy Schwarzschild solution:

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega^2\,, \tag{7}$$

where $d\Omega$ is the metric on the unit sphere and the lapse function f is given by:

$$f = 1 - \frac{2M}{r} + \alpha e^{-\frac{r}{M}}\,. \tag{8}$$

This solution satisfies dominant and strong energy conditions at $r > 2M$ and has the singularity at $r = 0$. We will use (7) to obtain the hairy regular solution in the next section. The parameter M (8) is related to the Schwarzschild black hole mass $\mathbf{M}$ by following relation:

$$M = \mathbf{M} + \frac{\alpha L}{2}\,. \tag{9}$$

Where the parameter L has the dimension of the length and is related to a new black hole charge. The impact of this charge on particle motion and thermodynamics has been studied in [13,14]. One should note that the Kretschmann scalar for the lapse function (8) is divergent at the limit $r \rightarrow 0$.

3. Hairy regular black hole

A minimal model of a regular black hole suggested by Hayward [4] is:

$$ds^2 = -\left(1 - \frac{2Mr^2}{r^3 + 2Ml^2}\right)dt^2 + \left(1 - \frac{2Mr^2}{r^3 + 2Ml^2}\right)dr^2 + r^2 d\Omega^2\,. \tag{10}$$

where l is a convenient encoding of the central energy density $\frac{3}{l^2}$, assumed positive. The solution (10) behaves as Schwarzschild one at the limit $r \to \infty$ and at the limit $r \to 0$ it behaves as:

$$g_{00} \simeq 1 - \frac{r^2}{l^2} \,. \tag{11}$$

Under this limit, the Einstein tensor has the cosmological constant form:

$$G \sim \lambda g \,, \tag{12}$$

and

$$\lambda = \frac{3}{l^2} \,. \tag{13}$$

Now, we modify the hairy Schwarzschild solution (7) to obtain a regular hairy black hole. First, let's write the metric in the advanced Eddington-Finkelstein coordinates:

$$ds^2 = -f(r)dv^2 + 2dvdr + r^2 d\Omega^2 \,. \tag{14}$$

Now, we assume that the lapse function has the form:

$$f(r) = 1 - \frac{2Mr^2}{r^3 + 2Ml^2} + \alpha e^{-\frac{r}{M}} \,. \tag{15}$$

The problem with this solution is that the Kretschmann scalar diverges at the limit $r \to 0$ indicating that there is a singularity at the centre of this black hole. Hence, one needs to modify the last term $\alpha e^{-\frac{r}{M}}$. One should find the modification which satisfies the following conditions:

- This term must be diveregent free at the limit $r \to 0$;
- It must vanish at the spatial infinity.

The following modification satisfies these conditions:

$$ds^2 = -\left(1 - \frac{2M\,r^2}{2l^2 M + r^3} + \alpha \exp\left(\frac{-2l^2 M - r^3}{M\,r^2}\right)\right) dv^2 + 2dvdr + r^2 d\Omega^2 \,. \tag{16}$$

The Kretschmann scalar has a horrific form, and we don't write it down here, but it has a finite limit at $r \to 0$:

$$\lim_{r\to 0} K \equiv R_{iklm} R^{iklm} = \frac{24}{l^4} \,, \tag{17}$$

which coinsides with Hayward model [4].

Now, we should write down the Einstein tensor to make sure that (16) is the combination of two separated Einstein equations. The Einstein tensor components at $\alpha = 0$ are given by:

$$\begin{aligned} \tilde{G}^0_0 &= -\frac{12 l^2 M^2}{(2l^2 M + r^3)^2} \\ \tilde{G}^1_1 &= -\frac{12 l^2 M^2}{(2l^2 M + r^3)^2} \\ \tilde{G}^2_2 &= -\frac{24 l^2 M^2 (l^2 M - r^3)}{(2l^2 M + r^3)^3} \\ \tilde{G}^3_3 &= -\frac{24 l^2 M^2 (l^2 M - r^3)}{(2l^2 M + r^3)^3} \end{aligned} \tag{18}$$

Now we should write down the Einstein tensor components for the metric:

$$ds^2 = -\left(1 + \alpha e^{-\frac{r^3+2l^2M}{Mr^2}}\right)dv^2 + 2dvdr + r^2 d\Omega^2\,, \tag{19}$$

they are:

$$\begin{aligned}
\hat{G}_0^0 &= \frac{\left(4l^2M + M\,r^2 - r^3\right)\alpha \mathrm{e}^{\frac{-2l^2M - r^3}{r^2M}}}{r^4M} \\
\hat{G}_1^1 &= \frac{\left(4l^2M + M\,r^2 - r^3\right)\alpha \mathrm{e}^{\frac{-2l^2M - r^3}{r^2M}}}{r^4M} \\
\hat{G}_2^2 &= \frac{\left(16l^4M^2 - 4l^2M^2r^2 - 8l^2M\,r^3 - 2M\,r^5 + r^6\right)\alpha \mathrm{e}^{\frac{-2l^2M - r^3}{r^2M}}}{2r^6M^2} \\
\hat{G}_3^3 &= \frac{\left(16l^4M^2 - 4l^2M^2r^2 - 8l^2M\,r^3 - 2M\,r^5 + r^6\right)\alpha \mathrm{e}^{\frac{-2l^2M - r^3}{r^2M}}}{2r^6M^2}
\end{aligned} \tag{20}$$

