Bound on non-commutative parameter based on gravitational measurements

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Introduction

Quantum gravity is one of the biggest problems in modern physics, one of the ideas to solve this problem adopts the same concept as quantum mechanics in the quantization of concerning the commutation relations between observables, where in this theory the coordinates of spacetime subject to the commutation relations between themselves $[x^\mu, x^\nu] = i\Theta^{\mu\nu}$, where Θ is the non-commutative (NC) parameter, which describes the fundamental cell discretizing the space-time.

Objectives

Our aim is to investigate the four classical experimental tests of general relativity in the NC gauge theory of gravity, and to estimate the lower bound on the NC parameter using the deformed Schwarzschild black hole (SBH) metric as background [1], then we compare our results to the one found in different approach of NC geometry [2, 3]. We start by the NC periastron advance of some planets orbit in our solar system, where for the deflection of light, red-shift, and time delay we use data of a typical primordial black hole at the early universe, and we use the scale factor a to get a physical result [2, 3].

Primary hypothesis

1. We use the space-space NC parameter Θ^{ij}, according to [2] the scale factor α^2 for a physical results $\Theta^{phy} = \sqrt{a^2\Theta^2}$, where the scale factor at the end of inflation $a = 10^{-29}$ [2].
2. Data of a typical primordial black hole $GM \sim 5 \times 10^{-4} m$ and radius $r = 1.5 \times 10^{-3} m$ [3].
3. We use the fact that the NC correction should be smaller than the accuracy of the measurements [3].

Orbital motion

1 Periastron advance

In our previous work [12], we derived the expression of the angle deviation after one revolution in the NC SBH metric given by:

$$\Delta\phi = \frac{6\pi GM}{c^2\alpha(1-e^2)} + \pi\Theta^2\left[\frac{m_0^2 v^2 c^2}{2GM\alpha(1-e^2)} - \frac{6m_0^2 v^2}{\alpha^2(1-e^2)^2} + \frac{3m_0^2 c^2}{2\alpha^2(1-e^2)^2}\right] \tag{1}$$

where α, e denote the major semi-axis and the eccentricity of the motion.

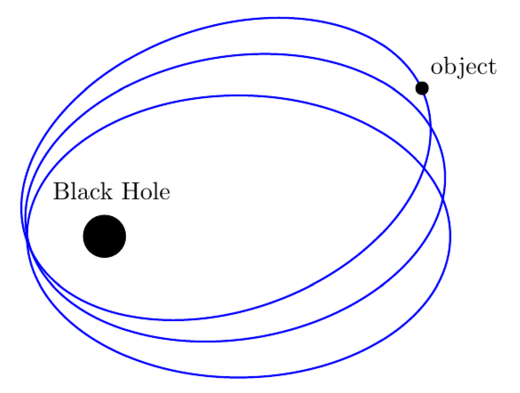

Figure 1: Periastron advance of orbit.

As numerical application we choose the problem of Mercury planet orbit, the lower bound on the NC parameter is computed using:

$$|\ \delta\phi_{NC}\ | \quad \leq \quad |\ \delta\phi_{GR} - \delta\phi_{obs}\ | \tag{2}$$

thus the NC parameter is of order:

$$\Theta^{phy} = \sqrt{\hbar\Theta} \leq 5.7876 \times 10^{-31} m \tag{3}$$

and for other planets in our solar system:

Planet	$\Delta\phi_{obs}$ $(\frac{arc-sec}{centry})$	$\Delta\phi_{GR}$ $(\frac{arc-sec}{centry})$	L.b of Θ^{phy} $(10^{-31} m)$
Mercury	42.9800 ± 0.0020	42.9805	≤ 05.7876
Venus	8.6247 ± 0.0005	8.6283	≤ 04.5239
Earth	3.8387 ± 0.0004	3.8399	≤ 04.0976
Mars	1.3565 ± 0.0004	1.3514	≤ 25.9393
Jupiter	0.0700 ± 0.0040	0.0623	≤ 01.5470
Saturn	0.0140 ± 0.0020	0.0137	≤ 02.1145

Light path

2 Red-shift

For an asymptotic observer at $r_2 \to \infty$, the measured red-shift for the NC SBH is given by $\hat{z}$:

$$\hat{z} = z\left[1 - \left(\frac{z+1}{z}\right)\left(\frac{88m^3 + m^2 r_1\left(-77 + 15\sqrt{1-\frac{2m}{r_1}}\right)}{32r_1^3(r_1-2m)^2} + \frac{-8mr_1^2\left(-2+\sqrt{1-\frac{2m}{r_1}}\right)}{32r_1^3(r_1-2m)^2}\right)\Theta^2\right] \tag{4}$$

where in the commutative case, the red-shift whose given by $z = \left(\sqrt{\left|\frac{g_{00}(r_2)}{g_{00}(r_1)}\right|} - 1\right)$.

We can fixe the bound on Θ:

$$\Theta^{phy} = \sqrt{\alpha^2\Theta^2} \leqslant 6.62 \times 10^{-34} m \tag{5}$$

