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SHOTA RUSTAVELI NATIONAL SCIENCE  
FOUNDATION OF GEORGIA

# Reconstruction, Analysis and Constraints of Cosmological Scalar Field $\phi$ CDM Models

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Based on the paper: Olga Avsajanishvili, Yiwen Huang, Lado Samushia, Tina Kahniashvili,

“*The observational constraints on the flat  $\phi$ CDM models*”, Eur. Phys. J. C78, 09, 773; 2018; arXiv:1711.11465

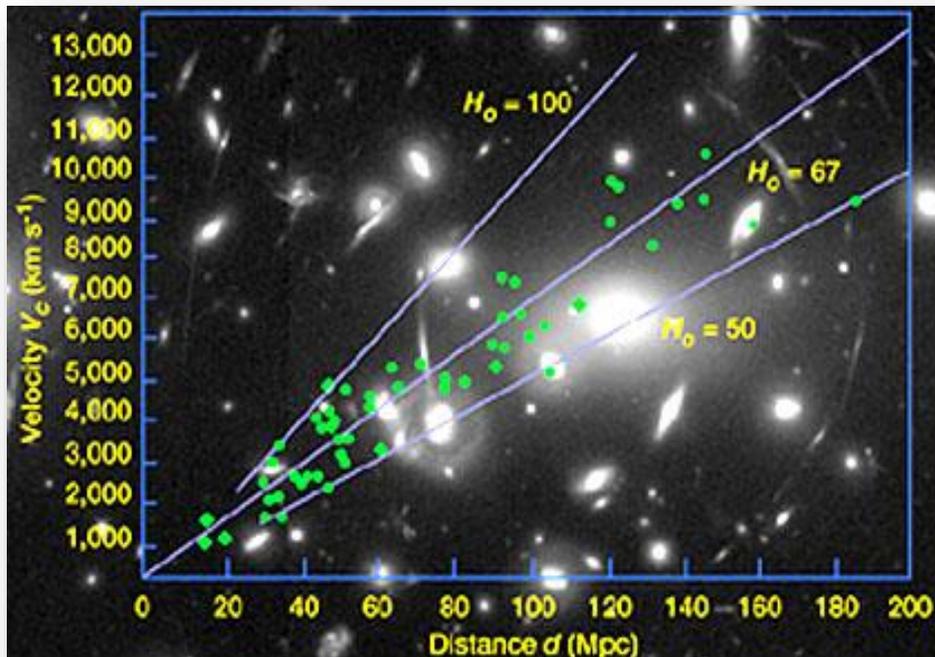
*The presentation is supported in part by the Rustaveli National Science Foundation of Georgia under Grant FR-19-8306*

# Hubble law

$$V = H_0 D$$

$H_0$  - Hubble constant

$$H_0 = 67.4 \text{ km s}^{-1} \text{Mpc}^{-1}$$



# Accelerated expansion of the Universe

## Nobel Prize in Physics 2011



Photo: Lawrence Berkeley National Lab

**Saul Perlmutter**



Photo: Belinda Pratten, Australian National University

**Brian P. Schmidt**



Photo: Scanpix/AFP

**Adam G. Riess**

The Nobel Prize in Physics 2011 was awarded "for the discovery of the accelerating expansion of the Universe through observations of distant supernovae" with one half to Saul Perlmutter and the other half jointly to Brian P. Schmidt and Adam G. Riess.

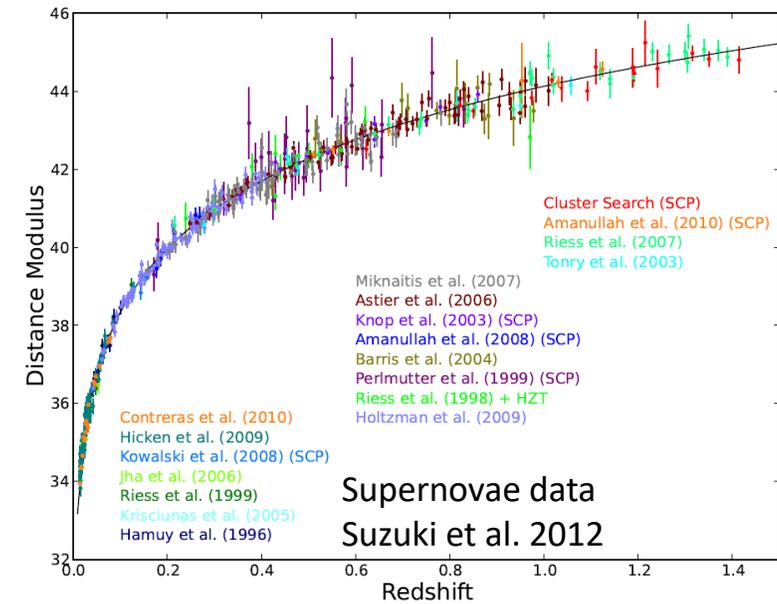
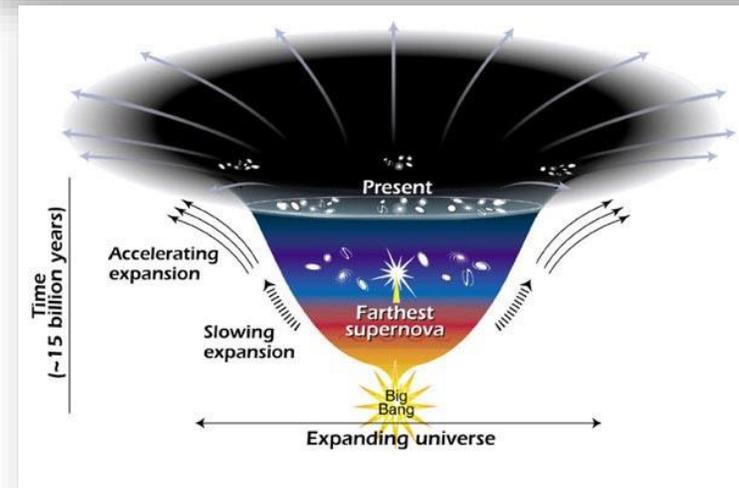


Figure 4. Hubble diagram for the Union2.1 compilation. The solid line represents the best-fit cosmology for a flat  $\Lambda$ CDM Universe for supernovae alone. SN SCP06U4 falls outside the allowed  $z_1$  range and is excluded from the current analysis. When fit with a newer version of SALT2, this supernova passes the cut and would be included, so we plot it on the Hubble diagram, but with a red triangle symbol.



**Dark Energy?**

**Modified Gravity?**

# Dark energy models

- **Cosmological constant,  $\Lambda$**
- **Scalar field (quintessence and phantom) models**
- **K-essence models**
- **Holographic dark energy models**
- **Barotropic fluid or Chaplygin gas models**
- **Coupled dark matter (neutrino)-dark energy models**

# Cosmological constant

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} (R + 2\Lambda) + S_M$$

- $\Lambda \rightarrow$  “concordance”  $\Lambda$ CDM model
- $\Lambda$ CDM model provides the best fit for cosmological observations
- $\Lambda$ CDM model is based on GTR for description of gravity in the Universe on large scales
- $\Lambda$ CDM model has:
  - fine tuning problem
  - coincidence problem

