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Bose-Einstein Condensate in Synchronous Coordinates †

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Abstract: Analytical spherically symmetric static solution to the set of Einstein and Klein-Gordon equations in a synchronous reference frame is considered. In synchronous reference frame, a static solution exists in the ultrarelativistic limit $p = -\varepsilon/3$. Pressure p is negative when matter tends to contract. The solution pretends to describe a collapsed black hole. The balance at the boundary with dark matter ensures the static solution for a black hole. There is a spherical layer inside a black hole between two "gravitational" radii r_g and $r_h > r_g$, where the solution exists, but it is not unique. In a synchronous reference frame $\det g_{ik}$ and g^{rr} do not change signs. The non-uniqueness of solutions with boundary conditions at $r = r_g$ and $r = r_h$ makes it possible to find the gravitational field both inside and outside a black hole. The synchronous reference frame allows one to find the rest mass of the condensate. In the model "$\lambda|\psi|^4$" total mass $M = 3(c^2/2k)r_h$ is three times of what a distant observer sees. This gravitational mass defect is spent for bosons to be in the bound ground state, and for the balance between elasticity and density of the condensate.

Keywords: black hole; dark matter; gravitating Bose-Einstein condensate; synchronous reference frame

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1. Introduction. In Schwarzschild coordinates

When we consider a gravitational field created by spherically symmetric matter, it is customary to proceed from the Schwarzschild metric [1]:

$$ds^2 = g_{ik}dx^i dx^k = e^{2F_0}\left(dx^0\right)^2 - e^{2F_1}dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \quad (1)$$

Exponential representation $g_{11} = -e^{2F_1}$ and $g_{00} = e^{2F_0}$ fixes signs of the metric tensor components $g_{00} > 0$, $g_{11} < 0$ and the determinant $g = \det g_{ik} < 0$. According to Einstein's hypothesis [2] $\det g_{ik}$ "always has a finite and negative value". Setting $g_{11} = -e^{2F_1}$ in the metric (1), we fix the sign of the component g_{11}. Thus, not caring about the presence or absence of a singularity, we exclude $g_{11} > 0$ from consideration as non-physical. In this case, the coordinate system turns out to be incomplete [3–5]. A critical mass M_{cr} arises (for neutron stars it is of the order of the Sun mass), so that for $M > M_{cr}$ regular static solutions to Einstein equations do not exist [6–8]. Shwartzshild's solution [1]

$$ds^2 = \left(1 - r_g / r\right)\left(dx^0\right)^2 - \left(1 - r_g / r\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{2}$$

is not regular in the center $r = 0$, though it describes a gravitational field in a vacuum far away from spherically symmetric matter, regardless of the mass, seen by a distant observer. For $M < M_{cr}$, metric (2) asymptotically coincides with the regular solution in [5] for $r >> r_g$.

It is believed that objects with a mass greater than the critical one are subject to unlimited compression [9]. The absence of static solutions in metric (1) for objects with a mass greater than the critical one contradicts the existence of the object with a mass seven orders of magnitude greater than the Sun mass [10] in the center of our Milky Way galaxy. "Unlimitedly collapsing objects" at the centers of galaxies are called black holes. Black holes in the centers of galaxies, like the galaxies themselves, exist as long as the universe exists. If we assume that the contraction is non-stop, then at the centers of galaxies we meet a singularity, contrary to Einstein's hypothesis that "this is nowhere to be found"[2]. If we take into account that in the process of unlimited compression with increasing pressure chemical reactions transform neutrons into more "elementary particles", then it can slow down and even stop compression. In order to find a regular static solution, describing the state of matter to which a collapse can lead (with no restriction on mass), I had abandoned the sign-fixing representation $g_{11} = -e^{2F_1}$. It is sufficient to restrict ourselves by a weaker condition of regularity: all invariants of the metric tensor are finite. With mass $M < M_{cr}$, in a static state, gravitational contraction is compensated by elasticity of fermions. With $M > M_{cr}$, elasticity of fermions can not resist the gravitational contraction. For a degenerate relativistic Fermi gas, the critical mass $M_{cr\,f} \sim M_{Pl}^3 / m_f^2$ [8]. The Planck mass $M_{Pl} = \sqrt{c\hbar / k} = 2.177 \times 10^{-5}$ g, m_f is the fermion mass, k is the gravitational constant. For neutron stars (neutron rest mass $m_f = 1.67 \times 10^{-24}$ g) the critical mass $M_{cr\,f} \sim 10^{33}$ g is of the order of the Sun mass.

