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Proceeding Paper

A Permutation-Based Mathematical Heuristic for Buy-Low-Sell-High †

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Abstract: Buy low sell high is one of the basic rules of thumb used by individuals for investment although it is not considered to be a constructive strategy. In this paper, we show how the appropriate representation of a minute-by-minute trading time series through ordinal (i.e., permutation) patterns and the use of a simple decision heuristic may surprisingly result in significant benefits. We don't compare our proposed approach to sophisticated methods in trading but show how a mathematical model adhering to the idea of bounded rationality may result in significant benefits.

Keywords: interdisciplinary mathematics; cognitive modeling; bounded rationality; permutations

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1. Introduction

The sophistication of current mathematical finance invites the question of whether complex models are necessary for reaching significant gains in financial markets. In this context, it may be interesting to study how simple models may support profitable decision-making even in situations, such as high-frequency trading, imbued with uncertainty. In this paper, we draw on the ideas of bounded rationality (e.g., [1]) and the surprising performance of simple models over complex ones (e.g., [2]) to show that a permutation-based representation of financial time series and the use of a simple version of the buy-low-sell-high strategy surprisingly allow us to successfully perform high-frequency (HF) minute by minute trading.

2. Materials and Methods

We used the minute-by-minute prices of various stocks [3] and represent the stock's prices as a time-series. Given a one-dimensional time series $S(t)$ of length N, we partition the series into overlapping blocks of length D (the embedding dimension) using a time delay τ. The elements in each block or vector are then sorted in ascending order and the vector is mapped into one of $D!$ permutations (i.e., π_i), each representing the ordinal pattern of the elements. For $D = 3$ there are six possible permutations: $\pi_1 = \{0,1,2\}$, $\pi_2 = \{0,2,1\}$, $\pi_3 = \{1,0,2\}$, $\pi_4 = \{1,2,0\}$, $\pi_5 = \{2,0,1\}$, $\pi_6 = \{2,1,0\}$. This results in a *symbolic sequence of permutations*: $\{\pi_s\}$ $s = 1 \ldots n$. The idea of mapping (i.e., representing) a time series of values into a time series of permutations may be highly relevant to prediction in natural environments [4]. For $D = 3$ and $\tau = 1$, *which are the focus of our study*, there are only *three legitimate transitions* for each permutation. What is important to realize is that the constraints imposed on the transition from one permutation to the next significantly reduce the uncertainty associated with the next permutation. Let us represent a time series of prices as a series of permutations length 3. Here we may adopt a buy-low-sell-high (BLSH) strategy: Whenever we observe a decline in the price from t_n to t_{n+1}, we buy N >= 1 units of the stock and aim to sell it in the next step (i.e., t_{n+2}) if an increase is observed. There are three permutations where a decline of a price is observed from t_n to t_{n+1}: $\pi_3 = \{1,0,2\}$, $\pi_5 = \{2,0,1\}$, $\pi_6 = \{2,1,0\}$. In two of these cases, we may have a profit from selling the stock at the third step. In one case (i.e., π_6) a failure is experienced when selling the stock in the third step. In two of the cases—permutations 3 and 5, the BLSH is a successful strategy, whereas in the third case, the one of permutation 6, it is a failure. Therefore, the Minimal Entrance Point (MEP) to a rational BLSH strategy as described above is that:

$$\mathbf{100} * \frac{\mathrm{n}(\pi 3 + \pi 5)}{\mathrm{n}(\pi 3 + \pi 5 + \pi 6)} \geq \mathbf{51} \tag{1}$$

meaning that the relative percent of permutations 3 and 5 in the time series of permutations, is equal to or greater than 51. The MEP is a simple heuristic for selecting stocks for trading and applying the simple BLSH strategy as described above. For example, we have analyzed the time series of 500 stocks (11 September 2017 to 16 February 2018), represented the time series as a series of permutations, and for each stock computed its MEP and the 95% Confidence Interval (CI) for the MEP. The data covers a time range of six months. Sorting the stocks according to the lower bound of their MEP 95% CI, we have identified the top seven stocks and used them for the analysis. According to the idea presented above, selecting stocks according to their MEP should result in a beneficial BLSH strategy. However, the MEP is a necessary but insufficient condition for a successful BLSH strategy. The next phase is to gain a quick estimation of the expected value of applying the BLSH. To compute the expected value, we must somehow estimate the outcome of selling the stock at the third component of each permutation in two scenarios: gain and loss. For simplicity, we assume that in the context of HF minute-by-minute trading, the average delta in price from minute to minute—Δ—is relatively stable and small. Therefore, for permutation 5 (i.e., 2, 0, 1) where a decline of the price is observed and then an increase, we may heuristically assume that the price increase is half the delta in price. Given our ignorance of the size of the decline from rank 2 to rank 0, it is simple to assume under our ignorance that the increase in price from rank 0 to rank 1 is half the size of the delta. In this context, buying a stock at the price of ω and selling at ω + 0.5Δ would leave us with a gain of 0.5Δ. According to the same logic, for permutation 3, the average expected gain is 1.5Δ. In all of the cases where we buy a stock after observing a price decline, the loss expected after observing another decline (permutation 6) is Δ. Following a simple expected value analysis:

$$\boldsymbol{EV} = \sum \boldsymbol{P}(\boldsymbol{Xi}) * \boldsymbol{Xi} \tag{2}$$

We should expect that in adopting a BLSH, where we gain whenever we encounter permutations 3 and 5 and lose whenever we encounter permutation 6, the expected value is positive, meaning that applying the BLSH strategy to the series of permutations under the constraint imposed by the MEP, is a beneficial strategy. In other words, if the MEP is higher enough and the difference in the stock price is small and consistent then applying the BLSH strategy to the stocks selected using the MEP heuristics should be (probabilistically) beneficial. In a HF trading based on the closing price of a minute-by-minute time series, there is a basic condition whereby we may calculate the *optimal proportion of our total bankroll to bet.* According to Kelly Criterion, the optimal proportion of our total bankroll to bet should be:

$$\boldsymbol{f}^* = \frac{\mathbf{bp-q}}{\mathbf{b}} \tag{3}$$

where p is the probability of success, q is the probability of failure and b is the odds or the amount we stand to win to the amount we stand to lose. b can be estimated by simulating the time series, but we may also assume that since in the short run of a one-minute difference, the benefits of winning are the same as those of losing. Given this assumption, we may set b to "1". If the opening price of a stock is $11.23 and p = 63% (rounded) the Kelly score is 50%. However, as Kelly represents the *limit* of a rational bet, a more cautious strategy should be to use a fraction of Kelly such as 0.33 of the f*. The proportional Kelly (0.33) score is 17%. In this case, our optimal betting size would be up to 17% of our total bankroll. This limit is important to the rational agent as in the above context, it provides her with a simple heuristic for deciding whether it is beneficial to enter the BLSH procedure described above. For example, if the price of a single stock is 11.23 USD and Kelly advises you to risk up to 17% of your total bankroll (BR), then your total bankroll or your "pocket size" must be at least:

$$\mathrm{BR} = \frac{\mathbf{100} * \boldsymbol{Price}}{\boldsymbol{f^*}} = \frac{\mathbf{100} * \mathbf{11.23}}{\mathbf{17}} = \mathbf{66\ USD} \tag{4}$$

for trading with a single stock at a time, which mean that unless you have a pocket size of $66, it is better not to enter the game given the above constraints. The overall heuristic presented so far may be generally described as follows: (1) Search for stocks satisfying the MEP criterion. (2) Rank the stocks according to their MEP score and select the top k. (3) For each stock, compute the proportional Kelly. (4) Use the price of a single unit to determine the minimal "pocket size" (i.e., BR). (5) If the minimal pocket size is higher than your actual pocket size, then refrain from playing the game, otherwise apply the BLSH strategy. (6) When applying the BLSH, buy whenever you observe a decline in price and sell immediately afterward. To test the above heuristic, we have analyzed the minute-by-minute stock prices of the seven stocks previously selected. We have used a simple heuristic:

For a given selected stock:

Define a bankroll (BR) which is the sum that you begin with. We use BR = 100 * the initial price of the stock.
Start with the first minute of your time series.
Your focal point, the minute you observe is always defined as n.

If you observe a decrease in price from t_{n-1} to t_n, use the relative Kelly of your BR to buy N stocks.

If you observe an increase from t_n to t_{n+1} THEN sell, update your BR, and restart the game from t_{n+2}. At this point, t_{n+2} is your focal point.

3. Results

See Table 1, where one can see that applying our procedure for minute-by-minute trading results in significant gains.

Table 1. The procedure's gains for the five selected stocks.

Stock	The Initial BR	Gain (USD)	%Gain
NWS	1345	+4478	355
AES	1124	+616	55
NWSA	1323	+391	30
CHK	336	+628	172
F	1140	+468	41
HPE	1325	+386	29
WU	1879	+360	27

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