# Dynamical scalar field $\phi$ CDM models

The equation of state parameter

$$w = p/\rho$$



$$w_{\Lambda} = -1$$

$\Lambda$ CDM model

$$w_{\phi}(t) \neq -1$$

$\phi$ CDM models

where  $\rho$  is an energy density,  $p$  is a pressure

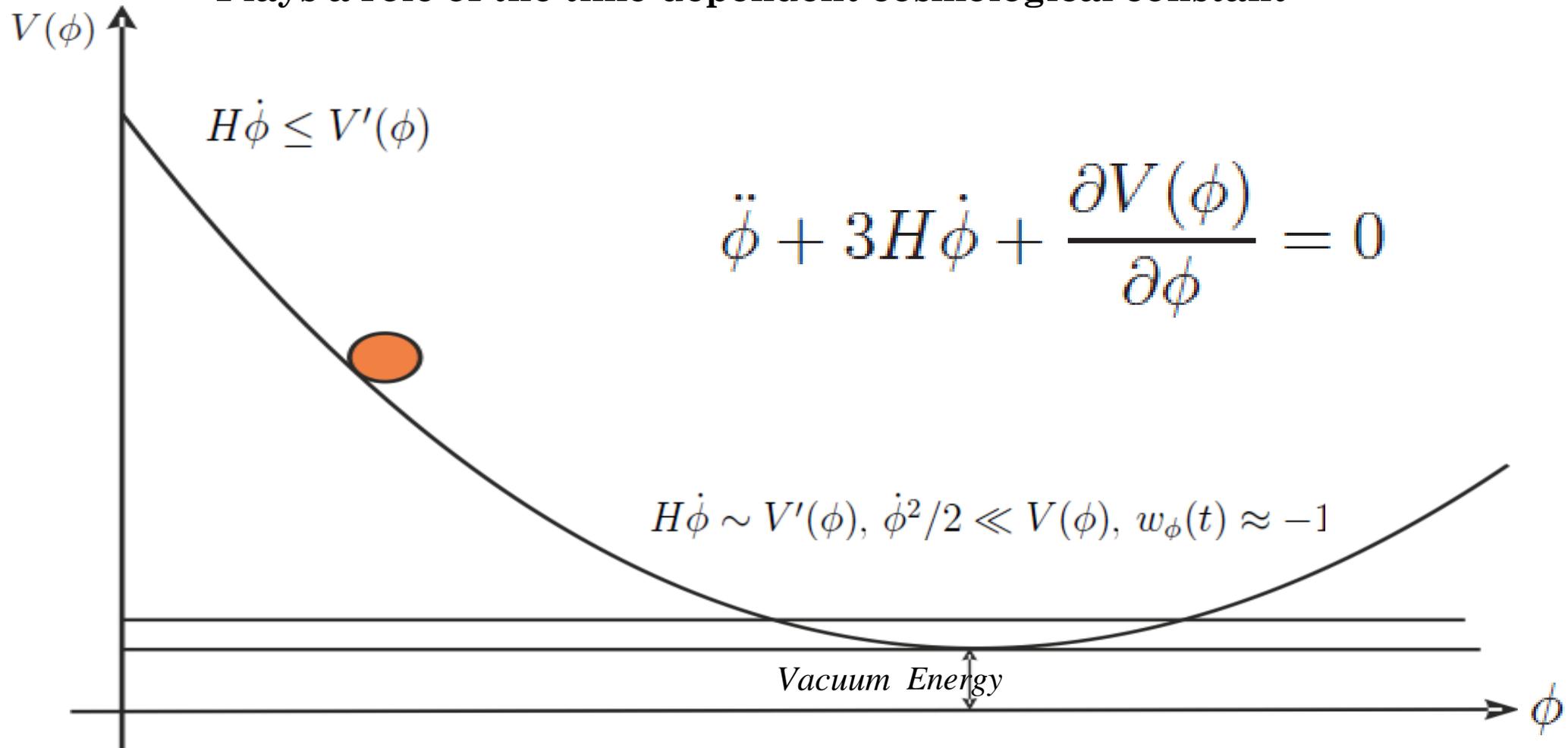
# Phantom and quintessence $\phi$ CDM models

$$S = \int d^4x \sqrt{-g} \left[ -\frac{M_{\text{pl}}^2}{16\pi} R + \mathcal{L}_\phi \right] + S_M$$

Phantom models	Quintessence models
$w_0 < -1$	$-1 < w_0 < -1/3$
$\mathcal{L}_\phi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$	$\mathcal{L}_\phi = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$
$\ddot{\phi} + 3H\dot{\phi} - \frac{\partial V(\phi)}{\partial\phi} = 0$	$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial\phi} = 0$

# Quintessence

- Spatially uniform dynamical scalar field
- Slowly rolls down to the minimum of its almost flat potential
- This model avoids the fine tuning problem of the  $\Lambda$ CDM model
- Plays a role of the time-dependent cosmological constant



# Subdivision of the quintessence models:

- **the thawing models** for which the evolution of the scalar field is fast compared to the Hubble expansion.

$$H < \sqrt{V''(\phi)} \quad - \text{underdamped}$$

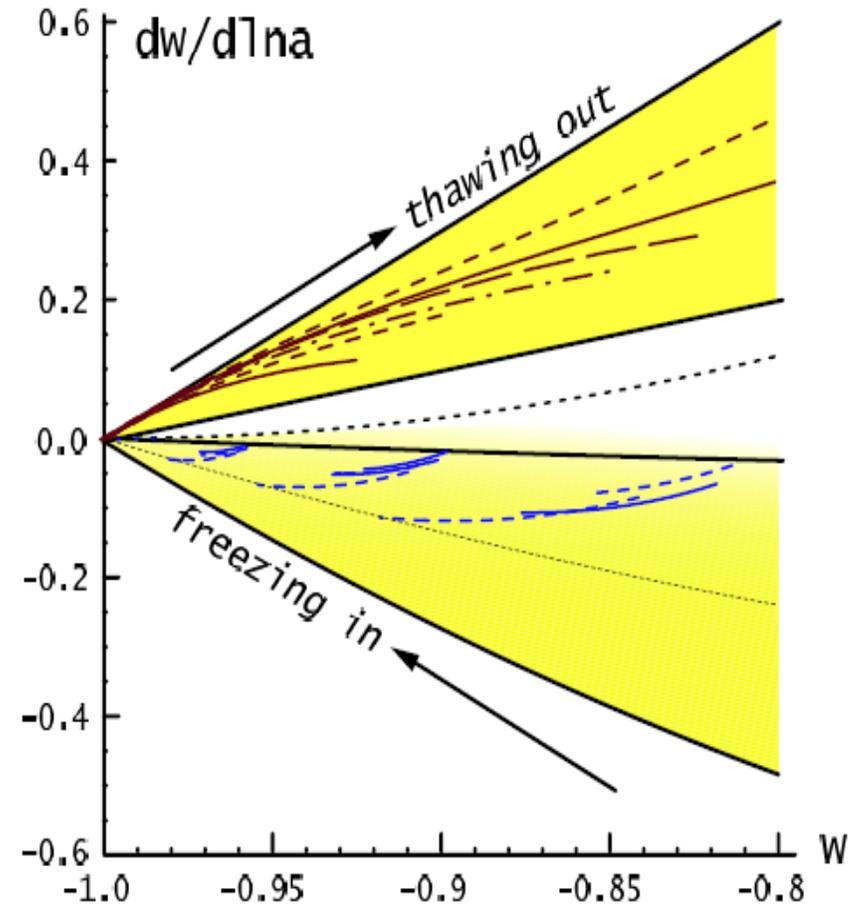
$$V(\phi) = M^{4-\alpha} \phi^\alpha, \alpha > 0;$$

$$V(\phi) = M^4 \exp(-\beta\phi/M_{pl}), \beta < \sqrt{24\pi}$$

- **the tracking (freezing) models** for which the evolution is slow compared to the Hubble expansion:

$$H > \sqrt{V''(\phi)} \quad - \text{overdamped}$$

$$V(\phi) = \frac{k}{2} M_{pl}^2 \phi^{-\alpha}, \alpha > 0$$



R.Caldwell & E.Linder, Phys. Rev. Lett. 95, 141301 (2005)