The static state of boson matter is energetically more preferable than that of fermionic matter. Unlike fermions, all bosons in equilibrium at zero temperature are in the ground state. This ultra-quantum state of matter is called a Bose-Einstein condensate. In equilibrium, concentrations of particles, transforming one into another in chemical reactions, depend on temperature and pressure, and do not depend on reaction channels ([11], §101). If we proceed from the modern Standard Model of "elementary" particles [12], then, in the state of equilibrium, massive Z- and W-bosons, the scalar Higgs boson, as well as bosonic quasiparticles of paired fermions (the Cooper effect [13]), can be dominant. The wave function of a condensate of neutral bosons is the classical scalar field ([14], §30).

Lagrangian L of a complex scalar field ψ: $L = g^{ik}\psi^*_{,i}\psi_{,k} - U\left(\psi^*\psi\right)$. With a large mass of the condensate, in the expansion of the potential

$$U\left(|\psi|^2\right) = (mc/\hbar)|\psi|^2 + (1/2)\lambda|\psi|^4 + \ldots \tag{3}$$

the main first term is the source of gravity, m is the rest mass of a boson. Leaving only the first term in the potential (3), we are dealing with an ideal gas of noninteracting bosons. The second and following terms are corrections that take into account non-gravitational interactions, including elasticity of a condensate. Taking into account only two first terms in expansion (3) with λ = const, we deal with a phenomenological model. This model can be named "$\lambda\psi^4$".

Equilibrium of a gravitating scalar field has been considered in a number of papers in relation to black holes and hypothetical boson stars, see [15]-[19] and references therein.

As in case of fermions, with the restriction $g_{11} = -e^{2F_1}$, equilibrium of a gravitating degenerate Bose gas exists only if the mass M of the condensate is less than the critical mass $M_{crb} \sim M_{Pl}{}^2 / m_b$ [20]. For Standard Model bosons (with the rest mass m_b about 100 GeV/c^2) the critical mass of the condensate is $M_{crb} \sim 10^{12}$ g. It is only about a million ton.

With no restriction $g^{rr} = -e^{-2F_1}$, a static solution to the set of Einstein and Klein-Gordon equations with mass $M > M_{cr}$ exists [21]. In Schwarzschild coordinates, there are two real gravitational radii in this solution. The metric component $g^{rr}(r)$ changes sign twice: at $r = r_g$ inside the condensate and at $r = r_h > r_g$ on its surface. Inside the spherical layer $r_g < r < r_h$ the metric component $g^{rr}(r) > 0$, and the signature of metric tensor is (+,+,–,–).

It follows from Einstein's equation (100.6) in [22] that if the energy density $\varepsilon(r_h) = T_0^0(r_h) = 0$ on the condensate surface, then $g^{rr}(r_h) = 0$ and $dg^{rr}(r_h)/dr = -1/r_h$. One can see from another Einstein's equation (100.4) in [22], that the pressure $p(r_h) = -T_r^r(r_h) = -1/\kappa r_h^2$ does not vanish on the surface of the condensate. Negative pressure means that gravitational forces are directed to compress the gas of bosons, and not to expand.

The sphere $r = r_h$ is the interface of a black hole and dark matter. The observed manifestations of dark matter, such as the rotation curves of galaxies, are adequately described by a longitudinal vector field [23]. A covariant divergence of a longitudinal vector field is a scalar. A Bose condensate wave function is a scalar also. Both satisfy the same Klein-Gordon equation, though the masses of their quanta are extremely different. Condition of regular continuity of pressure at the interface between a black hole and dark matter made it possible to determine the dependence of the plateau velocity (of galaxy rotation curves) on the mass of a black hole. See formula (68) in [21].

If in the potential (3) only the first term of expansion is used (ideal Bose gas with no elasticity), then the wave function of the condensate diverges logarithmically at the center [21]. A regular at the center static solution to the set of Einstein and Klein-Gordon equations with mass $M > M_{cr}$ exists in the model "$\lambda\psi^4$", provided that there is a balance of elasticity and density of the condensate [24]. In the Schwarzschild coordinates, a solution with boundary conditions of regularity at the center exists, but it is unique only in the interval $0 \le r < r_g$. Solutions with boundary conditions on spheres $r = r_g$ and $r = r_h$ where $g^{rr} = 0$ are not unique. This freedom makes it possible to find a solution with any mass $M > M_{cr}$, as well as to ensure the balance of a black hole with dark matter at the boundary.

