physical sciences forum

MDPI

Proceedings

LRS Bianchi-I Transit Cosmological Models in f(R,T) Gravity †

Siwaphiwe Jokweni [1], Vijay Singh [1], Aroonkumar Beesham [1,2,3,*] and Binaya Kumar Bishi [1,4]

1 Department of Mathematical Sciences, University of Zululand, Kwa-Dlangezwa 3886, South Africa
2 Faculty of Natural Sciences, Mangosuthu University of Technology, Jacobs 4026, South Africa
3 National Institute for Theoretical and Computational Sciences (NITheCS), South Africa
4 Department of Mathematics, Lovely Professional University, Phagwara,Panjab, 144401,India.
* Correspondence: beeshama@unizulu.ac.za
† Presented at the 2nd Electronic Conference on Universe, 16 February–2 March 2023; Available online: https://ecu2023.sciforum.net/.

Abstract: A locally rotationally symmetric Bianchi-I model is explored both in general relativity and in f(R,T) gravity, where R is the Ricci scalar and T is the trace of the energy-momentum tensor. Solutions have been found by means of a special Hubble parameter, yielding a hyperbolic hybrid scale factor. Some geometrical parameters have been studied. A comparison is made between solutions in general relativity and in f(R,T) gravity, where in both the theories, the models exhibit rich behavior from stiff matter to quintessence, phantom, then later mimicking the cosmological constant, depending on some parameters.

Keywords: LRS Bianchi I model; Hubble parameter; Modified theory of gravity; Dark energy

Citation: Jokweni, S.; Singh, V.; Beesham, A.; Bishi, B.K. LRS Bianchi-I transit cosmological models in f(R,T) gravity. *Phys. Sci. Forum* **2023**, *3*, x. https://doi.org/10.3390/xxxxx

Published: 18 February 2023

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Copyright: © 2023 by the authors. Submitted for possible open access publication under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

Observational data from refs. [1-3] suggest that the universe is currently in an accelerating epoch. A plethora of attempts have been made to explain this phenomenon, but neither of them is compelling. The first attempt is dark energy (DE), which is the hypothesis of exotic matter with the unique feature of anti-gravity due to highly negative pressure which hence accelerates the expansion of the universe [4]. On the other hand, there is insufficient information about DE from the ΛCDM model in general relativity (GR). The cosmological constant (CC) is the primary candidate for DE and the second candidate is modified gravity. Hence the shortcomings [5] from the ΛCDM model enable authors to consider other alternatives to fundamental theories of astrophysics and cosmology. These include the dynamical candidates of DE and modified theories of gravity, e.g., higher derivative theories, Gauss-Bonnet f(G) gravity, f(R) theory, f(T) and f(R,T) gravity theory. Harko et al [6] introduced f(R,T) gravity, where f(R,T) is an arbitrary function of the Ricci scalar R, and the trace T of the energy-momentum tensor.

Over the years, cosmologists have solved the field equations by means of assuming some cosmological parameters, i.e., Hubble parameter, scale factor, and even some form of deceleration parameter, based on the current understanding in cosmology that the universe has undergone stages of evolution, i.e., inflation, radiation, matter, and late time acceleration. Based on that, the notion of varying deceleration parameter, which changes signature from deceleration to acceleration, has been applied to many cosmological models. In ref. [7], the authors developed a hyperbolic scale factor. This form of scale factor has attracted a lot of attention over the years, where it was applied for both homogeneous and isotropic or anisotropic space-times through various contexts in cosmology, i.e., see ref. [8]. Recently, the Bianchi-I model with a perfect fluid with various cases of cosmological constant was considered [9].