# Chevallier-Polarsky-Linder (CPL) parameterization

$$w(a) = w_0 + w_a(1 - a)$$

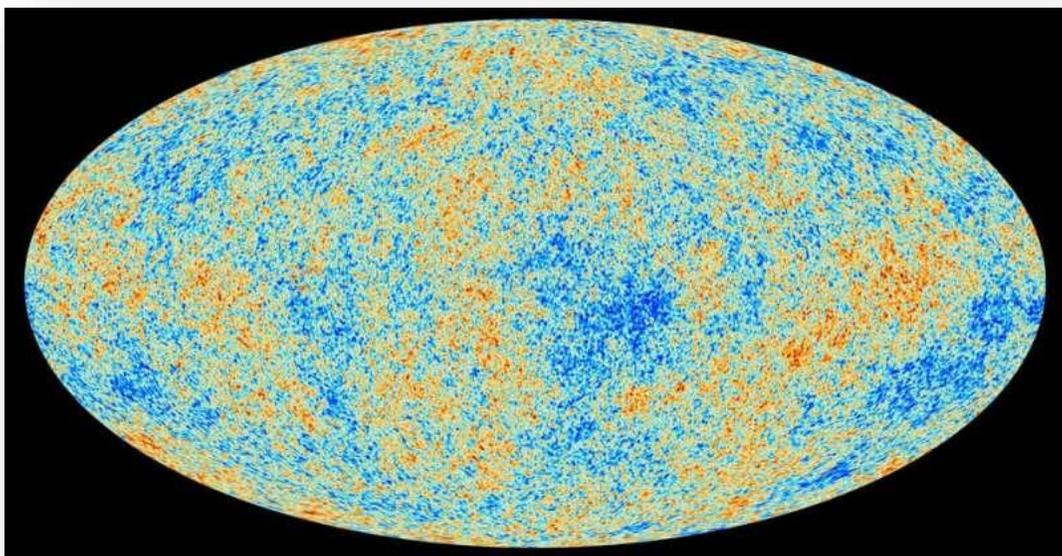
$$w_0 = w(a_0 = 1), w_a = (dw/dz)|_{z=1} = -a^{-2}(dw/da)|_{a=1/2}$$

$dw/da > 0$  for quintessence thawing models

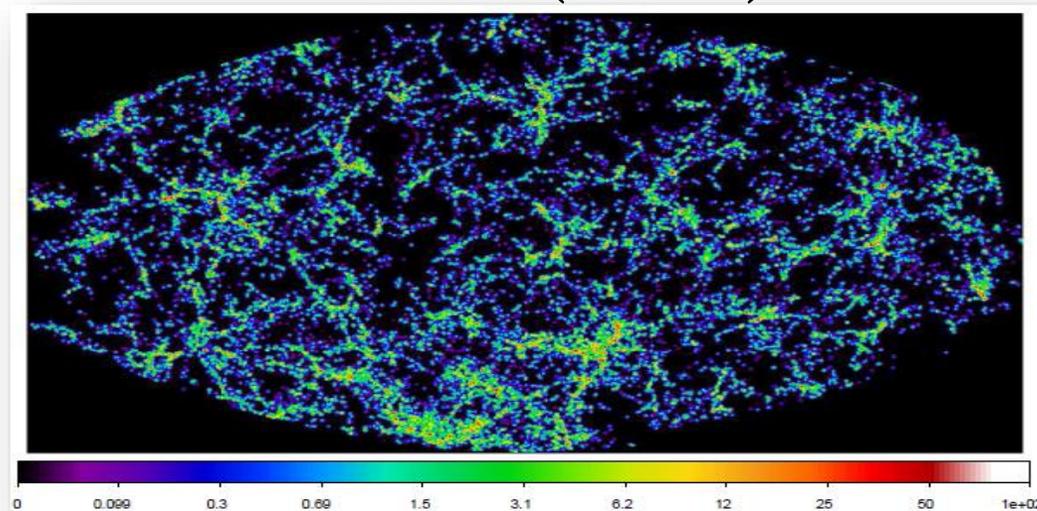
$dw/da < 0$  for quintessence freezing models

# What is the influence of the dark energy model on the evolution of large-scale structures?

CMB map from Planck space experiment

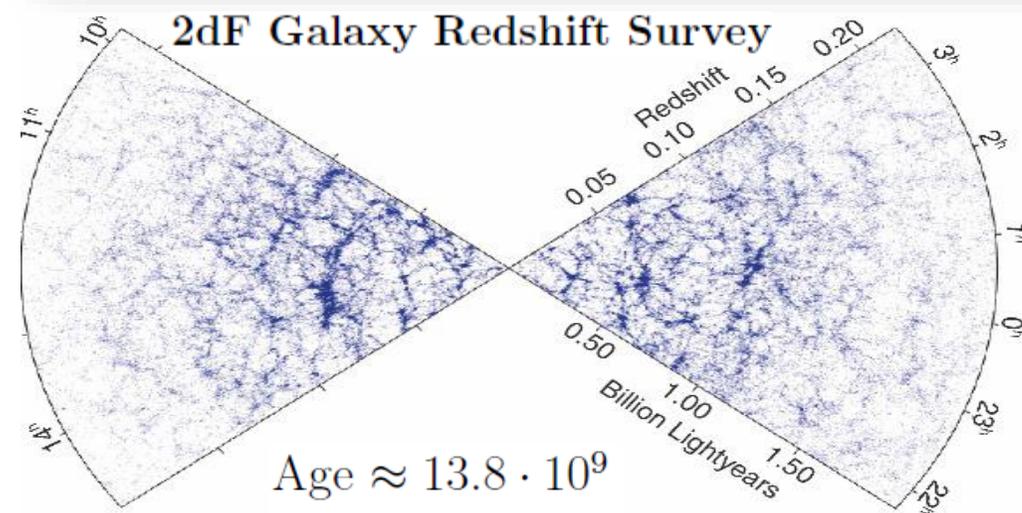


Cosmic Web (SDSS)



$$\frac{\Delta T}{T} \sim \frac{\Delta \rho}{\rho} \approx 5 \cdot 10^{-5}$$

$$\text{Age} \approx 3.8 \cdot 10^5 \text{ years}$$



# Influence of the $\varphi$ CDM model on the formation of the large-scale structure in the universe

The growth rate

$$f(a) = \frac{d \ln D(a)}{d \ln a}$$

The linear perturbation equation

$$D'' + \left( \frac{3}{a} + \frac{E'}{E} \right) D' - \frac{3\Omega_{m,0}}{2a^5 E^2} D = 0$$

The linear growth factor

$$D(a) = \frac{\delta(a)}{\delta(a_0)}$$

The matter density fluctuations

$$\delta = \frac{\rho(\vec{r}, t) - \langle \rho \rangle}{\langle \rho \rangle} \quad \delta(a_{\text{in}}) = \delta'(a_{\text{in}}) = 5 \cdot 10^{-5}$$

# $\gamma$ - parameterization

- Fractional matter density

$$\Omega_m(a) = \frac{\Omega_{m0} a^{-3}}{E^2}$$

- Growth rate function

$$f(a) = \frac{d \ln D(a)}{d \ln a}$$

- Fitting formula

$$f(a) \approx \Omega_m^\gamma(a), \quad \gamma \text{ is a growth index}$$

L.M.Wang & P.J. Steinhardt, *Astrophys. J.* 508, 483 (1998)

For the  $\Lambda$ CDM model  $\gamma \approx 0.55$ , while for the Dvali-Gabadadze-Porrati braneworld model  $\gamma \approx 0.68$

$$\gamma = \begin{cases} 0.55 + 0.05(1 + w_0 + 0.5w_a), & \text{if } w_0 \geq -1 \\ 0.55 + 0.02(1 + w_0 + 0.5w_a), & \text{if } w_0 < -1 \end{cases}$$

Erick Linder, *Phys. Rev. D* 72, 043529 (2005)

$$w_0 = w(a_0 = 1), \quad w_a = (dw/dz)|_{z=1} = -a^{-2}(dw/da)|_{a=1/2}$$

# Phantom Potentials

Name	Form	Reference
Fifth power	$V(\phi) = V_0\phi^5$	Scherrer & Sen (2008a)
Inverse square power	$V(\phi) = V_0\phi^{-2}$	Scherrer & Sen (2008a)
Exponent	$V(\phi) = V_0 \exp(\beta\phi), \beta = \text{const} > 0$	Scherrer & Sen (2008a)
Quadratic	$V(\phi) = V_0\phi^2$	Dutta & Scherrer (2009)
Gaussian	$V(\phi) = V_0(1 - \exp(\phi^2/\sigma^2)), \sigma = \text{const}$	Dutta & Scherrer (2009)
pseudo-Nambu-Goldstone boson (pNGb)	$V(\phi) = V_0(1 - \cos(\phi/\kappa)), \kappa = \text{const} > 0$	Frieman et al. (1995)
Inverse hyperbolic cosine	$V(\phi) = V_0(\cosh(\psi\phi))^{-1}, \psi = \text{const} > 0$	Dutta & Scherrer (2009)