In the Schwarzschild coordinates in the "$\lambda\psi^4$" model, static states of a black hole are determined by two free parameters. One of them characterizes elasticity of the condensate. It defines uniquely the density of a condensate ε in the center, and the inner gravitational radius r_g. Inside the sphere $r < r_g$, the equation of state of the condensate is

$$p = -\varepsilon/3 \quad (4)$$

while the energy density ε and metric component g^{00} are independent of r. The second free parameter ensures the existence of a regular static solution with arbitrary mass M in the range $M_{cr} < M < \infty$ [25].

2. In synchronous coordinates

A reference frame with $g_{00}=1,\ g_{0\alpha}=0$ is called synchronous. It is shown in [22], §97, that, on the one hand, it is possible to switch to a synchronous frame of reference in any space-time. On the other hand, it is argued that, generally speaking, "space-filling matter cannot be at rest with respect to a synchronous frame of reference." An exception "may occur only in special cases." This statement is based on the fact that in a synchronous frame of reference in statics the component of the Ricci tensor $R_0^0=0$, and the expression on the right side of the Einstein equation

$$R_0^0=\kappa(T_0^0-T/2)=\kappa(\varepsilon+3p)/2$$

"is positive with any distribution of matter". Note that the pressure p is positive when matter tends to expand, and negative when matter tends to contract. From the point of view stated in [22], §97, the state of a condensate, compressed to the ultra relativistic limit (4) by its own gravitational field, should be considered as an "exception in a special case".

It has been known since the time of Eddington [26] and Lemaître [27] that the gravitational radius r_g, on which the component $g^{rr}(r_g)=0$, is not a physical singularity in the Schwarzschild metric. In the problem 4 at the end of §100 in [22], the transformation of the Schwarzschild metric (2) to the conformal Euclidean form is given. The non-uniqueness of the solution to the system of Einstein and Klein-Gordon equations with boundary conditions exactly on the gravitational radii $r=r_g$ and $r=r_h$ where $g^{rr}(r)=0$ [25], is a feature of the Schwarzschild metric. In the conformal Euclidean form, $g^{rr}(r)$ does not vanish.

In a synchronous frame of reference, a static spherically symmetric metric

$$ds^2=\left(dx^0\right)^2-e^{2F_1(r)}dr^2-e^{2F_2(r)}\left(d\theta^2+\sin^2\theta\, d\varphi^2\right) \tag{5}$$

contains two functions $F_1(r)$ and $F_2(r)$, depending on one coordinate r. Unlike the Schwarzschild metric (1), the coordinate r is the true distance from the center. And the length of the central circle $\theta=\pi/2$ at a distance r from the center is $2\pi e^{F_2(r)}$. Radii $r=r_g$ and $r=r_h$ are playing an important role in a synchronous reference system: solutions to Einstein and Klein-Gordon equations with boundary conditions on these radii are not unique. However, the component $g^{11}(r)$ does not vanish now on the spheres $r=r_g$ and $r=r_h$. Therefore, as it is customary for everyone, I am using here exponential representations $g_{11}(r)=-e^{2F_1(r)},\ g_{22}(r)=-e^{2F_2(r)}$ in the metric (5). Substitution

$$dx=e^{F_1(r)}dr,\quad x(r)=\int_{r_0}^{r}e^{F_1(r)}dr,\quad F_2(r)=F_2(x(r)) \tag{6}$$

makes the metric (5) containing only one function $F_2(x)$:

$$ds^2=\left(dx^0\right)^2-dx^2-e^{2F_2(x)}\left(d\theta^2+\sin^2\theta\, d\varphi^2\right). \tag{7}$$

Ricci tensor is diagonal:

$$R_0^0=0,\quad R_1^1=2\left(F_2'^2+F_2''\right),\quad R_2^2=R_3^3=2F_2'^2+F_2''-e^{-2F_2}. \tag{8}$$

Energy is the integral of motion in a time-independent gravitational field. The wave function of the boson condensate in the state with a certain energy E per particle $\psi_E\left(x^0,x\right)=e^{iEx^0/\hbar c}\psi(x)$ satisfies the Klein-Gordon equation $\left(-\det g_{ik}\right)^{-1/2}\left(\sqrt{-\det g_{ik}}\,g^{lm}\psi_{,l}\right)_{;m}=-\left(\partial U/\partial|\psi|^2\right)\psi$. The radial part $\psi(x)$ obeys the equation

$$\psi''+2F_2'\,\psi'=\left[(mc/\hbar)^2+\lambda|\psi|^2-(E/\hbar c)^2\right]\psi. \tag{9}$$

Unlike the Schwarzschild metric, in equation (9) the coefficient at the highest derivative (unity) does not vanish anywhere.