2. The formalism of the model in $f(R, T)$ gravity

The general action of f (R,T) gravity with units in which$8\pi G = c = 1$is given by [6]

$$S = \frac{1}{2}\int [f(R,T) + 2L_m]\sqrt{-g}d^4x \tag{1}$$

After manipulation, for $f(R,T) = R + 2f(T)$, we get:

$$R_{ij} - \frac{1}{2}Rg_{ij} = T_{ij} - 2(T_{ij} + pg_{ij})f'(T) + f(T)g_{ij} \tag{2}$$

where a prime represents an ordinary derivative of *f(T)* with respect to *T*. In this work we have chosen $f(T) = \lambda T$, i.e., $f(R,T) = R + 2\lambda T$. The energy momentum tensor (EMT) of a perfect fluid is $T_{ij} = (\rho + p)u_i u_j - pg_{ij}$ with ρ and p the energy density and thermodynamic pressure, respectively. The trace of the energy-momentum tensor is$T = \rho - 3p$, Equation (8) then yields:

$$R_{ij} - \frac{1}{2}Rg_{ij} = (1 + 2\lambda)T_{ij} + \lambda(\rho - p)g_{ij}. \tag{3}$$

In this work, we have considered the LRS Bianchi-I spacetime:

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)[dy^2 + dz^2], \tag{4}$$

where A and B are the scale factors and functions of cosmic time t. The average scale factor for the metric (16) is defined as

$$a = (AB^2)^{\frac{1}{3}} \tag{5}$$

The average Hubble parameter is given by

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right), \tag{6}$$

where a dot denotes the derivative with respect to t.

It is crucial to mention that the coupling between geometry and matter in f (R,T) gravity adds some additional terms visible on the RHS of the field equations. These terms must be treated as matter that can be called “coupled matter”. Therefore, to distinguish between the main matter and coupled matter, we replace p with $p_m$and ρ with ρ_m which represents the primary or main matter.

Using the line element (4) and the energy-momentum tensor (EMT) of a perfect fluid, the field equations (3) yield:

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} = (1 + 3\lambda)\rho_m - \lambda p_m, \tag{7}$$

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\ddot{B}}{B} = -(1 + 3\lambda)p_m + \lambda\rho_m, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = -(1 + 3\lambda)p_m + \lambda\rho_m. \tag{9}$$

These equations consist of four unknowns, namely, A, B, p, ρ. Therefore, to find exact solutions, one supplementary constraint is required. We assume the following relationship between the Hubble parameter and cosmic time [7]:

$$H(t) = m + nCoth(t), \tag{10}$$

This yields:

$$a(t) = e^{mt}\left(Sinh(t)\right)^{n}, \tag{11}$$

where m, n are positive constants. Then the deceleration parameter (q) is given by

$$q = -1 + \frac{n}{\left(mSinh(t)+nCosh(t)\right)^{2}}, \tag{12}$$

and is illustrated in Figure 1.

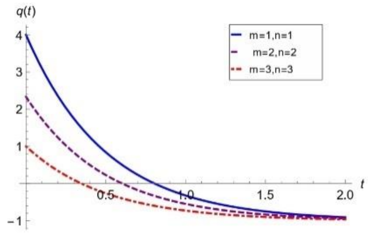

Figure 1. Deceleration parameter vs time.

To eschew repetition, $H(t)$, $a(t)$, $q(t)$ have been articulated in [7]. Then the energy density and pressure are given by:

$$\rho_m = \frac{3m^2}{1+4\lambda} - \frac{Q^2}{3(1+2\lambda)\left[e^{6mt}\left(Sinh(t)\right)^{6n}\right]} + \frac{3m^2\left(Csch(t)\right)^2\left(Cosh(t)\right)^2}{1+4\lambda} + \frac{3mn\left(Csch(t)\right)^2 Sinh(2t)}{1+4\lambda} + \frac{2\lambda\left(Csch(t)\right)^2}{(1+2\lambda)(1+4\lambda)} \tag{13}$$

$$p_m = -\frac{3m^2}{1+4\lambda} - \frac{Q^2}{3(1+2\lambda)\left[e^{6mt}\left(Sinh(t)\right)^{6n}\right]} - \frac{3m^2\left(Csch(t)\right)^2\left(Cosh(t)\right)^2}{1+4\lambda} - \frac{3mn\left(Csch(t)\right)^2 Sinh(2t)}{1+4\lambda} + \frac{2m\left(Csch(t)\right)^2(1+3\lambda)}{(1+2\lambda)(1+4\lambda)}, \tag{14}$$

where Q is a constant of integration.