# Quintessence Potentials

Name	Form	Reference
Ratra-Peebles	$V(\phi) = V_0 M_{\text{pl}}^2 \phi^{-\alpha}; \alpha = \text{const} > 0$	Ratra & Peebles (1988 <i>b</i> )
Ferreira-Joyce	$V(\phi) = V_0 \exp(-\lambda\phi/M_{\text{pl}}); \lambda = \text{const} > 0$	Ferreira & Joyce (1998)
Zlatev-Wang-Steinhardt	$V(\phi) = V_0(\exp(M_{\text{pl}}/\phi) - 1)$	Zlatev et al. (1999)
Sugra	$V(\phi) = V_0 \phi^{-\chi} \exp(\gamma\phi^2/M_{\text{pl}}^2);$ $\chi, \gamma = \text{const} > 0$	Brax & Martin (1999)
Sahni-Wang	$V(\phi) = V_0(\cosh(\zeta\phi) - 1)^g;$ $\zeta = \text{const} > 0, g = \text{const} < 1/2$	Sahni & Wang (2000)
Barreiro-Copeland-Nunes	$V(\phi) = V_0(\exp(\nu\phi) + \exp(v\phi));$ $\nu, v = \text{const} \geq 0$	Barreiro et al. (2000)
Albrecht-Skordis	$V(\phi) = V_0((\phi - B)^2 + A) \exp(-\mu\phi);$ $A, B = \text{const} \geq 0, \mu = \text{const} > 0$	Albrecht & Skordis (2000)
Urřena-López-Matos	$V(\phi) = V_0 \sinh^m(\xi M_{\text{pl}}\phi);$ $\xi = \text{const} > 0, m = \text{const} < 0$	Urena-Lopez & Matos (2000)
Inverse exponent potential	$V(\phi) = V_0 \exp(M_{\text{pl}}/\phi)$	Caldwell & Linder (2005)
Chang-Scherrer	$V(\phi) = V_0(1 + \exp(-\tau\phi)); \tau = \text{const} > 0$	Chang & Scherrer (2016)

# Phantom $\phi$ CDM model

- The first Friedmann's equation  $E(z) = H(z)/H_0 = (\Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{\phi}(z))^{1/2}$

- The scalar field equation  $\ddot{\phi} + 3H\dot{\phi} - \frac{\partial V(\phi)}{\partial\phi} = 0$

- The energy density of the scalar field  $\rho_{\phi} = -\frac{\dot{\phi}^2}{2} + V(\phi)$

- The pressure of the scalar field  $p_{\phi} = -\frac{\dot{\phi}^2}{2} - V(\phi)$

- The equation of state parameter  $w_{\phi} = \frac{-\dot{\phi}^2/2 - V(\phi)}{-\dot{\phi}^2/2 + V(\phi)}$

- We consider the flat universe

# Quintessence $\phi$ CDM model

- **The first Friedmann's equation**  $E(z) = H(z)/H_0 = (\Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{\phi}(z))^{1/2}$
- **The scalar field equation** 
$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$$
- **The energy density of the scalar field** 
$$\rho_{\phi} = \frac{\dot{\phi}^2}{2} + V(\phi)$$
- **The pressure of the scalar field** 
$$p_{\phi} = \frac{\dot{\phi}^2}{2} - V(\phi)$$
- **The equation of state parameter** 
$$w_{\phi} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}$$
- **We consider the flat universe**

# Criteria for Selection of Solutions

We have found the solutions describing the reliable universe, for which the following three criteria must be simultaneously fulfilled:

1. The matter and DE equality ( $\Omega_m = \Omega_\phi$ ) has happened relatively recently,  $a \in (0.6; 0.8)$ ;
2. The growth rate of the matter density fluctuations,  $f(a)$ , and the fractional matter density,  $\Omega_m(a)$ , are parametrized by the Linder  $\gamma$ -parametrization,  $f(a) \approx \Omega_m^\gamma(a)$ ;
3. The chosen current EoS parameters values should correspond to the expected current EoS parameter values for these models: for the phantom models  $w_0 < -1$ ; for the quintessence models  $-1 < w_0 < -0.75$ : for the freezing type  $dw/da < 0$ , for the thawing type  $dw/da > 0$ .

## Model Parameters and Initial Conditions for Phantom Potentials

Phantom potentials	Free parameters	
$V(\phi) = V_0\phi^5$	$H_0(50 \div 90)$ $\Omega_{m0}(0.25 \div 0.32)$ $V_0(10^{-3} \div 10^{-2})$	$\phi_0(3.37 \div 3.94)$ $\dot{\phi}_0(523 \div 563.6)$
$V(\phi) = V_0\phi^{-2}$	$H_0(50 \div 90)$ $\Omega_{m0}(0.25 \div 0.32)$ $V_0(30 \div 50)$	$\phi_0(2.83 \div 5.15)$ $\dot{\phi}_0(471.4 \div 600)$
$V(\phi) = V_0 \exp(\beta\phi)$	$H_0(50 \div 90)$ $\Omega_{m0}(0.25 \div 0.32)$ $V_0(1 \div 20)$	$\beta(0.08 \div 0.3)$ $\phi_0(0.2 \div 9.14)$ $\dot{\phi}_0(79.8 \div 830.9)$
$V(\phi) = V_0\phi^2$	$H_0(50 \div 90)$ $\Omega_{m0}(0.25 \div 0.32)$ $V_0(1 \div 20)$	$\phi_0(0.67 \div 2.8)$ $\dot{\phi}_0(191 \div 450)$
$V(\phi) = V_0(1 - \exp(\phi^2/\sigma^2))$	$H_0(50 \div 90)$ $\Omega_{m0}(0.25 \div 0.32)$ $V_0(5 \div 30)$	$\sigma(5 \div 30)$ $\phi_0(0.67 \div 2.8)$ $\dot{\phi}_0(191 \div 450)$
$V(\phi) = V_0(1 - \cos(\phi/\kappa))$	$H_0(50 \div 90)$ $\Omega_{m0}(0.25 \div 0.32)$ $V_0(1 \div 4)$	$\kappa(1.1 \div 2)$ $\phi_0(2.3 \div 3.37)$ $\dot{\phi}_0(420 \div 500)$
$V(\phi) = V_0(\cosh(\psi\phi))^{-1}$	$H_0(50 \div 90)$ $\Omega_{m0}(0.25 \div 0.32)$ $V_0(10^{-3} \div 10^2)$	$\psi(10^{-3} \div 1)$ $\phi_0(1.4 \div 2.3)$ $\dot{\phi}_0(310 \div 420.7)$