Lagrangian of a scalar field $L = g^{ik}\psi^*_{,i}\psi_{,k} - U(\psi^*\psi)$ does not depend on the derivatives of the metric tensor g_{ik}. The energy-momentum tensor of the condensate is derived easily using the formula $T_{ik} = -g_{ik}L + 2\,\partial L/\partial g_{ik}$:

$$T_0^0 = \left(\frac{E^2 + m^2c^4}{(\hbar c)^2} + \frac{1}{2}\lambda|\psi|^2\right)|\psi|^2 + |\psi'|^2 \quad T_1^1 = \left(-\frac{E^2 - m^2c^4}{(\hbar c)^2} + \frac{1}{2}\lambda|\psi|^2\right)|\psi|^2 - |\psi'|^2 \quad T_2^2 = T_3^3 = \left(-\frac{E^2 - m^2c^4}{(\hbar c)^2} + \frac{1}{2}\lambda|\psi|^2\right)|\psi|^2 + |\psi'|^2.$$

In the synchronous reference system $R_0^0 = 0$ (8). Therefore, it is convenient to work with the Einstein equations in the form

$$R_i^k = \kappa\left(T_i^k - \frac{1}{2}\delta_i^k\,T\right), \quad \kappa = \frac{8\pi}{c^4}k, \quad m^2c^4 \; k = 6{,}67\times 10^{-8}cm^3/g\times\sec^2. \tag{10}$$

It follows from (8) and (10):

$$T_0^0 - \frac{1}{2}T = \left(\frac{2E^2 - m^2c^4}{(\hbar c)^2} - \frac{1}{2}\lambda|\psi|^2\right)|\psi|^2 = 0 \tag{11}$$

In (11) E^2, λ, and m^2c^4 are constants. Therefore, the wave function of the condensate ψ is also a constant:

$$\psi = const, \quad \psi' = 0. \tag{12}$$

Using relations (9), (11), and (12), the energy of a boson *E* in the bound ground state and the balance between elasticity Λ and density $|\psi|^2$ of a condensate are determined:

$$E^2 = \frac{1}{3}m^2c^4, \quad \Lambda|\psi|^2 = -\frac{2}{3}\cdot \tag{13}$$

$\Lambda = (\hbar/mc)^2\lambda$ – parameter characterizing elasticity of a condensate in the model "$\lambda\psi^4$".
Taking into account (13), the energy-momentum tensor of the condensate

$$T_0^0 = \varepsilon = \left(\frac{mc}{\hbar}\right)^2|\psi|^2, \quad T_i^k = -\delta_i^k\,p = \frac{1}{3}\delta_i^k\left(\frac{mc}{\hbar}\right)^2|\psi|^2, \quad i > 0\cdot \tag{14}$$

corresponds to ultrarelativistic equation of state for the matter compressed by its own gravitational field. Einstein equations (10) with Ricci tensor (8) and energy-momentum tensor (14)

$$2\left(F_2'^2 + F_2''\right) = -2\kappa|p|, \tag{15}$$

$$2F_2'^2 + F_2'' - e^{-2F_2} = -2\kappa|p|\,. \tag{16}$$

define the metric function $F_2(x)$. These equations are not independent. Excluding F_2'', and subtracting (16) from (15), we get:

$$F_2'^2 - e^{-2F_2} = -\kappa|p|, \tag{17}$$

$$F_2'' + e^{-2F} = 0\,. \tag{18}$$

Since $\psi = const$ (12), the energy density ε and pressure p (14) are also constants. So, equation (18) is the derivative of equation (17). Multiplied by e^{2F_2}, equation (17) is reduced to