3 Deflection of light

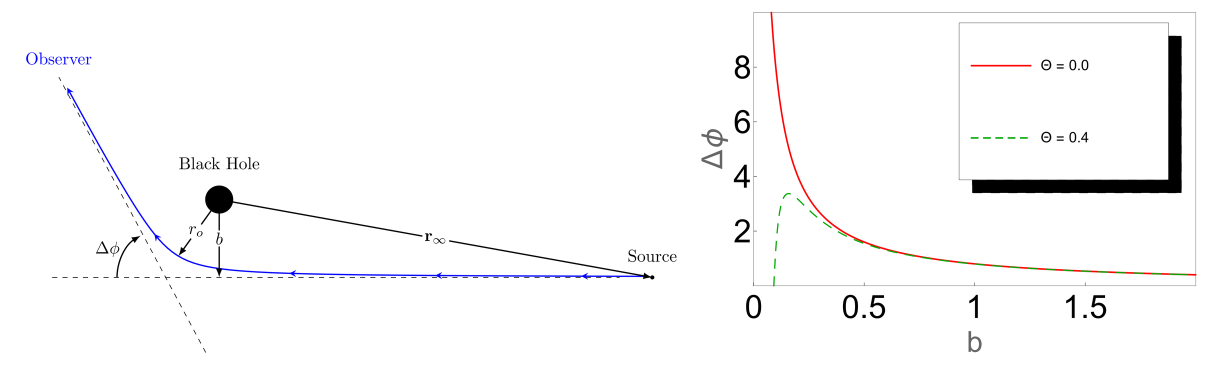

Figure 2: Deflection of light schema (left panel). Behavior of the NC deflection angle, with $\Theta = 0, 0.4$ (right panel).

The deflection of light passing near a strong gravitational field in the NC spacetime is given by:

$$\Delta\hat{\phi} = \frac{4GM}{c^2\, b} - \frac{5GM}{6c^2\, b^3}\Theta^2. \tag{6}$$

the lower bound of Θ in the case of $b = r$ is given by get:

$$\Theta^{phy} = \sqrt{\alpha^2\Theta^2} \leqslant 5.7 \times 10^{-34} m \tag{7}$$

Radar signal

4 Time delay

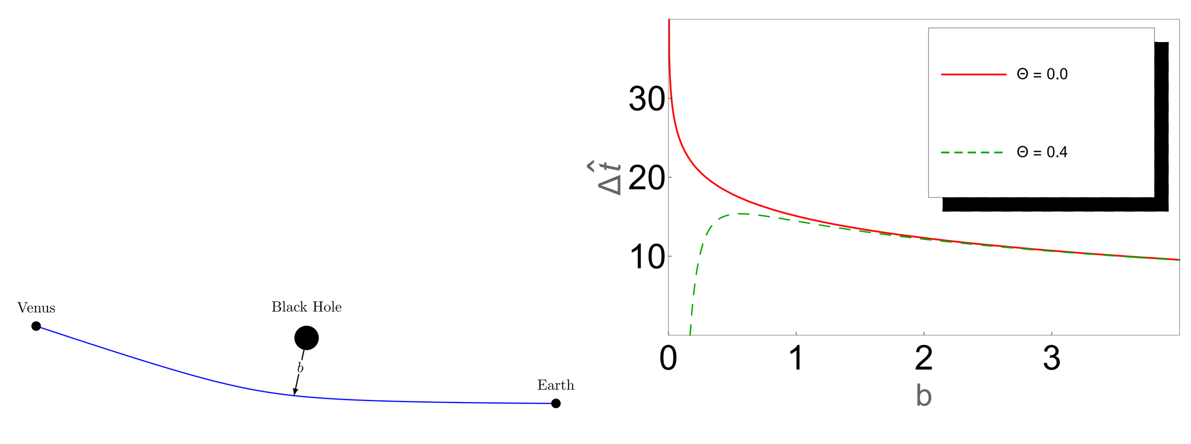

Figure 3: Schema of the Shapiro effect (left panel). Behavior of the NC time delay, with $\Theta =$ 0and0.4 (right panel).

The necessary time for a radar signal emitted from a point r_1 to another r_2 traveling near a massive object and returning to the emitting point in the NC space-time is given by:

$$\Delta\hat{t} \approx 4GM\left[ln\left(\frac{4r_1 r_2}{b^2}\right) + 1 - \frac{\Theta^2}{b^2}sin^2\theta\right] \tag{8}$$

we asumed the ration $\frac{4r_1r_2}{b^2}$ to be scale invariant: it has the same value for the solar system or for the microscopic black hole, so the lower bound is:

$$\Theta^{phy} = \sqrt{\alpha^2\Theta^2} \leqslant 2.57 \times 10^{-34} m \tag{9}$$

Discussion

Through this study, we used the NC Schwarzschild black hole metric [1] to investigate the four classical tests of general relativity in the NC spacetime. As primary results we show that the NC geometry removes the divergence behavior in deflection of light and time delay Fig. (1) and Fig. (2). The second significant result is the bound on the NC parameter using the periastron advance where the result shows that Θ^{phy} is in the same order in our solar system acts as a fundamental constant. For the rest of experimental tests, the bound of Θ^{phy} is watch better and is of order of $10^{-34} m$, compared to the bound in [2, 3] is where they found a θ bound in the order of $\sim 10^{-19} m$, our result is closer to the Planck scale and this is due to the use of the space-space NC parameter and the gauge theory of gravity. This confirms the quantization of the space-time with the fundamental length Θ^{phy}.
We summarize the results in the following table:

Experiment test	Physical bound on Θ^{phy} (m)
Periastron advance	$\Theta^{phy} \leqslant 5.7876 \times 10^{-31}$
Red-shift	$\Theta^{phy} \leqslant 6.62 \times 10^{-34}$
Deflection of light	$\Theta^{phy} \leqslant 5.7 \times 10^{-34}$
Time delay	$\Theta^{phy} \leqslant 2.57 \times 10^{-34}$

References

[1] A. Touati and S. Zaim. Geodesic equation in non-commutative gauge theory of gravity. *Chin. Phys. C*, 46(10):105101, 2022.

[2] S. Chingangbam, P.and Das et al. Constraint on noncommutative spacetime from planck data. *Phys. Rev. D*, 91(8):083503, 2015.

[3] M. Karimabadi, S. A. Alavi, and D. M. Yekta. Non-commutative effects on gravitational measurements. *Clas. Quan. Grav*, 37(8):085009, 2020.

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