## Model Parameters and Initial Conditions for Quintessence Potentials

Quintessence potentials	Free parameters	
$V(\phi) = V_0 M_{\text{pl}}^2 \phi^{-\alpha}$	$H_0(50 \div 90)$ $\Omega_{\text{m}0}(0.25 \div 0.32)$	$V_0(3 \div 5)$ $\alpha(10^{-6} \div 0.7)$
$V(\phi) = V_0 \exp(-\lambda\phi/M_{\text{pl}})$	$H_0(50 \div 90)$ $\Omega_{\text{m}0}(0.25 \div 0.32)$ $V_0(10 \div 10^3)$	$\lambda(10^{-7} \div 10^{-3})$ $\phi_0(0.2 \div 1.6)$ $\dot{\phi}_0(79.8 \div 338.9)$
$V(\phi) = V_0(\exp(M_{\text{pl}}/\phi) - 1)$	$H_0(50 \div 90)$ $\Omega_{\text{m}0}(0.25 \div 0.32)$ $V_0(10 \div 10^2)$	$\phi_0(1.5 \div 10)$ $\dot{\phi}_0(350 \div 850)$
$V(\phi) = V_0 \phi^{-x} \exp(\gamma\phi^2/M_{\text{pl}}^2)$	$H_0(50 \div 90)$ $\Omega_{\text{m}0}(0.25 \div 0.32)$ $V_0(10^{-2} \div 10^{-1})$ $\chi(4 \div 8)$	$\gamma(6.5 \div 7)$ $\phi_0(5.78 \div 10.55)$ $\dot{\phi}_0(680.6 \div 879)$
$V(\phi) = V_0(\cosh(\varsigma\phi) - 1)^g$	$H_0(50 \div 90)$ $\Omega_{\text{m}0}(0.25 \div 0.32)$ $V_0(5 \div 8)$ $\varsigma(0.15 \div 1)$	$g(0.1 \div 0.49)$ $\phi_0(1.8 \div 5.8)$ $\dot{\phi}_0(360 \div 685)$
$V(\phi) = V_0(\exp(\nu\phi) + \exp(v\phi))$	$H_0(50 \div 90)$ $\Omega_{\text{m}0}(0.25 \div 0.32)$ $V_0(1 \div 12)$	$\nu(6 \div 12)$ $\phi_0(0.014 \div 1.4)$ $\dot{\phi}_0(9.4 \div 311)$
$V(\phi) = V_0((\phi - B)^2 + A) \exp(-\mu\phi)$	$H_0(50 \div 90)$ $\Omega_{\text{m}0}(0.25 \div 0.32)$ $V_0(40 \div 70)$ $A(1 \div 40)$	$B(1 \div 60)$ $\mu(0.2 \div 0.9)$ $\phi_0(5.8 \div 8.45)$ $\dot{\phi}_0(681 \div 804.5)$
$V(\phi) = V_0 \sinh^m(\xi M_{\text{pl}}\phi)$	$H_0(50 \div 90)$ $\Omega_{\text{m}0}(0.25 \div 0.32)$ $V_0(1 \div 10)$ $m(-0.1 \div -0.3)$	$\xi(10^{-2} \div 1)$ $\phi_0(0.5 \div 2.5)$ $\dot{\phi}_0(190 \div 367)$
$V(\phi) = V_0 \exp(M_{\text{pl}}/\phi)$	$H_0(50 \div 90)$ $\Omega_{\text{m}0}(0.25 \div 0.32)$ $V_0(10^2 \div 10^3)$	$\phi_0(5.78 \div 10.55)$ $\dot{\phi}_0(680.6 \div 879)$
$V(\phi) = V_0(1 + \exp(-\tau\phi))$	$H_0(50 \div 90)$ $\Omega_{\text{m}0}(0.25 \div 0.32)$ $V_0(1 \div 10^2)$	$\tau(10 \div 10^2)$ $\phi_0(0.01 \div 0.075)$ $\dot{\phi}_0(9.4 \div 32)$

# MCMC Analysis

- **The normalized Hubble parameter:**  $E(z) = H(z)/H_0 = (\Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_\phi(z))^{1/2}$

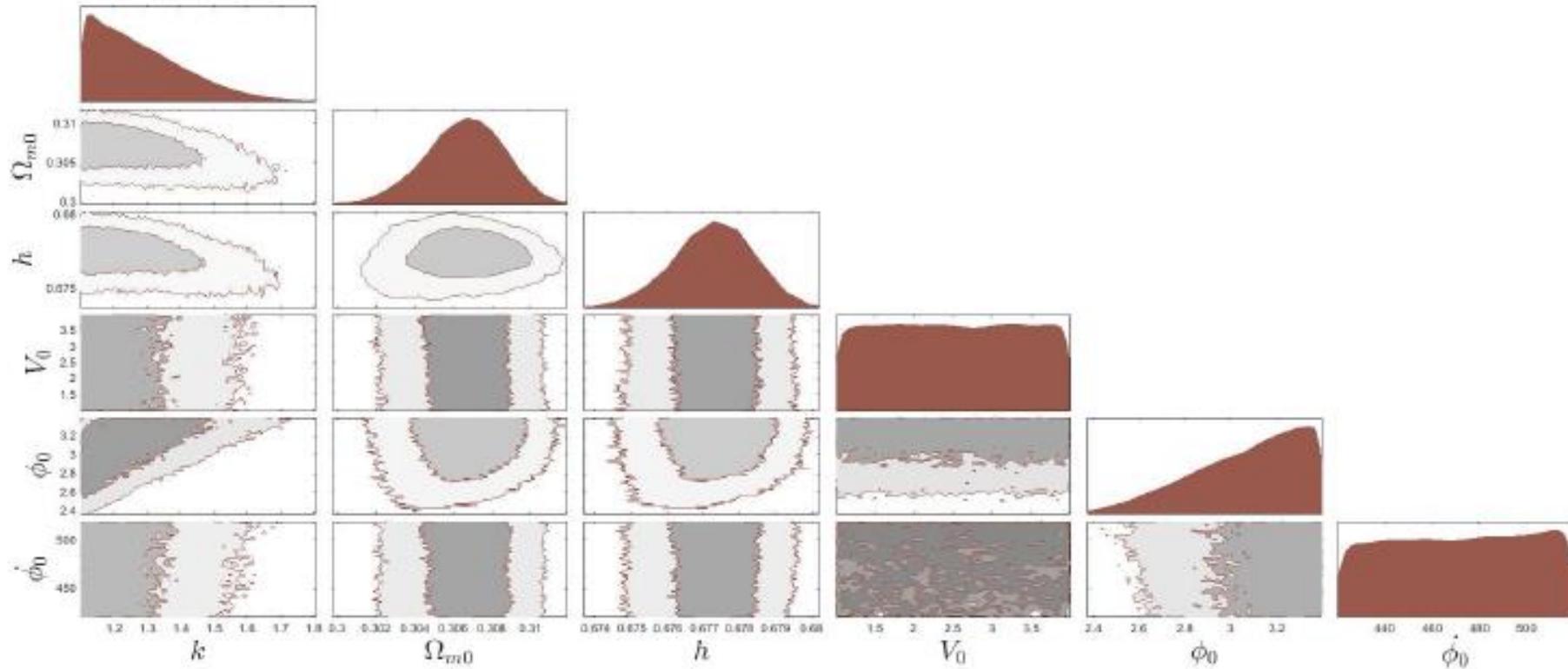
- **The angular diameter distance:** 
$$d_A(z) = \frac{1}{H_0(1+z)} \int_0^z \frac{dz'}{E(z')}$$

- **The combination of the growth rate of the matter density fluctuations and the matter power spectrum amplitude,  $f(a)\sigma_8(a)$  :**

$$\sigma_8(a) \equiv D(a)\sigma_8, \sigma_8 = 0.815 \text{ (Planck 2015 results. XIII)}$$