$$de^{F_2}/dx = \sqrt{1 - \kappa|p|\left(e^{F_2}\right)^2}\,. \tag{19}$$

The partial derivative $\partial\sqrt{1-\kappa|p|e^{F_2}}/\partial e^{F_2}$ suffers a discontinuity at $e^{F_2} = (\kappa|p|)^{-1/2}$. According to the existence and uniqueness theorem (see [28], §3) $e^{F_2} = (\kappa|p|)^{-1/2}$ is a solution of Eq. (19). But it is not unique: $e^{F_2} = (\kappa|p|)^{-1/2}\sin\left(\sqrt{\kappa|p|}(x - x_0)\right)$ is also a solution to Eq. (19). x_0 is the integration constant. The metric component $g_{22}(x) = -e^{2F_2(x)}$ (7) has two solutions: a constant $g_{22}(x) = -(\kappa|p|)^{-1}$ independent of *x*, and an oscillating function $g_{22}(x) = -(\kappa|p|)^{-1}\sin^2\left(\sqrt{\kappa|p|}(x - x_0)\right)$. These solutions coincide periodically at $x = x_0 + (\kappa|p|)^{-1/2}\pi(1/2 + n), \quad n = 0,1,2,\ldots$.

In accordance with (6), for the general metric (5), the oscillating solution

$$g_{22}(r) = -(\kappa|p|)^{-1}\sin^2\left[\sqrt{\kappa|p|}\int_{r_0}^{r} e^{F_1(x)}dx\right] \tag{20}$$

contains an arbitrary function $F_1(x)$, r_0 is an integration constant. At $\varepsilon \to 0$ (in a vacuum), solution (20) establishes the relation between g_{11} and g_{22}: $g_{11} = g_{22}'^2 / 4g_{22}$. One of these two functions is arbitrary.

In a simple case $F_1(x) = 0$, a regular solution at the center is $g_{22}(r) = -e^{2F_2(r)} = -(\kappa|p|)^{-1}\sin^2\left(\sqrt{\kappa|p|}\ r\right)$.

It is unique only in the interval $0 \le r < r_g$, $r_g = \pi/2\sqrt{\kappa|p|}$ is the internal "gravitational radius". In the region $r > r_g$, equation (19) with the boundary condition $e^{2F_2(r_g)} = (\kappa|p|)^{-1}$ is satisfied by both solutions. I prefer $g_{22}(r) = -(\kappa|p|)^{-1}$ at $r > r_g$, because there is no reason to choose the oscillating one.

In a synchronous reference frame, this is the same analytical solution as in [23] in Schwarzschild coordinates. It can be verified by putting $g_{22}(r) = -r^2$ in formula (20) and solving this equation with respect to $e^{F_1(x)}$. It will be obtained $g^{11}(x) = -e^{-2F_1(x)} = -1 + (\kappa\varepsilon/3)x^2$ as in formula (41) in [23] (up to notation). Both, in the Schwarzschild metric, and in the synchronous metric, the inner gravitational radius is the boundary of the central region in which the solution is unique and independent of the mass of the entire condensate. In the region $r > r_g$ the solution with the boundary condition $g_{22}(r_g) = (\kappa|p|)^{-1}$ is not unique. This ambiguity makes it possible to choose a solution corresponding to a given mass of a condensate. The difference lies in the fact that in the Schwarzschild coordinates at the branch points r_g and r_h the metric component $g^{11}(r) = 0$, but in the synchronous reference system (7) $g^{11}(r) = -1$, and nowhere does it vanish.

The total mass inside a sphere of radius r is obtained by integrating the energy density $T_0^0(r) = \varepsilon$:

$$M(r) = c^{-2}\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta\, d\theta \int_0^{r} T_0^0(r)|g_{22}(r)|dr .$$

Total mass is $M(r) = \dfrac{3c^2}{4k}\left(r - \dfrac{r_g}{\pi}\sin\dfrac{\pi r}{r_g}\right),\quad r \le r_g$, and $M(r) = \dfrac{3c^2}{2k}\left(r - \dfrac{1}{2}r_g\right),\quad r \ge r_g$.

The energy E of one particle in the condensate (13) is less than the rest energy of the same particle in a vacuum. In the "$\lambda\psi^4$" model, elastic collisions of particles occur without dissipation. This model shows that one third of the total energy is spent creating the bound state for bosons. The second third provides a balance of density and elasticity. And only one third of the original rest mass remains as the source of the gravitational field. If the boundary of the condensate $r_h >> r_g$, then the total mass of a black hole is three times greater than the Schwarzschild mass $M = (c^2/2k)r_h$. This is the composition of gravitational mass defect in the "$\lambda\psi^4$" model.