For a physical realistic cosmological model, the density ρ_m must be positive. Unfortunately, at early times, we find that the density ρ is negative. We shall comment on this later. Also, the pressure is negative throughout the evolution. The equation of state parameter (EoS), $\omega = p/\rho$, is given by:

$$\omega_m = \frac{-\frac{3m^2}{1+4\lambda} - \frac{Q^2}{3(1+2\lambda)\left[e^{6mt}\left(Sinh(t)\right)^{6n}\right]} - \frac{3m^2\left(Csch(t)\right)^2\left(Cosh(t)\right)^2}{1+4\lambda} - \frac{3mn\left(Csch(t)\right)^2 Sinh(2t)}{1+4\lambda} + \frac{2m\left(Csch(t)\right)^2(1+3\lambda)}{(1+2\lambda)(1+4\lambda)}}{\frac{3m^2}{1+4\lambda} - \frac{Q^2}{3(1+2\lambda)\left[e^{6mt}\left(Sinh(t)\right)^{6n}\right]} + \frac{3m^2\left(Csch(t)\right)^2\left(Cosh(t)\right)^2}{1+4\lambda} + \frac{3mn\left(Csch(t)\right)^2 Sinh(2t)}{1+4\lambda} + \frac{2\lambda\left(Csch(t)\right)^2}{(1+2\lambda)(1+4\lambda)}}$$

2.1. The behavior of coupled matter

As elucidated above, ρ_m and p_m do not represent the effective matter in this model of $f(R,T)$ gravity. The terms containing λ in Eqs. (7)–(9) can be assumed to be associated with the coupled matter. By separating these terms, the equations can be expressed as

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} = \rho_m + \rho_f \tag{16}$$

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\ddot{B}}{B} = -(p_m + p_f) \tag{17}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}=-(p_m+p_f) \tag{18}$$

where $\rho_f = \lambda\,(3\rho_m - p_m)$ and $p_f = \lambda(3p_m - \rho_m)$, respectively, represents the energy density and pressure of the coupled matter, and are obtained as:

$$\rho_f=\frac{12\lambda m^2}{1+4\lambda}+\frac{2\lambda Q^2}{3(1+2\lambda)\left[e^{6mt}\left(Sinh(t)\right)^{6n}\right]}+\frac{12\lambda m^2\left(Csch(t)\right)^2\left(Cosh(t)\right)^2}{1+4\lambda}+\frac{12\lambda mn\left(Csch(t)\right)^2 Sinh(2t)}{1+4\lambda}-\frac{2m\lambda\left(Csch(t)\right)^2}{(1+2\lambda)(1+4\lambda)} \tag{19}$$

$$p_f=-\frac{12\lambda m^2}{1+4\lambda}-\frac{2\lambda Q^2}{3(1+2\lambda)\left[e^{6mt}\left(Sinh(t)\right)^{6n}\right]}-\frac{12\lambda m^2\left(Csch(t)\right)^2\left(Cosh(t)\right)^2}{1+4\lambda}-\frac{12\lambda mn\left(Csch(t)\right)^2 Sinh(2t)}{1+4\lambda}+\frac{2m\lambda\left(Csch(t)\right)^2}{(1+2\lambda)(1+4\lambda)} \tag{20}$$

The EOS for the coupled matter is:

$$\omega_f=\frac{-\frac{12\lambda m^2}{1+4\lambda}-\frac{2\lambda Q^2}{3(1+2\lambda)\left[e^{6mt}\left(Sinh(t)\right)^{6n}\right]}-\frac{12\lambda m^2\left(Csch(t)\right)^2\left(Cosh(t)\right)^2}{1+4\lambda}-\frac{12\lambda mn\left(Csch(t)\right)^2 Sinh(2t)}{1+4\lambda}+\frac{2m\lambda\left(Csch(t)\right)^2}{(1+2\lambda)(1+4\lambda)}}{\frac{12\lambda m^2}{1+4\lambda}+\frac{2\lambda Q^2}{3(1+2\lambda)\left[e^{6mt}\left(Sinh(t)\right)^{6n}\right]}+\frac{12\lambda m^2\left(Csch(t)\right)^2\left(Cosh(t)\right)^2}{1+4\lambda}+\frac{12\lambda mn\left(Csch(t)\right)^2 Sinh(2t)}{1+4\lambda}-\frac{2m\lambda\left(Csch(t)\right)^2}{(1+2\lambda)(1+4\lambda)}}$$