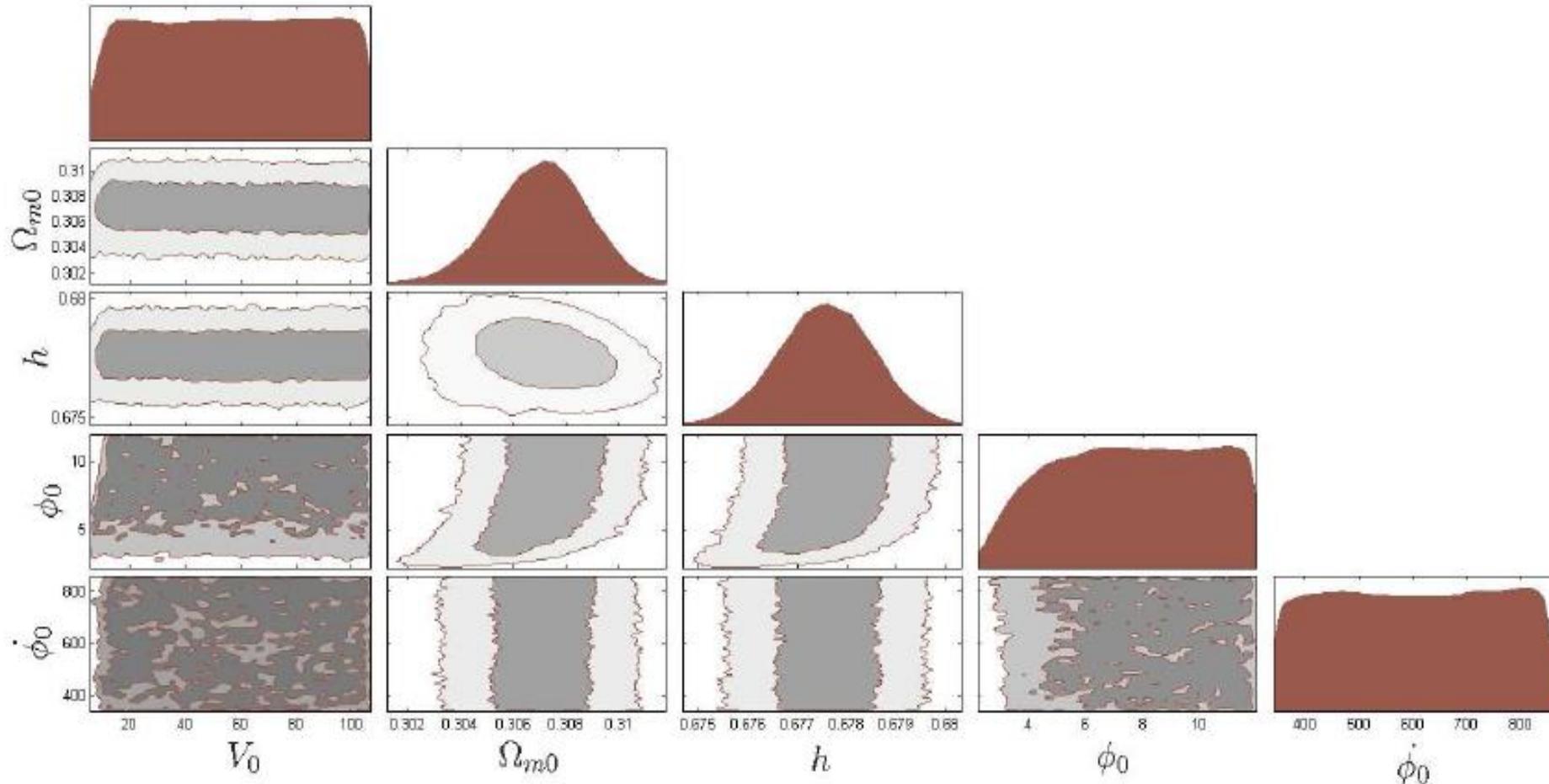
- **The range of the redshift values,  $z \in (0.15; 1.85)$**
- **The variances correspond to the predicted values for DESI data**

# MCMC Results



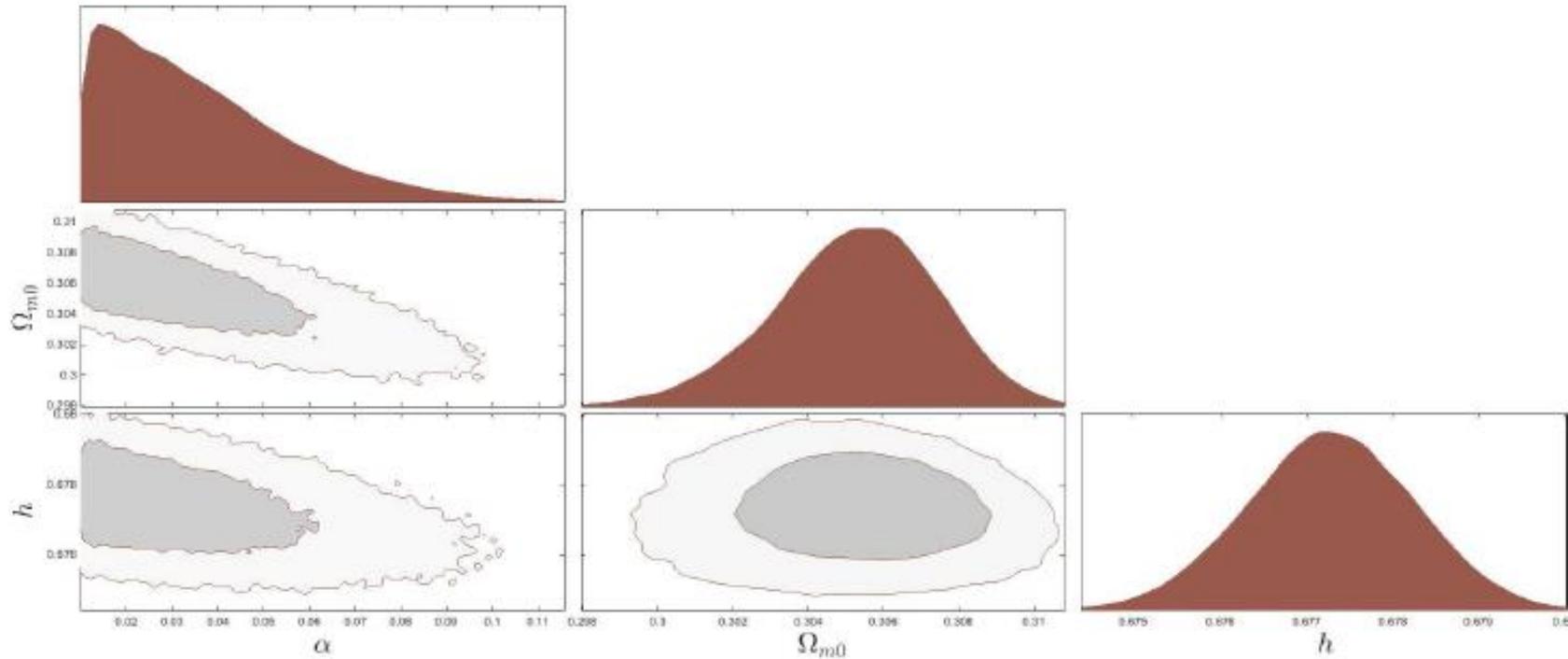
The  $2\sigma$  confidence level contours for pairs of the free parameters  $(k, \Omega_{m0}, h, V_0, \phi_0, \dot{\phi}_0)$ , for which the  $\phi$ CDM model with the phantom pseudo-Nambu-Goldstone boson potential  $V(\phi) = V_0(1 - \cos(\phi/\kappa))$  is in the best fit with the  $\Lambda$ CDM model.