3. Dark matter

The sphere $r = r_h$ is the boundary separating a black hole and dark matter. Manifestations of dark matter are adequately described using a longitudinal vector field ϕ_i [28]. Lagrangian:

$$L = \left(\phi^m_{;m}\right)^2 - V(\phi^n \phi_n). \tag{21}$$

For ideal gas of gravitating dark matter quanta the potential $V\left(\phi^n \phi_n\right) = V_0' \phi^n \phi_n$, $V_0' = dV(x)/dx$ at $x = 0$.

Dark matter energy-momentum tensor

$$T_i^k = \delta_i^k \left(\left(\phi^m_{;m}\right)^2 - V_0' \phi^m \phi_m\right) + 2V_0' \phi^k \phi_i. \tag{22}$$

Outside a black hole, the right side of Einstein equations (10), in accordance with (22), has the form

$$T_i^k - \frac{1}{2} T \delta_i^k = \begin{cases} -\left(\phi^m_{;m}\right)^2 - 2V_0' \left(\phi^r\right)^2, & i = r, \\ -\left(\phi^m_{;m}\right)^2, & i \neq r. \end{cases} \tag{23}$$

With the Ricci tensor (8) and tensor (23), the Einstein equations outside the black hole reduce to $R_0^0: \;\; \phi^m_{;m} = \left(\phi^r\right)' + 2F_2' \phi^r = 0,$ $R_1^1: \;\; F_2'^2 + F_2'' = -\kappa V_0' \left(\phi^r\right)^2$, $R_2^2: \;\; 2F_2'^2 + F_2'' - e^{-2F_2} = 0$

From equation $R_0^{0:}:$ we obtain $\phi^r(r) = Ce^{-2F_2(r)}$. C is the integration constant. Eliminating F_2'' from $R_1^1:$ and $R_2^2:$, we get the equation $F_2'^2 = e^{-2F_2} + \kappa V_0' C^2 e^{-4F_2}$. Multiplying by e^{4F_2}, we reduce this equation to the form $de^{2F_2}/dr = 2\sqrt{e^{2F_2} + \kappa V_0' C^2}$, determining the metric component $g_{22}(r)$ in the synchronous reference system:

$$g_{22}(r) = -e^{2F_2(r)} = \kappa V_0' C^2 - (r + D)^2, \quad r > r_h. \tag{24}$$

The dark matter Lagrangian (21) is required to have a negative V_0'. Note that $V_0' < 0$ is also a condition of regularity at the center (formula (39) in [28]). Two integration constants $C^2 = -\left(\kappa^2 |p| V_0'\right)^{-1}$ and $D = -r_h$ in (24) provide a smooth transition of the gravitational field through the boundary between a black hole and dark matter. In view of $r_g = \pi/2\sqrt{\kappa|p|}$ it is convenient to express $g_{22}(r)$ in terms of the gravitational radii r_g and r_h:

$$g_{22}(r) = \left\{-\left(2r_g/\pi\right)^2 \sin^2\left(\pi r/2r_g\right),\; r \le r_g; \quad -\left(2r_g/\pi\right)^2,\; r_g \le r \le r_h; \quad -\left(2r_g/\pi\right)^2 - \left(r - r_h\right)^2,\; r > r_h\right\}. \tag{25}$$

Solution (25) for r_g =1 and r_h =5 is demonstrated in Figure 1. Since $g_{11}(r_h) \neq 0$ in the synchronous reference frame, there is no reason to call r_h an event horizon.

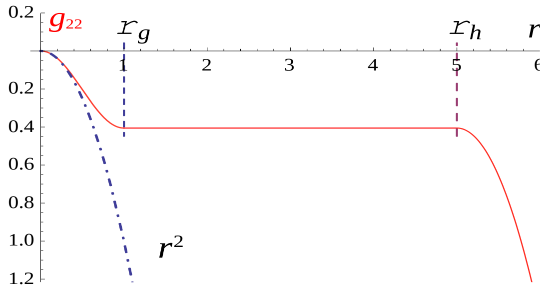

Figure 1. Red line is $g_{22}(r)$ (25). r_g =1, r_h =5.

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