2.2 State-finder parameter, energy conditions and Stability

2.2.1 State-finder analysis

The state finder pair {r, s} allows the examining the features of DE for the model, and to compare with the ΛCDM model. They rely on the third derivative of the scale factor, as they were introduced in ref. [10]. In this model, they are given by

$$r=\frac{\dddot{a}}{aH^2}=\frac{3mn+m^2+n(2-3n+3m^2)Coth(t)+3mn(n-1)\left(Coth(t)\right)^2+n(2-3n+n^2)\left(Coth(t)\right)^3}{(m+nCoth(t))^3}$$

$$s=\frac{r-1}{3(q-\frac{1}{2})}=\frac{4n(3m+2)Coth(t)}{3(m+nCoth(t))[3n^2-3m^2-4n+3(n^2+m^2)Cosh(2t)+6mnSinh(2t)]}$$

We observe from the above that as $t \to 0$, {r, s} → {0, 0.6}, and as $t \to \infty$, {r, s} → {1, 0}. In this model, initially we have $r < 1$and $s > 0$, which imply quintessence and phantom, respectively. At late times, the model mimics the ΛCDM model.

2.2.2 Energy Conditions

The behaviour of the energy conditions such as the Weak Energy Condition (WEC: $\rho_m \geq 0, \rho_m + p_m \geq 0$), Dominant Energy Condition (DEC: $\rho_m \geq |p_m|$) and Strong Energy Condition (SEC: $\rho_m + p_m \geq 0, \rho_m + 3p_m \geq 0$), have been studied. They are illustrated below:

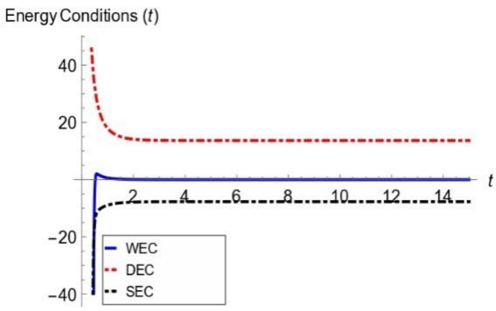

Figure 2. Behaviour of energy conditions.

2.2.3 Stability of the models through the speed of sound

It is crucial to study the stability of the theory, and here, we make use of the technique of speed of sound to study stability [11]. Numerous other techniques can be used to understand the stability of the solutions/methods [12]. The speed of sound is given by $v_s^2 = \frac{dp}{d\rho}$. If $v_s^2 < 0$ or $v_s^2 > 1$, the system is unstable, whereas if $0 \leq v_s^2 = \frac{dp}{d\rho} \leq 1$, the system is stable. In this model

$$v_s^2 = -\frac{378[3+3Coth(t)+3(Coth(t))^2] - 2106e^{-18t}\left(Csch(t)\right)^{18}[1+Coth(t)] - 18Coth(t)\left(Csch(t)\right)^2[-20+63((Cosh(t))^2+Sinh(2t))]}{36[126(3+3Coth(t)+3(Coth(t))^2)] + 351e^{-18t}\left(Csch(t)\right)^{18}[1+Coth(t)] - 3Coth(t)\left(Csch(t)\right)^2[-1+126((Cosh(t))^2+Sinh(2t))]}$$

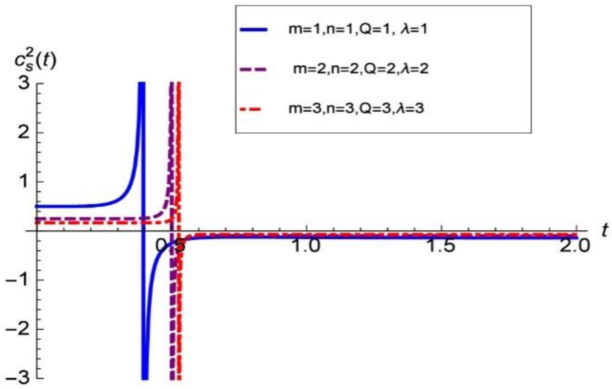

Figure 3. Squared of sound speed against time.