# MCMC Results



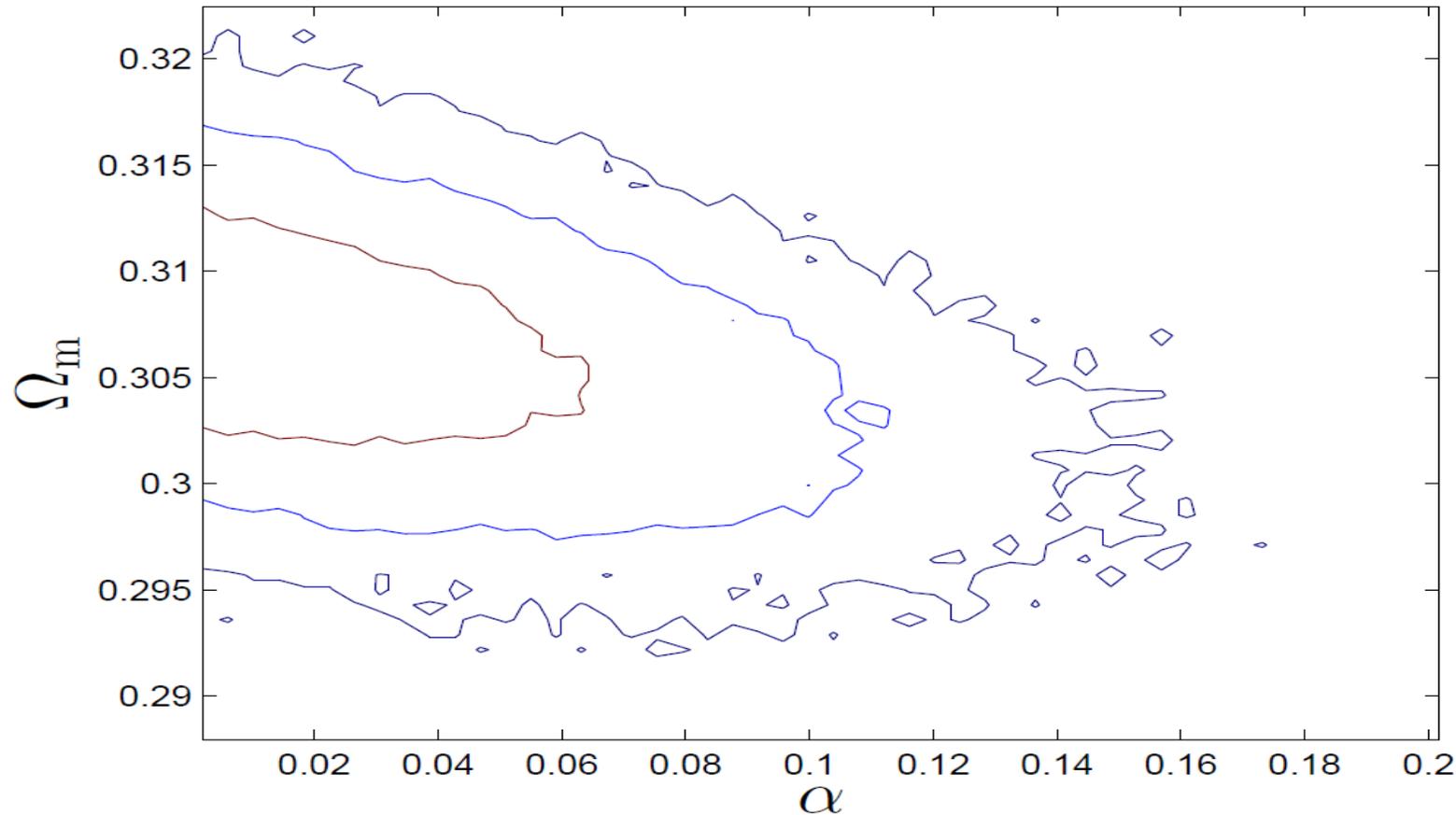
The  $2\sigma$  confidence level contours for pairs of the free parameters ( $V_0$ ,  $\Omega_{m0}$ ,  $h$ ,  $\phi_0$ ,  $\dot{\phi}_0$ ), for which the  $\phi$ CDM model with the Zlatev-Wang-Steinhardt potential  $V(\phi) = V_0(\exp(M_{\text{pl}}/\phi) - 1)$  is in the best fit with the  $\Lambda$ CDM model.

# MCMC Results



The  $2\sigma$  confidence level contours plots for various pairs of the free parameters ( $\alpha$ ,  $\Omega_{m0}$ ,  $h$ ), for which the  $\phi$ CDM model with the Ratra-Peebles potential  $V(\phi) = V_0 M_{\text{pl}}^2 \phi^{-\alpha}$  is in the best fit with the  $\Lambda$ CDM model.

# MCMC analysis with upcoming DESI data



Carrying out the MCMC analysis with upcoming DESI data, we obtained ranges of  $\alpha$  and  $\Omega_m$  parameters,  $0 < \alpha \leq 0.16$  and  $0.296 < \Omega_m < 0.32$  at  $3\sigma$  confidence level, where the Ratra-Peebles  $\phi$ CDM model is compliance with the  $\Lambda$ CDM model.

# Bayesian Statistics

- Akaike information criterion

$$AIC = -2 \ln \mathcal{L}_{\max} + 2k,$$

$$\mathcal{L}_{\max} \propto \exp(-\chi_{\min}^2/2)$$

$k$  is the number of observations

Akaike, H. (1974), *IEEE Transactions on Automatic Control* **19**(06), 716–723

- Schwarz information criterion

$$BIC = -2 \ln \mathcal{L}_{\max} + k \ln N,$$

$N$  is the number of the model parameters

Schwarz, G. E. (1978), *Annals of Statistics* **06**(02), 461–464

- Bayes evidence

$$\mathcal{E} = \int d^3 p \mathcal{P}(p),$$

$\mathcal{P}$  is the posterior likelihood

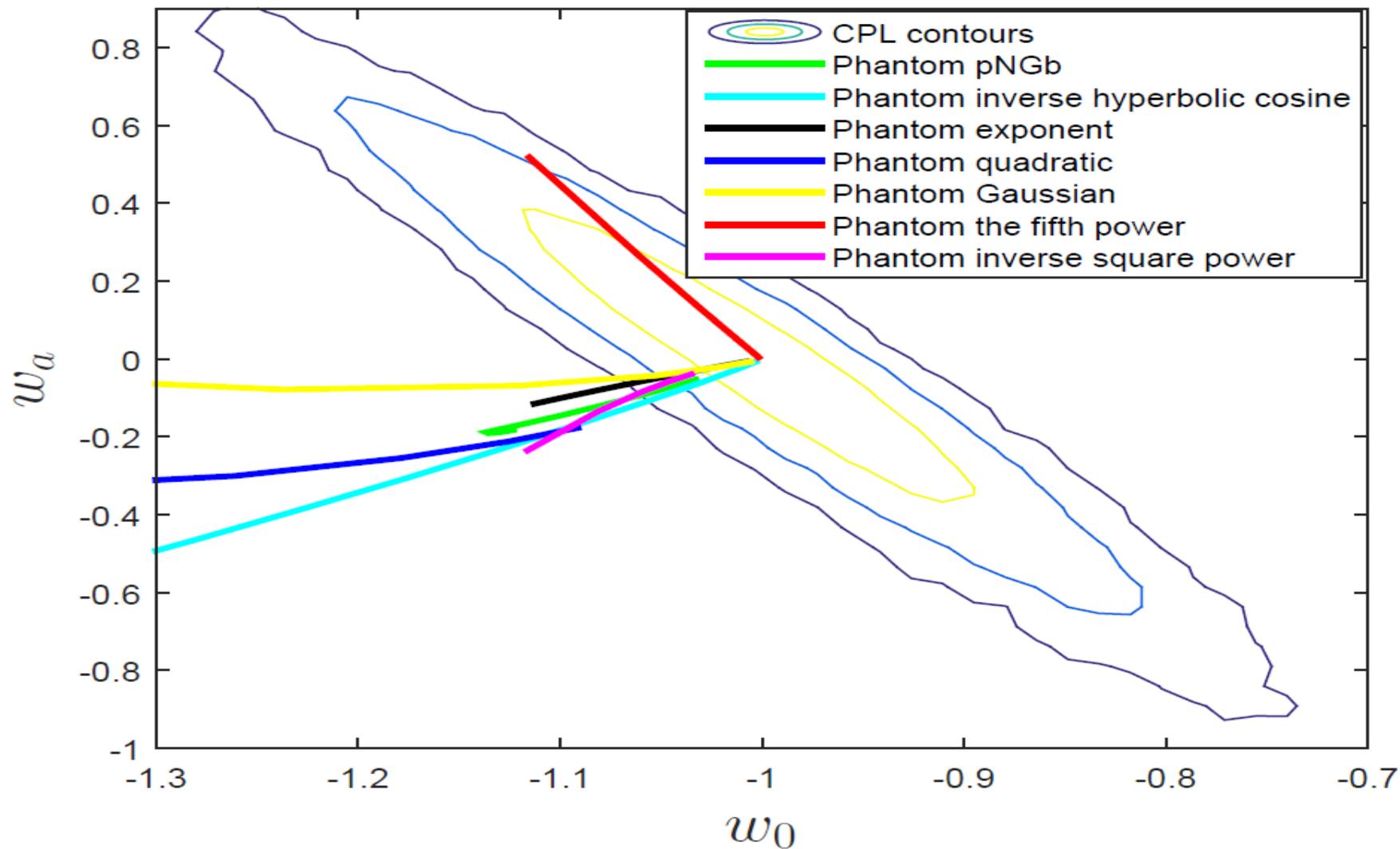
## Bayesian Statistics for the Phantom Potentials