If we consider the sum $\tilde{G}_k^i + \hat{G}_k^i$, then we obtain the Einstein tensor components for the metric (16):

$$\begin{aligned}
G_0^0 &= \frac{16\alpha\left(l^2M + \frac{r^3}{2}\right)^2\left(\left(l^2 + \frac{r^2}{4}\right)M - \frac{r^3}{4}\right)\mathrm{e}^{\frac{-2l^2M - r^3}{M\,r^2}} - 12l^2M^3r^4}{r^4M\left(2l^2M + r^3\right)^2} \\
G_1^1 &= \frac{16\alpha\left(l^2M + \frac{r^3}{2}\right)^2\left(\left(l^2 + \frac{r^2}{4}\right)M - \frac{r^3}{4}\right)\mathrm{e}^{\frac{-2l^2M - r^3}{M\,r^2}} - 12l^2M^3r^4}{r^4M\left(2l^2M + r^3\right)^2} \\
G_2^2 &= \frac{1}{2r^6\left(2l^2M + r^3\right)^3M^2}\left(128\alpha\left(l^2M + \frac{r^3}{2}\right)^3\left(\left(l^4 - \frac{1}{4}l^2r^2\right)M^2 +\right.\right. \\
&\left.\left. + \left(-\frac{1}{2}l^2r^3 - \frac{1}{8}r^5\right)M + \frac{r^6}{16}\right)\mathrm{e}^{\frac{-2l^2M - r^3}{M\,r^2}} - 48l^4M^5r^6 + 48l^2M^4r^9\right) \\
G_3^3 &= \frac{1}{2r^6\left(2l^2M + r^3\right)^3M^2}\left(128\alpha\left(l^2M + \frac{r^3}{2}\right)^3\left(\left(l^4 - \frac{1}{4}l^2r^2\right)M^2 +\right.\right. \\
&\left.\left. + \left(-\frac{1}{2}l^2r^3 - \frac{1}{8}r^5\right)M + \frac{r^6}{16}\right)\mathrm{e}^{\frac{-2l^2M - r^3}{M\,r^2}} - 48l^4M^5r^6 + 48l^2M^4r^9\right)
\end{aligned} \tag{21}$$

Thus, we have shown that the model is the combination of two matter fields. Now we should find out if there is an energy exchange between these sources. For this purpose, one needs to consider the energy-momentum tensor at the limit $\alpha \to 0$ and consider only $\tilde{T}^{ik}_{;k}$ because $T^{ik}_{;k} = 0$ in virtue of the Einstein equation. The energy-momentum tensor components of the pure Hayward model are given by:

$$\begin{aligned}
\tilde{T}_{00} &= \frac{12M^2l^2\left(2l^2M - 2M\,r^2 + r^3\right)}{\left(2l^2M + r^3\right)^3} \\
\tilde{T}_{01} &= -\frac{12M^2l^2}{\left(2l^2M + r^3\right)^2} \\
\tilde{T}_{22} &= -\frac{24r^2M^2l^2\left(l^2M - r^3\right)}{\left(2l^2M + r^3\right)^3} \\
\tilde{T}_{33} &= -\frac{24r^2M^2l^2\left(l^2M - r^3\right)\sin(\theta)^2}{\left(2l^2M + r^3\right)^3}
\end{aligned} \tag{22}$$

If there is energy exchange between two sources $\tilde{T}_{ik}$ and Θ_{ik}, then the following condition is held:

$$\tilde{T}^{ik}_{;k} = -\alpha\Theta^{ik}_{;k}\,. \tag{23}$$

The covariant derivative is taken with regard to the metric (16). Considering the covariant derivative of (22) one obtains:

$$\tilde{T}^{ik}_{;k} = -\frac{12M\,l^2 e^{\frac{-2l^2M-r^3}{M\,r^2}}\,\alpha\left(8l^4M^2+4l^2M^2r^2+2l^2M\,r^3-4M\,r^5-r^6\right)}{\left(2l^2M+r^3\right)^3 r^3}\delta^i_0\,. \tag{24}$$

And one can see that there is an energy exchange between two sources. However, this energy exchange is negligible because the minimal geometrical deformation assumes $\alpha \ll 1$ and according to the Hayward model l is the Planck scale. Moreover, the expression (24) is regular at the limit $r \to 0$:

$$\lim_{r\to 0}\tilde{T}^{ik}_{;k} = 0\,. \tag{25}$$

As we pointed out above, there is an energy exchange between these sources, and this exchange is negligible. In particle physics, there are several scenarios in which a small energy exchange between dark and ordinary matters is allowed. For this reason, we can still think about Θ_{ik} as a possible dark matter candidate. However, a deeper investigation should be done in this direction, and we leave this question for future research.

4. Conclusion

In this paper, we have modified the hairy Schwarzschild solution to obtain the model of a hairy regular black hole. This model is regular in the centre of a black hole, and the Kretschmann scalar is the same, in the limit $r \to 0$ as in the Hayward model. We have shown that there is an energy exchange between two sources and the energy-momentum tensor T_{ik} is conserved, but its parts $\tilde{T}_{ik}$ and Θ_{ik} are not conserved separately. The main question about the formation and evaporation of this black hole is for future research. However, we can consider the hairy Vaidya solution for this purpose, which gravitational decoupling has been considered in [15].

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