We find that for the values (m = 1, n = 1, Q =1, $\lambda = 1$) , (m = 2, n = 2, Q =2, $\lambda = 2$), (m = 3, n = 3, Q =3, $\lambda = 3$), our model is stable (during the early phase of the model), whereas, at present, it is unstable. This is illustrated in Figure 3.

3. Discussions

We have studied an LRS Bianchi I model with the special Hubble parameter $H = m + nCoth(t)$, yielding $q = -1 + \frac{n}{(mSinh(t)+nCosh(t))^2}$, where m, n are positive constants. We find that the model transits from early deceleration ($q = -1 + \frac{1}{n} > 0$ for $1 < n < 0$) to late time acceleration ($t \to \infty$, $q = -1$) in line with observations [1-3]. In any physically realistic cosmological model, the energy density (ρ_m, ρ_f) ought to be positive throughout the evolution. Here, we observe that for ($m, n, Q, \lambda > 0$) initially, both densities are negative but later positive. Due to the complicated nature of the expressions, we have not checked the density for all values of the parameters. So, it is quite possible that the density could be positive for some parameters, and we are investigating this further, and hope to report elsewhere. However, our results do indicate that models have to be checked very carefully to ensure that all reasonable conditions are met. The pressure is negative, which is associated with late-time acceleration.

The DEC is satisfied, but not the SEC. Again, this is in keeping with the late-time acceleration of the model (Figure 2). Initially, the model starts evolving as stiff matter (see Figure 3). It then evolves to semi-realistic matter ($\omega_m > 1$), radiation ($\omega_m = 1/3$) and dust ($\omega_m = 0$). It then transits to quintessence ($\omega_m = -1/3$) , crossing the phantom divide line ($\omega_m < -1$), and finally mimics the cosmological constant ($\omega_m = -1$). To avoid a similar discussion, we note that both normal and coupled matter behave in a similar way as illustrated in Figure 3. The other important aspect of the model is the effective matter for $\lambda = 0$, in which the solutions of GR are recovered. The effective matter behaves like in $f\,(R,T)$ gravity due to similar metric potentials in both theories. We can conclude that even in $f\,(R,T)$, the model accelerates at late times.

The material presented here is brief, and an extended version will be presented elsewhere.

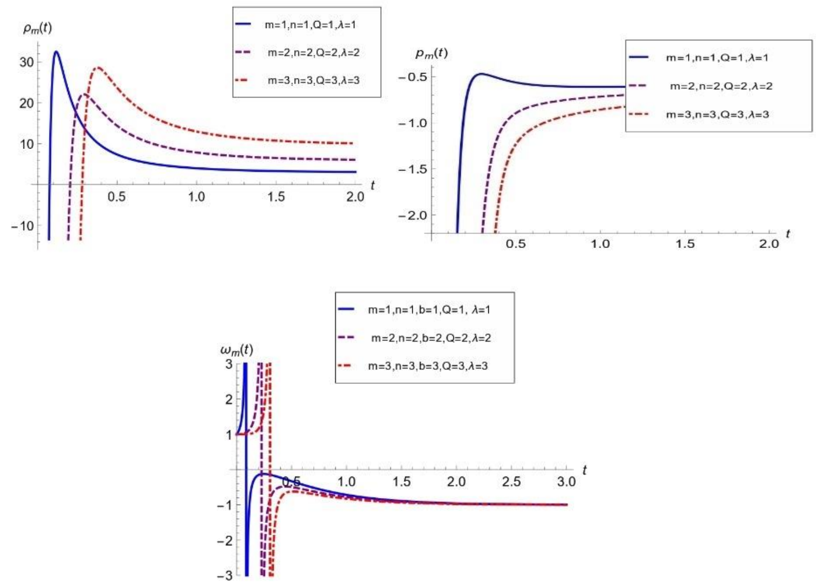

Figure 3. Behaviour of density, pressure and EoS parameter, respectively.