Phantom potentials	AIC	BIC	Bayes factor
$V(\phi) = V_0\phi^5$	10.0	18.7	0.0921
$V(\phi) = V_0\phi^{-2}$	10.0	18.7	0.0142
$V(\phi) = V_0 \exp(\beta\phi)$	22.4	12.0	0.0024
$V(\phi) = V_0\phi^2$	10.0	18.7	0.0808
$V(\phi) = V_0(1 - \exp(\phi^2/\sigma^2))$	12.0	22.4	0.0113
$V(\phi) = V_0(1 - \cos(\phi/\kappa))$	12.0	22.4	0.0061
$V(\phi) = V_0(\cosh(\psi\phi))^{-1}$	12.0	22.4	0.0056

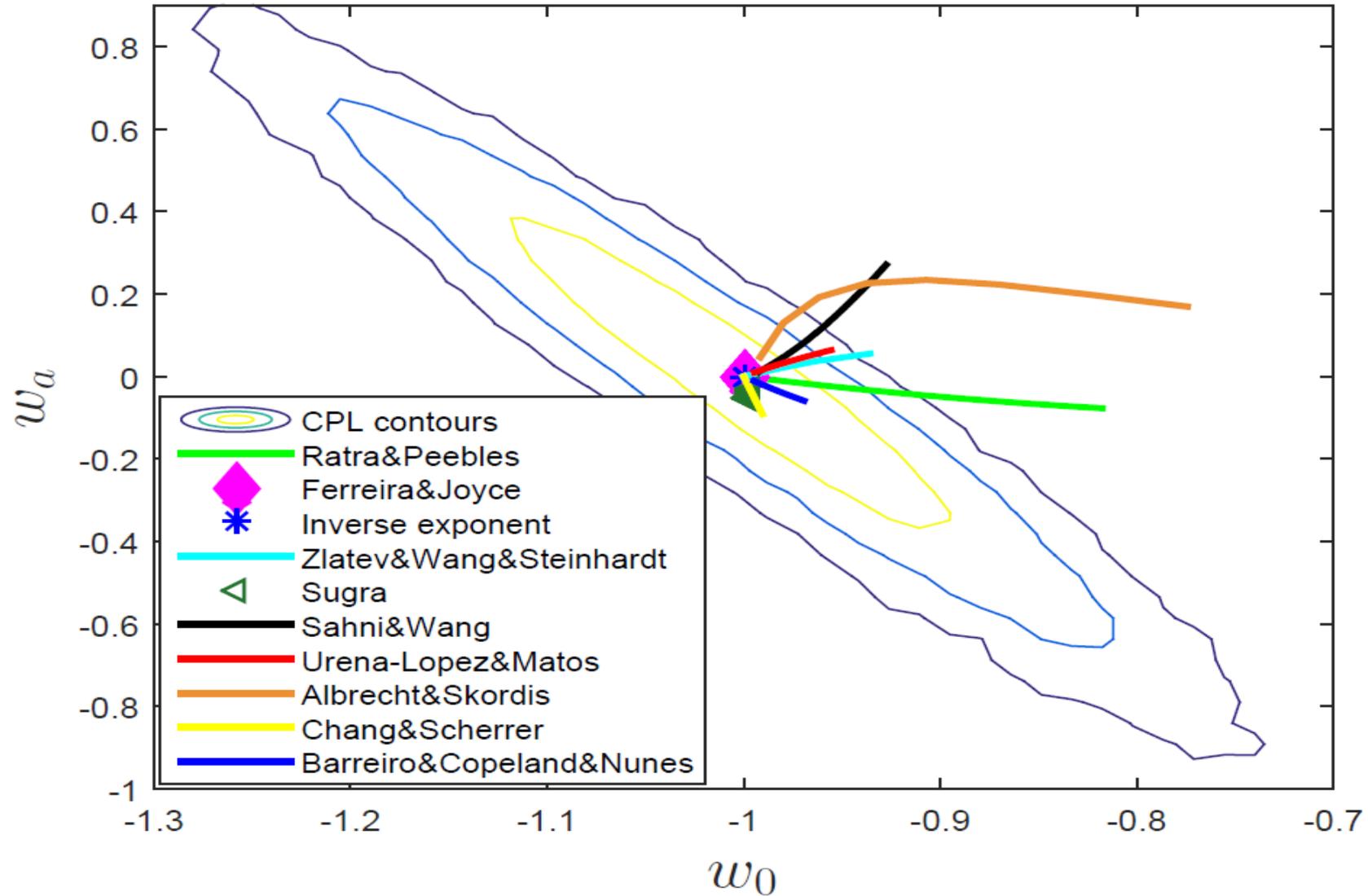
## Bayesian Statistics for the Quintessence Potentials

Quintessence potentials	AIC	BIC	Bayes factor
$V(\phi) = V_0 M_{\text{pl}}^2 \phi^{-\alpha}$	10	18.7	0.5293
$V(\phi) = V_0 \exp(-\lambda\phi/M_{\text{pl}})$	12	22.4	0.0059
$V(\phi) = V_0(\exp(M_{\text{pl}}/\phi) - 1)$	10	18.7	0.0067
$V(\phi) = V_0 \phi^{-x} \exp(\gamma\phi^2/M_{\text{pl}}^2)$	14	26.2	0.0016
$V(\phi) = V_0(\cosh(\zeta\phi) - 1)^g$	14	26.2	0.0012
$V(\phi) = V_0(\exp(\nu\phi) + \exp(v\phi))$	14	26.2	0.0053
$V(\phi) = V_0((\phi - B)^2 + A) \exp(-\mu\phi)$	16	29.9	0.0034
$V(\phi) = V_0 \sinh^m(\xi M_{\text{pl}}\phi)$	14	26.2	0.0014
$V(\phi) = V_0 \exp(M_{\text{pl}}/\phi)$	10	18.7	0.0077
$V(\phi) = V_0(1 + \exp(-\tau\phi))$	12	22.4	0.0024

# Phantom $\phi$ CDM Models in the CPL Phase Space



# Quintessence $\phi$ CDM Models in the CPL Phase Space



# Subclasses of the Potentials

Attractor solution	Usual solution
Sugra	Zlatev-Wang-Steinhardt
Ur̃ena-López-Matos	Sahni-Wang
Albrecht-Scordis	quadratic
Chang-Scherer	Gaussian
Barreiro-Copeland-Nunes	fifth power
pNGb	inverse square
inverse hyperbolic cosine exponent	

# Subclasses of the Potentials

Undistinguished from $\Lambda$ CDM model	Undistinguished or Distinguished from $\Lambda$ CDM model	Distinguished from $\Lambda$ CDM model
Ferreira-Joyce inverse exponent Sugra Chang-Scherrer Ur̃ena-López-Matos Barreiro-Copeland-Nunes fifth power	Ratra-Peebles Zlatev-Wang-Steinhardt Albrecht-Skordis Sahni-Wang pNGb inverse hyperbolic cosine exponent Gaussian inverse square	phantom quadratic

# Conclusion

- 1) We reconstructed quintessence and phantom scalar field  $\phi$ CDM models using the phenomenological method developed by us.
- 2) Using the Bayesian statistical analysis, we could not uniquely identify the preferable  $\phi$ CDM models compared to the fiducial  $\Lambda$ CDM model based on the predicted DESI data, and the  $\Lambda$ CDM model is a true model.
- 3) Mapping  $\phi$ CDM models in the phase space of the CPL- $\Lambda$ CDM contours, we could identify the subclasses of these models, which:
  - i) have the attractor and usual solutions,
  - ii) can be distinguished from the  $\Lambda$ CDM model,
  - iii) cannot be distinguished from the  $\Lambda$ CDM model,
  - iv) can be either distinguished or undistinguished from the  $\Lambda$ CDM model at present epoch.

**Thank You for Your Kind Attention!**