Author Contributions: Conceptualization, S.J. and A.B.; methodology, S.J.; software, S.J.; validation, S.J., A.B. and B.K.B.; formal analysis, S.J.; investigation, S.J.; resources, B.K.B.; data curation, A.B.; writing—original draft preparation, S.J.; writing—review and editing, A.B.; visualization, S.J.; supervision, A.B.; project administration, A.B.; funding acquisition, A.B. All authors have read and agreed to the published version of the manuscript.

Funding: This work is based on research supported wholly/ in part by the National Research Foundation of South Africa (NRF), grant numbers (118511).

Institutional Review Board Statement: Not applicable

Informed Consent Statement: Not applicable

Data Availability Statement: Not applicable

Acknowledgments: we thank the reviewers for their valuable comments which led to an improvement in the paper.

Conflicts of Interest: The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

References

1. Perlmutter, S.; Aldering, G.; Goldhaber, G.; Knop, R.A.; Nugent, P.G.; Castro, P.G.; Deustua, S.; Fabbro, S.; Goobar, A.; Groom, D.E.; et al. Measurements of Ω and Λ from 42 High-Redshift Supernovae. *Astrophys. J.***1999**, *517*, 565–586.
2. Riess, A.G.; Filippenko, A.V.; Challis, P.; Clocchiatti, A.; Diercks, A.; Garnavich, P.M.; Gilliland, R.L.; Hogan, C.J.; Jha, S.; Kirshner, R.P.; et al. Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *Astron. J.***1998**, *116*, 1009–1038.

3. Allen, S.W.; Schmidt, R.W.; Ebeling, H.; Fabian, A.C.; Van Speybroeck, L. Constraints on dark energy from Chandra observations of the largest relaxed clusters. *Mon. Not. R. Astron. Soc.* **2004**, *353*, 457.
4. Copeland, E.J.; Sami, M.; Tsijikawa, S. Dynamics of dark energy. *Int. J. Mod. Phys. D***2006**, *15*, 1753.
5. Zlatev, I.; Wang, L.; Steinhardt, P.J. Quintessence, cosmic coincidence, and the cosmological constant.*Phys. Rev. Lett.***2008**, *82*, 896.
6. Harko, T.; Lobo, F.S.N.; Nojiri, S.; Odintsov, S.D. f(R,T) gravity. *Phys. Rev. D***2011**, *84*, 24020.
7. Nagpal, R.; Singh, J.K.; Beesham, A. and Shabani, H. Cosmological aspects of a hyperbolic solution in f(R,T) gravity. Annals of Phys. **2019,** 405, pp.234-255.
8. Singh, J.P.;Baghel, P.S.; and Singh, A. Perfect fluid Bianchi typeI cosmological models with time-dependent cosmological term Lambda. Int. J. Mod. Phys. D**2020**, 15(08), p.1850132.
9. Nagpal, R.; Singh, J.K.; Beesham, A. and Shabani, H. Cosmological aspects of a hyperbolic solution in f(R,T) gravity. Annals of Physics. **2019**, 405, pp.234-255.
10. Alam, U.;Sahni, V.; Deep Saini, T.; and Starobinsky, A.A. Exploring the expanding universe and dark energy using the Statefinder diagnostic. Mon. Not. Royal Astron. Soc. **2003,** 344(4), pp.1057-1074.
11. Zhao, G.B.; Xia, J.Q.; Li, M.; Feng, B. and Zhang, X. Perturbations of the quintom models of dark energy and the effects on observations. Phys. Rev. D**2005**, 72(12), p.123515.
12. Sadeghi, J.;Setare, M.R.; Amani, A.R. and Noorbakhsh, S.M. Bouncing universe and reconstructing vector field. Physics Letters B. **2010**,685(4- 5), pp.229-234.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.